# DESIGN MODEL OF A VACUUM-ASSISTED HYDRAULIC BRAKING SYSTEM

Ramprasad S. Krishnamachari
Design Laboratory
Department of Mechanical Engineering and Applied Mechanics
The University of Michigan, Ann Arbor
Ann Arbor, MI 48109-2125

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Ramprasad S. Krishnamachari
Graduate Student

Design Laboratory
Department of Mechanical Engineering
and Applied Mechanics
The University of Michigan
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## ABSTRACT

A mathematical model of the design problem of an automotive brake system is developed in this article. A description of the overall system followed by a detailed description of the individual subsystems is presented. The design model of the individual subsystems is then developed. The overall system design model that includes the different subsystem models and the models that describe the performance of the entire system is then presented.

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## INTRODUCTION

A vacuum assisted hydraulic brake system is one of the common brake systems used in cars. Shown in Fig. 1 is a typical vacuum assisted hydraulic brake system for a car. It consists of many subsystems starting from a pedal linkage using which the driver applies the brakes. The pedal linkage is connected to the input of the vacuum booster that assists the driver in applying more effort to stop the vehicle. The booster has a diaphragm that is open to atmosphere on one side, and has vacuum on the other side. The vacuum is achieved using a vacuum pump.

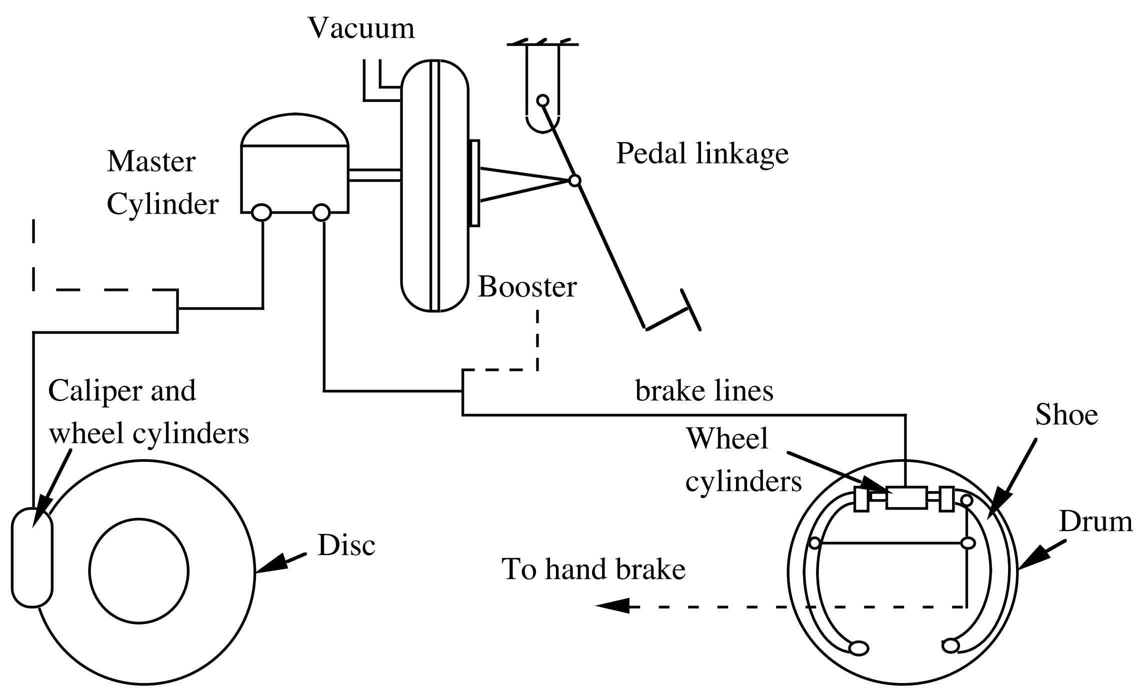


Fig. 1 Vacuum assisted hydraulic brake system

The difference in the pressure on either sides of the diaphragm provides the assisting force. The force from the booster is used to pressurize the fluid in the master cylinder. The pressurized fluid is conveyed to the wheel cylinders of the front and the rear brakes. At the front wheel, the wheel cylinders apply pressure to the brake pads that apply force to the brake disc. At the rear, this force is applied to the drum through the brake shoes. The brake shoes at the rear are also connected to the parking brake mechanism.

The design of the brake system for an automotive involves designing the system such that the vehicle it is installed satisfies the federal braking standards. The present system is designed to satisfy FMVSS - 105 standards (CFR, 1995). For a vehicle to pass through these standards it has to pass through several tests. They can be broadly classified into the following categories: (i) tests that require stopping distances be within specified limits under different road conditions, and from different speeds, (ii) tests that require stopping distances be within limits in the event of a partial failure, (iii) tests that require stopping distances be within limits without power assists devices and using parking brakes, (iv) tests that require stopping distances be with limits with different shoe conditions, (v) the system components should not fail under the application of a specified maximum load, and the system should function at specified maximum and minimum temperatures.

**In addition to the specified standards there are other requirements that a brake system has to satisfy for marketability such as** the **limits on** the **application force used by the driver, feel of the brakes etc. It is also required that the front wheels lock up before the rear wheels to mainta**in the stability of the vehicle during braking.

## SYSTEM DESIGN PROBLEM

The design problem of a vacuum assisted brake system for a typical front wheel driven vehicle (GM responses **to Docket to 85-06**) is presented in this section. The parameters assumed for designing such a system is presented in Table 1. The most important consideration in designing such a system is to choose the proportioning of the braking effort between the front and the rear wheels such that the required deceleration rate is obtained, and the front wheels lock up before the rear ones when the system is fully functional.

Shown in Fig. 2 are the regions in which the front and rear braking forces must lie for the required deceleration levels necessary for stopping the vehicle. The deceleration levels are obtained assuming that the stopping distance is calculated using Newton laws of motion (see also Gillespie, 1992). In the current formulation we design a system with a proportioning of the rear and the front braking force about a value of 0.2. With this it is always the front wheels that would lock up before the rear wheels as can be seen in the Fig. 2. Fluid at the same pressure is supplied to the front and the rear brakes but a different force is applied at the front discs and rear shoes by virtue of different wheel cylinder areas. The same proportioning effort can also be achieved using a proportioning valve that helps in having the front and rear lines carry the brake fluid at different pressures.

Table 1 Vehicle Parameters for Brake Design

| Parameter | Unladen | Laden |
|---|---|---|
| Front weight, Kg | 907.2 | 1088.6 |
| Rear Weight, Kg | 453.6 | 816.5 |
| CG Height, mm | 533.4 | 508 |
| CG to front axle, mm | 889 | 1143 |
| CG to rear axle, mm | 1778 | 1524 |
| Wheelbase, mm | 2667 | 2667 |
| Tire rolling radius, m | 0.29 | 0.29 |

## COMPONENTS IN THE SYSTEM

The brake system components need to be designed such that the functional requirements outlined in the earlier section are satisfied, and none of the components fail before their intended life of operation.  In this section a brief description of the main components in the brake system is presented along with how they function.

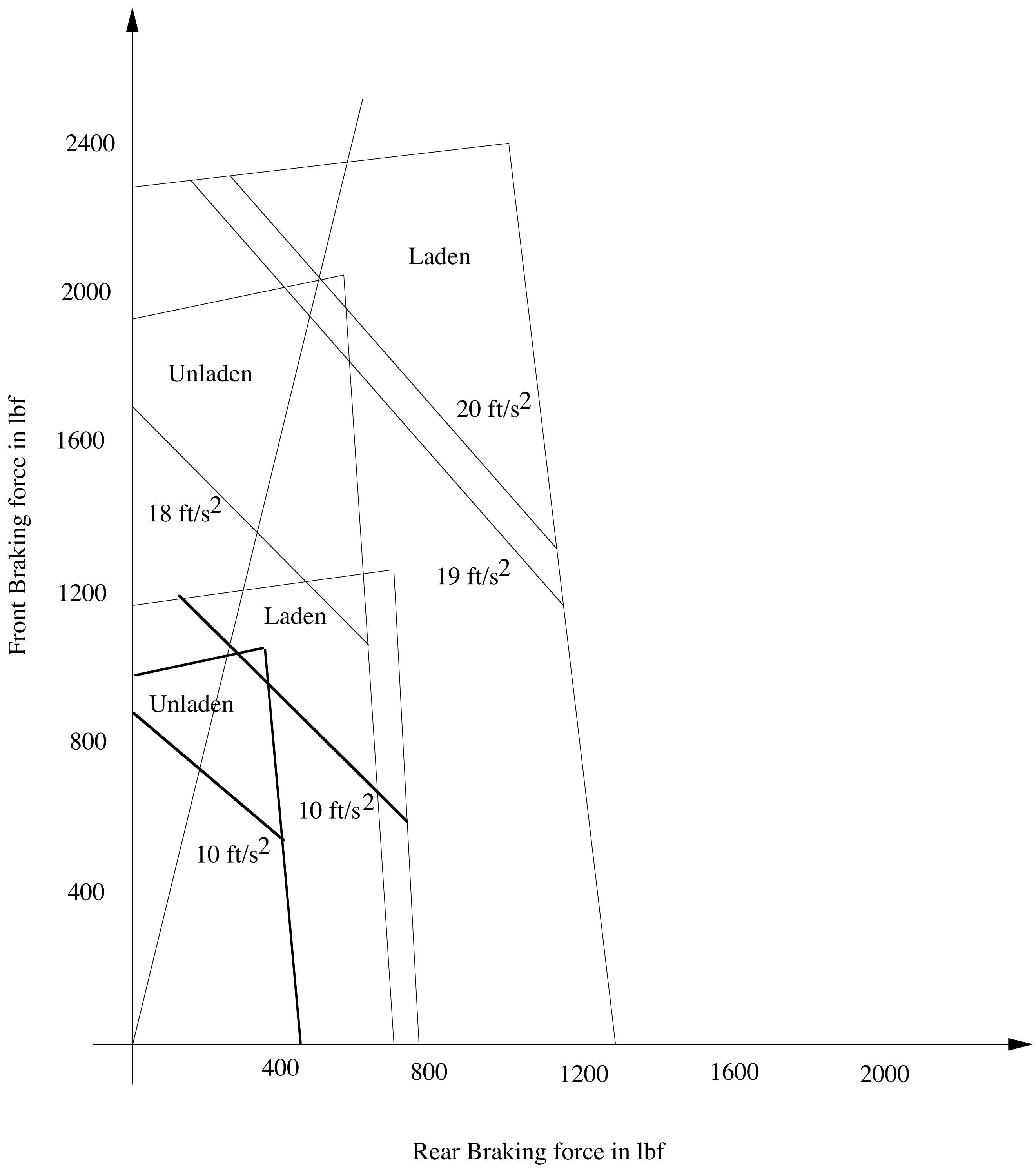

Fig.2  Proportioning of the brake effort between the front and rear brakes

The variables in their design along with the design requirements the components have to satisfy are also outlined. Detailed description of all the design variables, and the design criteria is presented in the subsequent section.

## PEDAL LINKAGE

Shown in Fig. 3 is an example of a typical brake pedal linkage. The pedal linkage consists of two members (members 1 and 2). Member 1 is used in the application of the braking force or is the foot pedal. Member 2 pivoted to member 1 is used to apply the braking force to the vacuum booster.

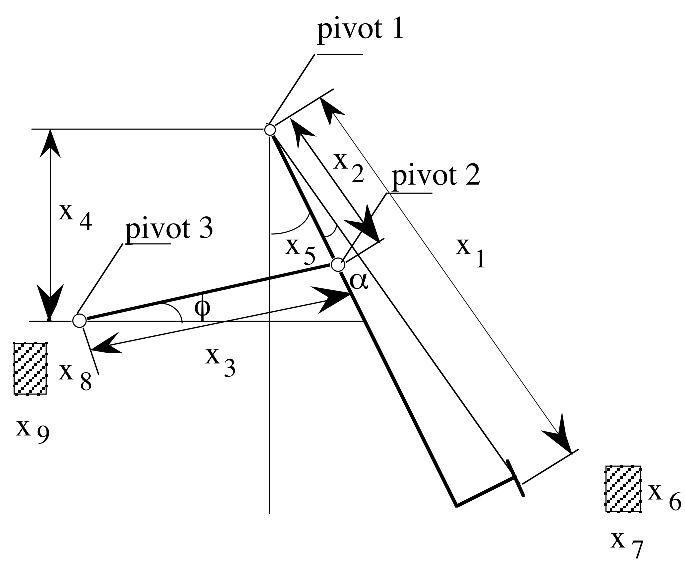


Fig.3  Brake Pedal Linkage

The variables that describe the different geometry's of the linkage such as length, cross-section, resting angles etc. are also shown in Fig. 3. Application of a force at the foot pedal moves member 1 about pivot 1 from its resting angle $x_5$. This movement results in the movement of pivot 3 along a straight line as it coupled to the booster input that is constrained to move along a straight line. The lever ratio arising out of a increased length of $x_1$ compared with $x_2$ results in an increased force applied to the booster input. It is also necessary that dimensions of the linkage be such that the pedal travel at the foot be within specified limits, the pedal does not fail under fail under a maximum load or by fatigue. It is also necessary that member 2 does not buckle under the maximum compressive load. The pivots 1-3 are essentially pins that are expected to withstand shear and bending loads arising from the application of the foot pedal load. The length and diameter of the pins 1-3 are denoted by the variables - {($x_{10}$, $x_{11}$), ($x_{12}$, $x_{13}$), ($x_{14}$, $x_{15}$)} respectively.

## VACUUM BOOSTER

Shown in Fig. 4 (see Gerdes et al. 1993) is a typical line sketch of the vacuum booster. It consists of two chambers - the apply chamber and the vacuum chamber separated by a diaphragm. The external force to the booster is applied through the push rod, and is assisted by the vacuum on the other side of the diaphragm. The vacuum chamber is connected to engine manifold that serves to create the required vacuum.

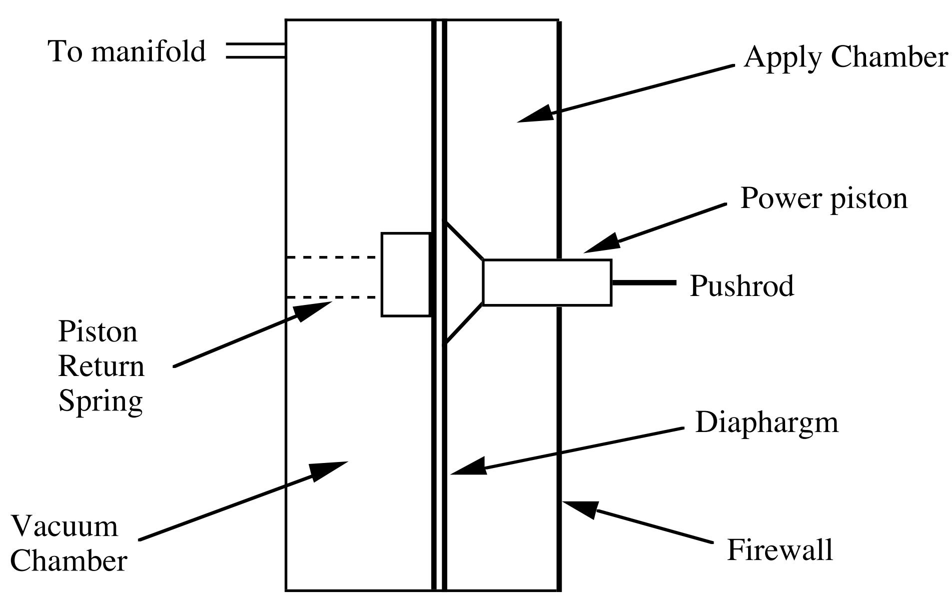

Fig.4 Vacuum Booster [Gerdes and Maciuca et al. (1993)]

The operation of the vacuum booster is illustrated in Figs. 5 (a-c) that represent the three stages of booster operation (see Gerdes et al. 1993). In the initial release stage, Fig. 5 (a), the vacuum and apply chambers are connected with the pushrod and powerpiston resting on their limit stops. Once the force applied to the pushrod is sufficient to overcome the preload on the pushrod return spring, the pushrod begins to move to the left. After some initial displacement, the pushrod seals the vacuum chamber. Further motion of the pushrod causes the control valve to open as in Fig. 5 (b), releasing the atmospheric air into the apply chamber. The resultant force on the diaphragm, after compensating for the piston return spring pre-load, causes the power piston to move to the left.

The master cylinder responds to this displacement with a force on the reaction disc. The disc apportions the reaction force between the pushrod and the power piston according to their respective areas of contact. The reaction disc acts like an incompressible fluid allowing for the driver to get a feel of how much foot pedal effort is applied. The force on the pushrod moves the reaction piston to the right causing the control valve to close. At this point, the hold stage, the booster is in equilibrium with a specific pressure difference maintained between two chambers as in Fig. 5 (c). A decrease in the pushrod force results in the connection of the vacuum and apply chambers and a consequent return to the release stage.

Note the different components of the booster listed in Figs. 4 and 5. The design variables that represent their geometry mainly come from the dimensions that affect the performance of the booster. The booster push rod diameter, the diameter of the diaphragm, maximum effective vacuum, reaction piston diameter, reaction disc diameter, power piston diameter, diameter of the thin section of the push rod correspond to the design variables – $\{x_{39}, \ldots, x_{41}\}$ respectively in the mathematical model to be developed.

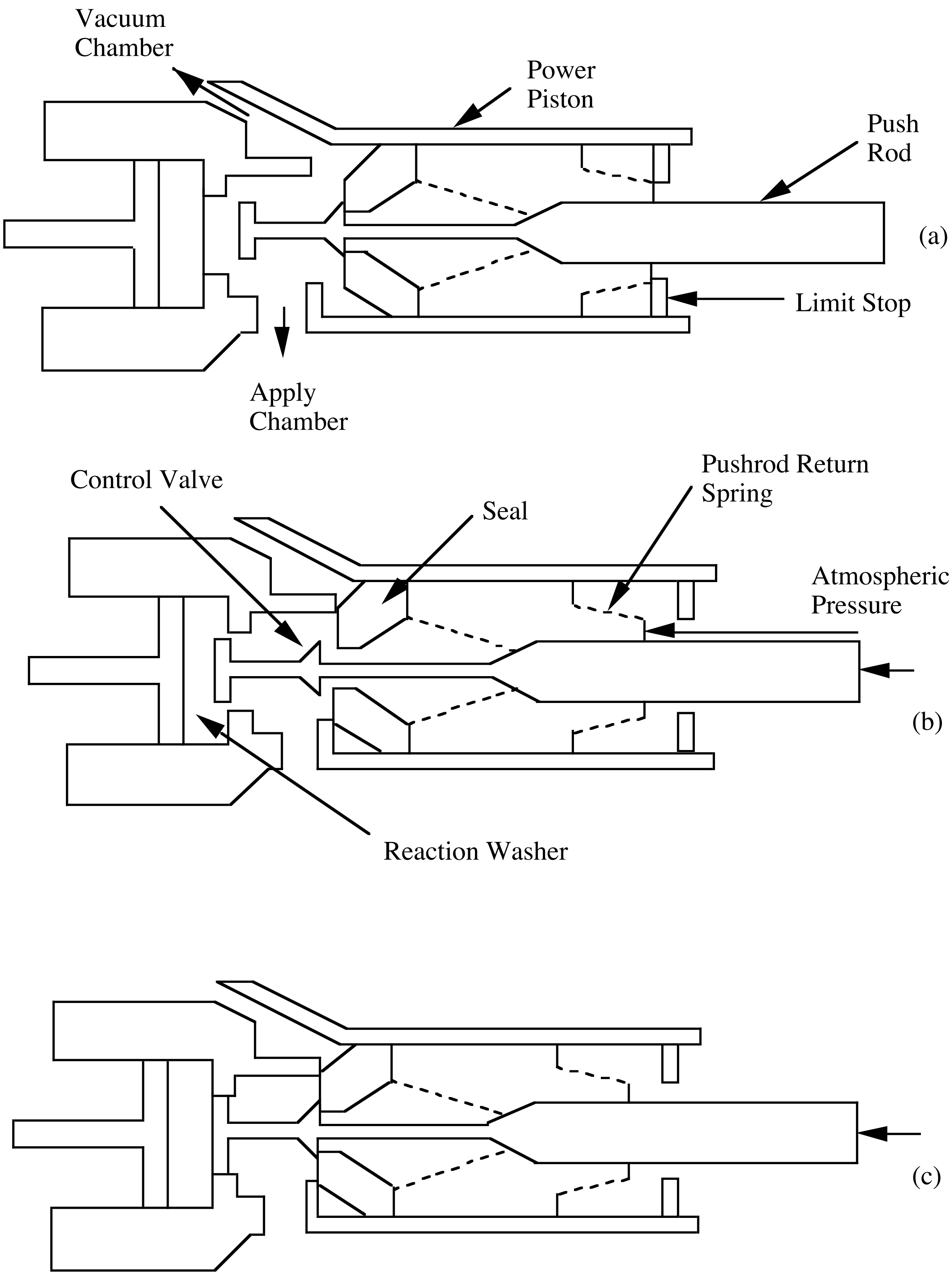


Fig.5 Operation of Vacuum Booster [Gerdes et al. 1993]

## MASTER CYLINDER

Shown in Fig. 6 is a typical design of a master cylinder used in a braking system (see Gerdes et al. 1993). It consists of two chambers that are connected to the front and the rear brake lines. The brake fluid is displaced by motion of the primary piston flows freely into the reservoir through the compensating port. Once piston travel becomes sufficient to close this port, the pressure begins to build up and the brake fluid is forced into the primary brake lines. The building of pressure in the primary port causes pressure to build up in the secondary lines thereby forcing liquid into secondary lines. The primary piston diameter, primary piston rod diameter, secondary piston diameter, secondary piston rod diameter, primary piston rod length, clearing length of the primary piston rod, secondary piston rod length, clearing length of the secondary piston correspond to the variables – $\{x_{45},..., x_{53}\}$ respectively in the mathematical model to be developed.

In the event of a failure e.g., of the rear line, the pressure does not build up in the front line until the secondary piston strikes the wall. Similarly in the event of failure of the front lines the primary piston moves to the left and physically moves the secondary piston to the left. The geometry of the piston rods, and the pistons determine the performance of the master cylinder.

## BRAKES

The pressurized fluid from the master cylinder is used to apply the front and the rear brakes. The front brake subsystem (see Fig. 6) mainly consists of the brake rotors, the wheel cylinders, the brake pads. The rotor is attached to the wheels, and the application of the brakes results in the application of the braking force being applied to the rotors using the brake pads connected to the wheel cylinders. The rear brake subsystem (Fig. 7) consists of the brake drum, the wheel cylinders, the brake shoes, and the parking brake mechanism. The drum is attached to the wheels and the application of the foot pedal force results in the application of the braking force to the drums using the brake shoes connected to the wheel cylinders.

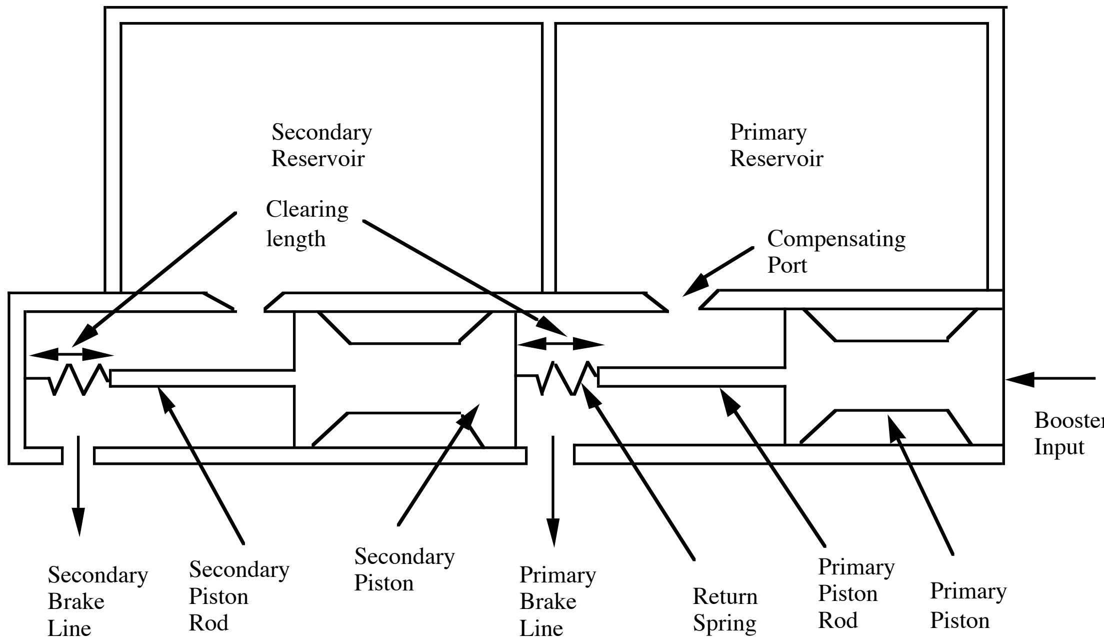


Fig.6 Master Cylinder Construction [Gerdes et al. 1993]

The parking brake mechanism consists mainly of two links that are connected to the brake shoes. The application of the hand actuated parking brake results in the opening of the brake shoes, and braking action obtained on the drums. The wheel cylinders are contained in brake calipers that holds the wheel cylinders in place. The calipers remain stationary and are connected to the main body of the vehicle. The distance of the cross rod connection point from the pivot in the parking brake (see Fig. 8), length of the pull rod, length of the pull rod pivot point from the anchor pin, the width of the pull rod, height of the pull rod, width of the cross rod, height of the cross rod, the outer diameter of the disc, width of the disc hub, thickness of the disc hub, the length of the disc pads, width of disc pads, diameter of the front wheel cylinder, length of the front wheel cylinder piston rod, diameter of the front wheel cylinder piston rod, thickness of the drum, inside width of the drum, inside diameter of the drum, radius of the axis of the brake lining (Fig. 8), diameter of the rear wheel cylinder, length of the rear wheel cylinder piston rod, the diameter of the rear wheel cylinder piston rod correspond to the variables - $\{x_{16},..., x_{38}\}$ respectively in the mathematical model of the design problem to be developed.

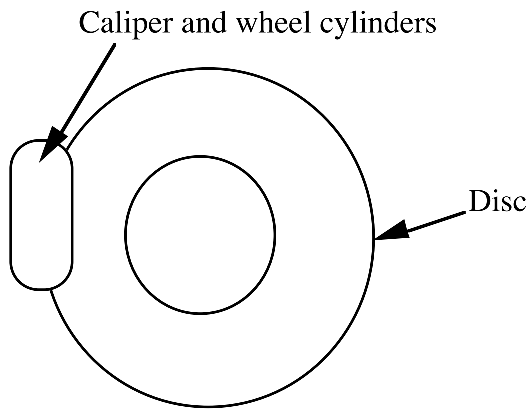


Fig.7  Brake disc assembly

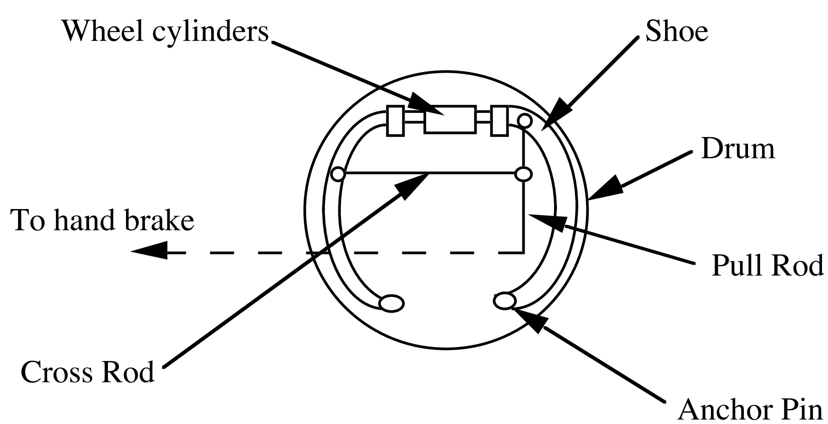


Fig.8  Brake drum assembly

## BRAKE SYSTEM DESIGN PROBLEM - PRELIMINARIES

The design of the brake system involves designing the system such that it can achieve the required performance as required by federal standards as well as satisfy some of the basic marketability criteria. In addition, the system should not fail before the intended life of operation. The design criteria considered in the design of this system are: (i) Foot pedal linkage - pedal travel be within limits, and no failure of the foot pedal takes place either under maximum load or by fatigue, the pins that connect the different parts of the linkage do not fail under maximum load or by fatigue, and the pedal provides for the required pedal ratio, (ii) Booster - the components of the booster do not fail under maximum load or fatigue, the booster is able to supply the required booster ratio, (iii) Master cylinder - the components do not fail under maximum load or by fatigue, the required volume of fluid is available, the system does not fail when there is failure of one of the brake lines, (iv) brake rotors (discs or drums) - do not fail under thermal stress, maximum temperature limits are not exceeded, (v) wheel cylinders - do not fail under maximum load or fatigue, (vi) parking brake mechanism - able to hold the vehicle at the required grade, and provide necessary deceleration when the main system fails, and the components do not fail under maximum load.

The development of the mathematical model of the design problem based on the above requirements first requires the description of the design variables and parameters. Once such a description is obtained the design model consisting of the defining equations and the design criteria can be developed from those. The description of the design variables is given in Table 2, and the design parameters in Table 3.

Table 2 - Description of Design Variables

| |
|---|
| $x_1$ – length of member 1 of the pedal linkage, mm |
| $x_2$ – length between pivot 1 and pivot 2, mm |

$x_3$ – length of member 2, mm

$x_4$ – vertical length between pivot 1 and pivot 3, mm

$x_5$ – resting position of the pedal or the maximum angle, deg

$x_6$ – thickness of member 1, mm

$x_7$ – width of member 1, mm

$x_8$ – thickness of member 2, mm

$x_9$ – width of member 2, mm

$x_{10}$ – length of pin 1, mm

$x_{11}$ – diameter of pin 1, mm

$x_{12}$ – length of pin 2, mm

$x_{13}$ – diameter of pin 2, mm

$x_{14}$ – length of pin 3, mm

$x_{15}$ – diameter of pin 3, mm

$x_{16}$ – distance of the cross rod connection point from pivot in the parking brake, mm

$x_{17}$ – length of the pull rod, mm

$x_{18}$ – length of the pull rod pivot point from the anchor pin, mm

$x_{19}$ – width of the pull rod, mm

$x_{20}$ – height of the pull rod, mm

$x_{21}$ – width of the cross rod, mm

$x_{22}$ – height of the cross rod, mm

$x_{23}$ – outer diameter of the disc, mm

$x_{24}$ – thickness of the disc, mm

$x_{25}$ – width of the disc hub, mm

$x_{26}$ – thickness of the disc hub, mm

$x_{27}$ – length of the disc pad, mm

$x_{28}$ – width of the disc pad, mm

$x_{29}$ – diameter of the front wheel cylinder , mm

$x_{30}$ – length of the front wheel cylinder piston rod, mm

$x_{31}$ – diameter of the front wheel cylinder piston rod, mm

$x_{32}$ – thickness of the drum, mm

$x_{33}$ – inside width of the drum, mm

$x_{34}$ – inside diameter of the drum, mm

$x_{35}$ – radius of the axis of the lining, mm

$x_{36}$ – diameter of the rear wheel cylinder , mm

$x_{37}$ – length of the rear wheel cylinder piston rod, mm

$x_{38}$ – diameter of the rear wheel cylinder piston rod, mm

$x_{39}$ – booster push rod diameter, mm

$x_{40}$ – diameter of the diaphragm, mm

$x_{41}$ – maximum effective vacuum, $N/cm^2$

$x_{42}$ – reaction piston diameter, mm

$x_{43}$ – reaction disc diameter , mm

$x_{44}$ – power piston diameter, mm

$x_{45}$ – diameter of the thin section of the push rod, mm

$x_{46}$ – master cylinder primary piston diameter, mm

$x_{47}$ – master cylinder primary piston rod diameter, mm

$x_{48}$ – master cylinder secondary piston diameter mm

$x_{49}$ – master cylinder secondary piston rod diameter, mm

$x_{50}$ – master cylinder primary piston rod length, mm

$x_{51}$ – clearing length of the primary piston , mm

$x_{52}$ – master cylinder secondary piston rod length, mm

$x_{53}$ – clearing length of the secondary piston, mm

Intermediate variables: Description

$x_{54}$ – braking force on the front wheels at regular foot load (100 lbs), N

$x_{55}$ – braking force on the rear wheels at regular foot load, mm

$x_{56}$ – pedal travel at regular foot load, mm

$x_{57}$ – pedal travel at maximum foot load, mm

$x_{58}$ – pedal travel at front line failure and at regular foot load, mm

$x_{59}$ – pedal travel at rear line failure and at regular foot load, mm

$x_{60}$ – angle made by the foot pedal at maximum foot load (250 lbs), deg

$x_{61}$ – free pedal travel, mm

$x_{62}$ – maximum elastic pedal travel, mm

$x_{63}$ – maximum pedal travel due to thermal expansion, mm

$x_{64}$ – maximum total pedal travel mm

$x_{65}$ – angle made by the foot pedal at regular foot load, deg

$x_{66}$ – angle made by member 2 at regular foot load, deg

$x_{67}$ – input force to the booster at regular foot load, N

$x_{68}$ – pedal ratio at regular foot load, UD

$x_{69}$ – Booster output at regular foot load, N

$x_{70}$ – input force to the booster at maximum foot load, N

$x_{71}$ – Booster output at maximum foot load, N

$x_{72}$ – front wheel cylinder force at regular foot load, N

$x_{73}$ – rear wheel cylinder force at regular foot load, N

$x_{74}$ – front wheel cylinder force at maximum foot load , N

$x_{75}$ – rear wheel cylinder force at maximum foot load, N

$x_{76}$ – pedal ratio at maximum foot load, UD

$x_{77}$ – approximate pedal ratio at any foot load, UD

$x_{78}$ – cross sectional area of the master cylinder, sq. mm

$x_{79}$ – line pressure at regular foot load, $N/mm^2$

$x_{80}$ – line pressure at maximum foot load, $N/mm^2$

$x_{81}$ – brake torque factor front disc, UD

$x_{82}$ – brake torque factor rear drum, UD

$x_{83}$ – front axle braking torque at regular foot load, Nmm

$x_{84}$ – rear axle braking torque at regular foot load, Nmm

$x_{85}$ – effective radius of the disc, mm

$x_{86}$ – parking brake force at the wheels at regular hand brake load, UD

$x_{87}$ – gain of the mechanical portion of the parking brake, UD

$x_{88}$ – maximum bending stress in the pull rod, N/sq.mm

$x_{89}$ – maximum compressive stress in the cross rod, N/sq.mm

$x_{90}$ – critical buckling load (Euler), N

$x_{91}$ – critical buckling load (Johnson), N

$x_{92}$ – position parameter-1 for cross rod, mm

$x_{93}$ – position parameter-2 for cross rod, mm

$x_{94}$ – position parameter-3 for cross rod, mm

$x_{95}$ – volume of the disc, $mm^3$

$x_{96}$ – increase in temperature of the disc by sudden braking, $^oF$

$x_{97}$ – maximum axial load on member 2, N

$x_{98}$ – mass of the disc, kgs

$x_{99}$ – cooling parameter for the disc, UD

$x_{100}$ – surface area of the disc, sq.mm

$x_{101}$ – temperature of the disc by sudden braking, $^oF$

$x_{102}$ – temperature of the disc by repeated braking, $^oF$

$x_{103}$ – temperature of the disc by continued braking, $^oF$

$x_{104}$ – thermal stress in the disc (radial), N/sq.mm

$x_{105}$ – thermal stress in the disc (longitudinal), N/sq.mm

$x_{106}$ – stress in the disc pads, N/sq.mm

$x_{107}$ – stress in the front wheel cylinder push rod, N/sq.mm

$x_{108}$ – critical buckling load on the front wheel cylinder push rod, N/sq.mm

$x_{109}$ – volume of the drum, $mm^3$

$x_{110}$ – increase in temperature of the drum by sudden braking, $^oF$

$x_{111}$ – brake factor rear brakes, UD

$x_{112}$ – mass of the drum, kgs

$x_{113}$ – cooling parameter for the drum, UD

$x_{114}$ – surface area of the drum, sq.mm

$x_{115}$ – temperature of the drum by sudden braking, $^oF$

$x_{116}$ – temperature of the drum by repeated braking, $^oF$

$x_{117}$ – temperature of the drum by continued braking, $^oF$

$x_{118}$ – thermal stress in the drum (radial), N/sq.mm

$x_{119}$ – thermal stress in the drum (longitudinal), N/sq.mm

$x_{120}$ – stress in the rear wheel cylinder push rod, N/sq.mm

$x_{121}$ – critical buckling load on the rear wheel cylinder push rod, N/sq.mm

$x_{122}$ – brake shoe angular parameter-1, deg

$x_{123}$ – brake shoe angular parameter-2, deg

$x_{124}$ – brake shoe angular parameter-3, deg

$x_{125}$ – brake shoe angular parameter-4, deg

$x_{126}$ – moment of inertia of the front wheel cylinder push rod, $mm^4$

$x_{127}$ – moment of inertia of the rear wheel cylinder push rod, $mm^4$

$x_{128}$ – maximum stroke of the vacuum booster, mm

$x_{129}$ – area of the diaphragm, sq.mm

$x_{130}$ – area of the power piston, sq.mm

$x_{131}$ – area of the reaction disc, sq.mm

$x_{132}$ – area of the reaction piston, sq.mm

$x_{133}$ – maximum stress in the thin section of the booster, N/sq.mm

$x_{134}$ – stress in the thin section of the booster at regular foot load, N/sq.mm

$x_{135}$ – alternating stress in the thin section of the booster, N/sq.mm

$x_{136}$ – mean stress in the thin section of the booster, N/sq.mm

$x_{137}$ – buckling load for thin section of the push rod, N

$x_{138}$ – moment of inertia of thin section of the push rod, $mm^4$

$x_{139}$ – area of the primary piston, sq.mm

$x_{140}$ – area of the secondary piston, sq.mm

$x_{141}$ – area of the primary piston rod, sq.mm

$x_{142}$ – area of the secondary piston rod, sq.mm

$x_{143}$ – displacement of master front cylinder piston at regular foot load, mm

$x_{144}$ – displacement of master rear cylinder piston at regular foot load, mm

$x_{145}$ – buckling limiting load on the primary piston rod, N

$x_{146}$ – buckling limiting load on the secondary piston rod, N

$x_{147}$ – stress in the primary piston rod at regular foot load, N/sq.mm

$x_{148}$ – stress in the secondary piston rod at regular foot load, N/sq.mm

$x_{149}$ – alternating stress in the primary piston rod, N/sq.mm

$x_{150}$ – alternating stress in the secondary piston rod, N/sq.mm

$x_{151}$ – mean stress in the primary piston rod, N/sq.mm

$x_{152}$ – mean stress in the secondary piston rod, N/sq.mm

$x_{153}$ – maximum bending stress in member 1, N/sq.mm

$x_{154}$ – maximum axial stress in member 1, N/sq.mm

$x_{155}$ – net maximum normal stress in member 1, N/sq.mm

$x_{156}$ – area of member 1, sq.mm

$x_{157}$ – moment of inertia of member 1, sq.mm

$x_{158}$ – bending stress in member at regular foot load, N/sq.mm

$x_{159}$ – axial stress in member 1 at regular foot load, N/sq.mm

$x_{160}$ – net normal stress in member 1 at regular foot load, N/sq.mm

$x_{161}$ – alternating stress in member 1, N/sq.mm

$x_{162}$ – mean stress in member 1, N/sq.mm

$x_{163}$ – maximum compressive stress in member 2, N/sq.mm

$x_{164}$ – area of member 2, N/sq.mm

$x_{165}$ – buckling limit of member 2, N/sq.mm

$x_{166}$ – moment of inertia of member 2, N/sq.mm

$x_{167}$ – alternating stress in member 2, N/sq.mm

$x_{168}$ – mean stress in member 2, N/sq.mm

$x_{169}$ – maximum shear stress in pin 1, N/sq.mm

$x_{170}$ – shear stress in pin 1 at regular foot load, N/sq.mm

$x_{171}$ – maximum bending stress in pin 1, N/sq.mm

$x_{172}$ – bending stress in pin 1 at regular foot load, N/sq.mm

$x_{173}$ – combined bending and shear in pin 1 at regular foot load, N/sq.mm

$x_{174}$ – alternating combined stress in pin 1, N/sq.mm

$x_{175}$ – mean combined stress in pin 1, N/sq.mm

$x_{176}$ – maximum shear stress in pin 2, N/sq.mm

$x_{177}$ – shear stress in pin 2 at regular foot load, N/sq.mm

$x_{178}$ – maximum bending stress in pin 2, N/sq.mm

$x_{179}$ – bending stress in pin 2, N/sq.mm

$x_{180}$ – combined bending and shear in pin 2 at regular foot load, N/sq.mm

$x_{181}$ – alternating combined stress in pin 2, N/sq.mm

$x_{182}$ – mean combined stress in pin 2, N/sq.mm

$x_{183}$ – maximum shear stress in pin 3, N/sq.mm

$x_{184}$ – shear stress in pin 3 at regular foot load, N/sq.mm

$x_{185}$ – maximum bending stress in pin 3, N/sq.mm

$x_{186}$ – bending stress in pin 3, N/sq.mm

$x_{187}$ – combined bending and shear in pin 3 at regular foot load, N/sq.mm

$x_{188}$ – alternating combined stress in pin 3, N/sq.mm

$x_{189}$ – mean combined stress in pin 3, N/sq.mm

$x_{190}$ – compressive stress in the front wheel cylinder push rod at regular foot load, N/sq.mm

$x_{191}$ – alternating stress in the front wheel cylinder push rod at regular foot load, N/sq.mm

$x_{192}$ – mean stress in the front wheel cylinder push rod at regular foot load, N/sq.mm

$x_{193}$ – compressive stress in the rear wheel cylinder push rod at regular foot load, N/sq.mm

$x_{194}$ – alternating stress in the rear wheel cylinder push rod at regular foot load, N/sq.mm

$x_{195}$ – mean stress in the rear wheel cylinder push rod at regular foot load, N/sq.mm

$x_{196}$ – geometry parameter - disc brake, UD

$x_{197}$ – geometry parameter - drum brake, UD

$x_{198}$ – booster diaphragm force at particular vacuum level, N

$x_{199}$ –booster ratio, UD

$x_{200}$ – regular elastic pedal travel, mm

$x_{201}$ – regular thermal pedal travel, mm

$x_{202}$ – area of the front wheel cylinder, sq.mm

$x_{203}$ – area of the rear wheel cylinder, sq. mm

$x_{204}$ – radial expansion of the drum, mm

$x_{205}$ – angle made by member 2 at maximum foot load, N

$x_{206}$ – distance between the anchor pin and wheel cylinder, mm

$x_{207}$ – distance between the anchor pin and centerline, mm

$x_{208}$ – moment of inertia of the cross section of the pull rod, $mm^4$

$x_{209}$ – maximum bending moment in the pull rod, Nmm

$x_{210}$ – length of the cross rod, mm

$x_{211}$ – radius of gyration of the cross rod, $mm^2$

$x_{212}$ – moment of inertia of the front wheel cylinder push rod, $mm^4$

$x_{213}$ – moment of inertia of the rear wheel cylinder push rod, $mm^4$

$x_{214}$ – moment of inertia of the master cylinder primary piston rod, $mm^4$

$x_{215}$ – moment of inertia of the master cylinder secondary piston rod, $mm^4$

$x_{216}$ – maximum normal stress in the primary piston rod, $N/mm^2$

$x_{217}$ – maximum normal stress in the secondary piston rod, $N/mm^2$

$x_{218}$ – area of pin 1, $mm^2$

$x_{219}$ – moment of inertia of pin 1, $mm^4$

$x_{220}$ – area of pin 2, $mm^2$

| |
|---|
| $x_{221}$ – moment of inertia of pin 2, $mm^4$ |
| $x_{222}$ – area of pin 3, $mm^2$ |
| $x_{223}$ – moment of inertia of pin 3, $mm^4$ |
| $x_{224}$ – displacement of master front cylinder piston at maximum foot load, mm |
| $x_{225}$ – displacement of master rear cylinder piston at maximum foot load, mm |
| $x_{226}$ – elastic pedal travel at the disc at regular load, mm |
| $x_{227}$ – elastic pedal travel at the drum at regular load, mm |
| $x_{228}$ – elastic pedal travel at the disc at maximum load, mm |
| $x_{229}$ – elastic pedal travel at the drum at maximum load, mm |
| $x_{230}$ – thermal pedal travel at the disc, mm |
| $x_{231}$ – thermal pedal travel at the drum, mm |
| $x_{232}$ – liquid consumed in the front lines at regular load, $mm^3$ |
| $x_{233}$ – liquid consumed in the front lines at maximum load, $mm^3$ |
| $x_{234}$ – liquid consumed in the rear lines at regular load |
| $x_{235}$ – liquid consumed in the rear lines at maximum load |

Table 3 - Description of design parameters

| Design Parameters: Description |
|---|
| $p_1$ – Required force at front wheels at regular foot load, N |
| $p_2$ – Required force at rear wheels at regular foot load, N |
| $p_3$ – angle at the foot pedal, degree |

$p_4$ – pedal travel clearance in linkage, losses in master cylinder, and seal expansion, mm

$p_5$ – lining clearance - disc, mm

$p_6$ – lining clearance - drum, mm

$p_7$ – coefficient of linear expansion of the disc, units

$p_8$ – fluid volume lost by trapped air and other losses per 1000 psi, cu.mm

$p_9$ – fluid volume lost by flexible hose expansion per 1000 psi, cu.mm

$p_{10}$ – disc caliper spread per 1000 lbs, mm

$p_{11}$ –disc caliper spread per 1000 lbs, mm

$p_{12}$ – disc brake lining pad compression per 100 psi, mm

$p_{13}$ –drum brake lining pad compression per 100 psi, mm

$p_{14}$ – frictional coefficient between the pad and the disc, UD

$p_{15}$ – Average hand braking force, N

$p_{16}$ – parking brake spring factor, UD

$p_{17}$ – mechanical efficiency of the parking brake from hand force to cable force, UD

$p_{18}$ – efficiency of the actuating mechanism inside the brake, UD

$p_{19}$ – axle radius, mm

$p_{20}$ – central diameter of the disc, mm

$p_{21}$ – coefficient of rolling friction, UD

$p_{22}$ – weight of the car, N

$p_{23}$ – density of air, $Kg/m^3$

$p_{24}$ – $\gamma_b$ lumped factor for inertia, UD

$p_{25}$ – frontal area of the car, sq. mm

$p_{26}$ – assumed initial velocity for sudden stop, m/s

$p_{27}$ – assumed deceleration value for sudden stop, m/sec$^2$

$p_{28}$ – density of the disc material, Kg/mm$^3$

$p_{29}$ – number of brake applications, UD

$p_{30}$ – convective heat transfer coefficient, Nm/hKm$^2$

$p_{31}$ – intermediate cooling time, h

$p_{32}$ – grade for parking, deg

$p_{33}$ – grade assumed to test constant braking, deg

$p_{34}$ – distance for constant braking, m

$p_{35}$ – velocity for constant braking, m/s

$p_{36}$ – Young's Modulus for the disc material, N/m$^2$

$p_{37}$ – Poison's ratio for the disc, UD

$p_{38}$ – Young's Modulus for the front wheel cylinder push rod material, N/m$^2$

$p_{39}$ – initial temperature of the disc, $^o$F

$p_{40}$ – density of the drum material, Kg/mm$^3$

$p_{41}$ – convective heat transfer coefficient, Nm/hKm$^2$

$p_{42}$ – Young's Modulus for the drum material, N/m$^2$

$p_{43}$ – Poison's ratio for the drum, UD

$p_{44}$ – Young's Modulus for the rear wheel cylinder push rod material, N/m$^2$

$p_{45}$ – initial temperature of the drum, $^o$F

$p_{46}$ – coefficient of friction between the drum and shoes, UD

$p_{47}$ – stroke of the foot pedal at the maximum foot load, mm

$p_{48}$ – Young's modulus of the push rod material, $N/m^2$

$p_{49}$ – Young's modulus of the master cylinder piston rod material, $N/m^2$

$p_{50}$ – Young's modulus of member 2, $N/m^2$

$p_{51}$ – regular pedal travel, mm

$p_{52}$ – pedal travel at partial failure, mm

$p_{53}$ – minimum deceleration level at partial failure, $m/sec^2$

$p_{54}$ – minimum deceleration level with parking brake, $m/sec^2$

$p_{55}$ – master cylinder primary piston rod diameter, mm

$p_{56}$ – volume limit on the disc, $mm^3$

$p_{57}$ – volume limit on the drum, $mm^3$

$p_{58}$ – Temperature limits for sudden braking, $^oF$

$p_{59}$ – Temperature limits for repeated braking, $^oF$

$p_{60}$ – Temperature limits for continued braking, $^oF$

$p_{61}$ – maximum stroke of the foot pedal till it becomes vertical, mm

$p_{62}$ – volume limit on the booster, $mm^3$

$p_{63}$ – minimum effective vacuum limit in booster, $N/cm^2$

$p_{64}$ – maximum effective vacuum limit in booster, $N/cm^2$

$p_{65}$ – volume limit on the liquid in the front master cylinder, $mm^3$

$p_{66}$ – volume limit on the liquid in the rear master cylinder, $mm^3$

$p_{67}$ – maximum line pressure, $N/m^2$

$p_{68}$ – safety factor for parking force, UD

$p_{69}$ – yield limit on for pull rod in parking brake, N/m$^2$

$p_{70}$ – safety factor for pull rod yielding, UD

$p_{71}$ – yield limit on for cross rod in parking brake, N/m$^2$

$p_{72}$ – safety factor for cross rod yielding, UD

$p_{73}$ – yield limit on disc material, N/m$^2$

$p_{74}$ – safety factor for disc yielding, UD

$p_{75}$ – yield limit on wheel cylinder piston rod material, N/m$^2$

$p_{76}$ – safety factor for wheel cylinder piston rod yielding, UD

$p_{77}$ – yield limit on drum material, N/m$^2$

$p_{78}$ – safety factor for drum yielding, UD

$p_{79}$ – $k_a$ for wheel cylinder piston, UD

$p_{80}$ – $k_b$ for wheel cylinder piston, UD

$p_{81}$ – $k_c$ for wheel cylinder piston, UD

$p_{82}$ – $k_d$ for wheel cylinder piston, UD

$p_{83}$ – $k_e$ for wheel cylinder piston, UD

$p_{84}$ – Se' for wheel cylinder piston, N/m$^2$

$p_{85}$ – Se for wheel cylinder piston, N/m$^2$

$p_{85}$ – $S_{ut}$ for wheel cylinder piston, N/m$^2$

$p_{86}$ – $k_a$ for booster push rod, UD

$p_{87}$ – $k_b$ for booster push rod, UD

$p_{88}$ – $k_c$ for booster push rod, UD

$p_{89}$ – $k_d$ for booster push rod, UD

$p_{90}$ – $k_e$ for booster push rod, UD

$p_{91}$ – Se' for booster push rod, N/m$^2$

$p_{92}$ – Se for booster push rod, N/m$^2$

$p_{93}$ – $S_{ut}$ for booster push rod, N/m$^2$

$p_{94}$ – $k_a$ for master cylinder piston rod, UD

$p_{95}$ – $k_b$ for master cylinder piston rod, UD

$p_{96}$ – $k_c$ for master cylinder piston rod, UD

$p_{97}$ – $k_d$ for master cylinder piston rod, UD

$p_{98}$ – $k_e$ for master cylinder piston rod, UD

$p_{99}$ – Se' for master cylinder piston rod, N/m$^2$

$p_{100}$ – Se for master cylinder piston rod, N/m$^2$

$p_{101}$ – $S_{ut}$ for master cylinder piston rod, N/m$^2$

$p_{102}$ – $k_a$ for member 1, UD

$p_{103}$ – $k_b$ for member 1, UD

$p_{104}$ – $k_c$ for member 1, UD

$p_{105}$ – $k_d$ for member 1, UD

$p_{106}$ – $k_e$ for member 1, UD

$p_{107}$ – Se' for member 1, N/m$^2$

$p_{108}$ – Se for member 1, N/m$^2$

$p_{109}$ – $S_{ut}$ for member 1, N/m$^2$

$p_{110}$ – $k_a$ for member 2, UD

$p_{111}$ – $k_b$ for member 2, UD

$p_{112}$ – $k_c$ for member 2, UD

$p_{113}$ – $k_d$ for member 2, UD

$p_{114}$– $k_e$ for member 2, UD

$p_{115}$ – Se' for member 2, N/m$^2$

$p_{116}$ – Se for member 2, N/m$^2$

$p_{117}$ – $S_{ut}$ for member 2, N/m$^2$

$p_{118}$ – $k_a$ for pin 1, UD

$p_{119}$ – $k_b$ for pin 1, UD

$p_{120}$ – $k_c$ for pin 1, UD

$p_{121}$ – $k_d$ for pin 1, UD

$p_{122}$ – $k_e$ for pin 1, UD

$p_{123}$ – Se' for pin 1, N/m$^2$

$p_{124}$ – Se for pin 1, N/m$^2$

$p_{125}$ – $S_{ut}$ for pin 1, N/m$^2$

$p_{126}$ – $k_a$ for pin 2, UD

$p_{127}$ – $k_b$ for pin 2, UD

$p_{128}$ – $k_c$ for pin 2, UD

$p_{129}$ – $k_d$ for pin 2, UD

$p_{130}$ – $k_e$ for pin 2, UD

$p_{131}$ – Se' for pin 2, N/m$^2$

$p_{132}$ – Se for pin 2, N/m$^2$

$p_{133}$ – $S_{ut}$ for pin 2, N/m$^2$

$p_{134}$ – $k_a$ for pin 3, UD

$p_{135}$ – $k_b$ for pin 3, UD

$p_{136}$ – $k_c$ for pin 3, UD

$p_{137}$ – $k_d$ for pin 3, UD

$p_{138}$ – $k_e$ for pin 3, UD

$p_{139}$ – Se' for pin 3, N/m$^2$

$p_{140}$ – Se for pin 3, N/m$^2$

$p_{141}$ – $S_{ut}$ for pin 3, N/m$^2$

$p_{142}$ – Tire radius, mm

$p_{143}$ – $k_p$, UD

$p_{144}$ – displacement gain between hand force and cable force, UD

$p_{145}$ – Young's Modulus of the cross rod, N/m$^2$

$p_{146}$ – buckling parameter, UD

$p_{147}$ – specific heat of the disc material, Units

$p_{148}$ – fraction of the braking on the front brakes, UD

$p_{149}$ – ambient temperature, $^o$F

$p_{150}$ - central diameter of the drum, mm

$p_{151}$ – specific heat of the drum material, Units

$p_{152}$ – coefficient of linear expansion of the drum, units

$p_{153}$ - length of thin section of the push rod, mm

$p_{154}$ - time taken for sudden stop from a fixed speed at a fixed deceleration, secs

$p_{155}$ - mechanical efficiency of the booster linkages, UD

$p_{156}$ - spring factor required to for moving push rod return spring, UD

$p_{157}$ - yield limit of the disc pads, $N/mm^2$

$p_{158}$ - safety factor on disc pads, UD

$p_{159}$ - yield limit on booster push rod material, $N/mm^2$

$p_{160}$ - safety factor on booster push rod material design, UD

$p_{161}$ - safety factor on booster push rod buckling, UD

$p_{162}$ - yield limit on primary master cylinder piston rod material, $N/mm^2$

$p_{163}$ - safety factor on primary master cylinder piston rod material design, UD

$p_{164}$ - safety factor on primary master cylinder piston rod buckling, UD

$p_{165}$ - yield limit on secondary master cylinder piston rod material, $N/mm^2$

$p_{166}$ - safety factor on secondary master cylinder piston rod material design, UD

$p_{167}$ - safety factor on secondary master cylinder piston rod buckling, UD

$p_{168}$ - yield limit on member 1 material, $N/mm^2$

$p_{169}$ - safety factor on member 1 material design, UD

$p_{170}$ - yield limit on member 2 material, $N/mm^2$

$p_{171}$ - safety factor on member 2 material design, UD

$p_{172}$ - safety factor on member 2 buckling, UD

$p_{173}$ - yield limit on pin 1 material, $N/mm^2$

$p_{174}$ - safety factor on pin 1 material design, UD

$p_{175}$ - yield limit on pin 2 material, $N/mm^2$

$p_{176}$ - safety factor on pin 2 material design, UD

$p_{177}$ - yield limit on pin 3 material, $N/mm^2$

$p_{178}$ - safety factor on pin 3 material design, UD

$p_{179}$ - axial expansion of the disc, mm

$p_{180}$ - average energy spent in a sudden stop per unit time, J/s

$p_{181}$ - braking time keeping constant velocity of 20 mph for 8 km.

$p_{182}$ - 250 lbs load, lbs

$p_{183}$ - 100 lbs load, lbs

$p_{184}$ - minimum booster ratio, UD

$p_{185}$ - maximum booster ratio, UD

$p_{186}$ - minimum angle of the foot pedal, deg

$p_{187}$ - maximum angle of the foot pedal, deg

$p_{188}$ - maximum pedal travel, mm

## MATHEMATICAL MODEL

In this section we first develop the mathematical model of the defining quantities. The design criteria are then stated using the defined quantities. The general design problem essentially consists of these defining equations and the associated design criteria. The defining equations and the design criteria have been obtained based on design requirements, and using the information presented in Limpert (1992), Puhn (1985), Gerdes et al. (1993). Note that several assumptions are made in developing this model. It is beyond the scope of this report to describe the derivation of the different defining equations and criteria. Most of them can be easily be obtained based on their definition and common engineering principles.

**Defining Equations**

$x_{54} = x_{83}/p_{142}$

$x_{55} = x_{84}/p_{142}$

$x_{56} = x_{61} + x_{200} + x_{201}$

$x_{57} = x_{61} + x_{62} + x_{63}$ (at normal temperatures)

$x_{58} = x_{56} + x_{51}\ x_{68}$

$x_{59} = x_{56} + x_{53}\ x_{68}$

$x_{60} = (p_{143}\ (x_5) - p_{181})/p_{143}$

$x_{61} = p_4 + 2(x_{196}\ p_5\ x_{202} + x_{197}\ p_6\ x_{203})\{x_{77}/x_{78}\}$

$x_{62} = \{p_8 + p_9 + 2(x_{202}{}^2 p_{10} + x_{203}{}^2 p_{11}) + 20\ (p_{12}x_{202} + p_{13}x_{203})\}[(145\ x_{80}\ x_{76})/\ (1000\ x_{78})] +$
$\{p_8 + p_9\}[(145\ x_{80}\ x_{76})/\ (1000\ x_{78})]$

$x_{63} = 2(x_{196}\ p_{179}\ x_{202} + x_{197}\ x_{204}\ x_{203})\{x_{77}/x_{78}\}$

$x_{64} = x_{61} + x_{62} + x_{63}$

$x_{65} = (p_{143}\ (x_5) - p_{182})/p_{143}$

$x_{66} = \sin^{-1}\{(x_4 - x_2 \cos (x_{65})]/x_3\}$

$x_{67} = x_{68}\ p_{182} \cos (x_{66})$

$x_{68} = (x_1 \cos (p_3) \cos (x_{66}))/(x_2 \cos (x_{65} - x_{66}))$

$x_{69} = x_{199}\ x_{67}$

$x_{70} = x_{76}\ p_{181} \cos (x_{205})$

$x_{71} = x_{199}\ x_{70}$

$x_{72} = x_{79}\ \ x_{202}$

$x_{73} = x_{79}\ \ x_{203}$

$x_{74} = x_{80}\ \ x_{202}$

$x_{75} = x_{80}\ \ x_{203}$

$x_{76} = (x_1 \cos(p_3) \cos(x_{205}))/(x_2 \cos(x_{60} - x_{205}))$

$x_{77} = x_1/x_2$

$x_{78} = x_{139}$

$x_{79} = x_{69}/x_{78}$

$x_{80} = x_{71}/x_{78}$

$x_{81} = p_{14}\ x_{85}$

$x_{82} = x_{111}\ (x_{34})$

$x_{83} = 2\ x_{81}\ x_{72}$

$x_{84} = 2\ x_{82}\ x_{73}$

$x_{85} = (1/3)[x_{23}^2 + p_{20}\ x_{23} + x_{23}^2]/(x_{23} + p_{20})$

$x_{86} = 2\ x_{82}\ (p_{15}\ p_{16}\ p_{17}\ p_{18}\ p_{144})\ (x_{87})\ [(0.5\ x_{34})/p_{142}]$

$x_{87} = 0.5\{(x_{17}/(x_{16}\ x_{206}))(x_{18} - x_{16}) + (x_{18}/(x_{16}x_{206})(x_{17} - x_{16})]$

$x_{88} = [(x_{209})\ (x_{20}/2)]/\ (x_{208})$

$x_{89} = (p_{15}p_{17}p_{144})(x_{17}/x_{16})/(x_{21}\ x_{22})$

$x_{90} = (\pi^2)p_{145}[(1/12)\ x_{21}\ x_{22}^3\ ]/(x_{210}^2)$

$x_{91} = x_{21}\ x_{22}\ \{[\ p_{71} - \ (x_{210}\ p_{71}/(2\pi x_{211}))^2]\ /\ (p_{146}p_{145})\}$

$x_{92} = 0.35\ x_{35}$

$x_{93} = 0.5\ x_{35}$

$x_{94} = 0.9\ x_{35}$

$x_{95} = (\pi/4)[(p_{20} + 2\ x_{26})^2 - p_{20}^2)x_{25} + (x_{23}^2 - p_{20}^2)x_{24}]$

$x_{96} = [(p_{180}\ p_{154}\ \ /\ (x_{98}\ p_{147}))(0.5\ p_{148})$

$x_{97} = x_{76}\ p_{181}$

$x_{98} = x_{95}\ p_{28}$

$x_{99} = (p_{41}\ x_{100}\ p_{31}) / (p_{147}\ p_{28}\ x_{95})$

$x_{100} = [\pi\ (p_{20} + 2\ x_{26})\ x_{25} + \pi/4\ (x_{23}^2 - p_{20}^2)2.0]$

$x_{101} = x_{96} + p_{39}$

$x_{102} = p_{39} + \{[\ 1 - \exp(-\ p_{29}\ x_{99})](x_{96})\}/[1 - \exp(-\ x_{99})]$

$x_{103} = [p_{39} - p_{149} - (p_{180}/p_{41}\ x_{100})] \exp(-(p_{41}\ x_{100}\ p_{31})/\ (p_{147}\ x_{98})) + p_{149}$

$+ (p_{180}/p_{41}\ x_{100})$

$x_{104} = (p_{36}\ /(1 - p_{37}))\ p_7\ x_{96}$

$x_{105} = (p_{36}\ /(1 - p_{37}))\ p_7\ x_{96}$

$x_{106} = x_{74} / (x_{27}\ x_{28})$

$x_{107} = x_{74} / [(\pi/4)\ (x_{31})^2]$

$x_{108} = [\pi^2\ p_{38}\ x_{212}]/(x_{30}^2)$

$x_{109} = (\pi/4)[(x_{34} + 2\ x_{32})^2)x_{33} + ((x_{34} + 2\ x_{32})^2 - p_{150}^2)x_{32}]$

$x_{110} = [(p_{180}\ p_{154} / (x_{112}\ p_{151}))(0.5\ (1 - p_{148}))$

$x_{111} = (p_{46}\ x_{206} / (0.5\ x_{34}))/\{(x_{35}/(0.5\ x_{34}))[(0.0174\ x_{122} - \sin(x_{122}) \cos(x_{125}))/(4 \sin(x_{122}/2) \sin(x_{125}/2))] + p_{46}(1 + (x_{35}/(0.5\ x_{34})) \cos(x_{122}/2) \cos(x_{125}/2))\}$

$+ (p_{46}\ x_{206} / (0.5\ x_{34}))/\{(x_{35}/(0.5\ x_{34}))[(0.0174\ x_{122} - \sin(x_{122}) \cos(x_{125}))/(4 \sin(x_{122}/2) \sin(x_{125}/2))] - p_{46}(1 + (x_{35}/(0.5\ x_{34})) \cos(x_{122}/2) \cos(x_{125}/2))\}$

$x_{112} = p_{40}\ x_{109}$

$x_{113} = (p_{41}\ x_{114}\ p_{31}) / (p_{151}\ p_{40}\ x_{109})$

$x_{114} = [(\pi)(x_{34} + 2\ x_{32})(x_{33} + x_{32})+ (\pi/4)\ ((x_{34} + 2\ x_{32})^2 - p_{150}^2)]$

$x_{115} = x_{110} + p_{45}$

$x_{116} = p_{45} + \{[ 1 - \exp(- p_{29} x_{113})](x_{110})\}/[1 - \exp(- x_{113})]$

$x_{117} = [p_{45} - p_{149} - (p_{180}/p_{41} x_{114})] \exp(-(p_{41} x_{114} p_{181})/ (p_{151} p_{40} x_{109}))$

$+ p_{149} + (p_{180}/p_{41} x_{114})$

$x_{118} = (p_{42} /(1 - p_{43})) p_{152} x_{110}$

$x_{119} = (p_{42} /(1 - p_{43})) p_{152} x_{110}$

$x_{120} = x_{75} / [(\pi/4) (x_{38})^2]$

$x_{121} = [\pi^2 p_{38} x_{213}]/(x_{37}^2)$

$x_{122} = 2 \tan^{-1}[(x_{206})/(2 x_{35})]$

$x_{123} = (180 - x_{122})/2 + \cos^{-1}[(x_{206})/(2 x_{35})]$

$x_{124} = x_{122} + x_{123}$

$x_{125} = x_{124} + x_{123}$

$x_{126} = (\pi/64) (x_{31}^4)$

$x_{127} = (\pi/64) (x_{38}^4)$

$x_{128} = (203.2/x_{77})$

$x_{129} = (\pi/4)(x_{40}^2)$

$x_{130} = (\pi/4)(x_{44}^2)$

$x_{131} = (\pi/4)(x_{43}^2)$

$x_{132} = (\pi/4)(x_{42}^2)$

$x_{133} = [x_{80} x_{132}]/[(\pi/4)x_{45}^2)$

$x_{134} = [x_{79} x_{132}]/[(\pi/4)x_{45}^2)$

$x_{135} = x_{134}/2$

$x_{136} = x_{134}/2$

$x_{137} = (\pi^2/4)(p_{48} x_{138})/(p_{153}^2)$

$x_{138} = (\pi/64)(x_{42}^4)$

$x_{139} = (\pi/4)(x_{46}^2)$

$x_{140} = (\pi/4)(x_{48}^2)$

$x_{141} = (\pi/4)(x_{47}^2)$

$x_{142} = (\pi/4)(x_{49}^2)$

$x_{143} = x_{232}/ (x_{139} - x_{141})$

$x_{144} = x_{234}/ (x_{140} - x_{142})$

$x_{145} = (\pi^2)(p_{49}\, x_{214})/x_{50}^2$

$x_{146} = (\pi^2)(p_{49}\, x_{215})/x_{52}^2$

$x_{147} = x_{79}$

$x_{148} = x_{79}$

$x_{149} = x_{147} / 2$

$x_{150} = x_{147}/ 2$

$x_{151} = x_{148} / 2$

$x_{152} = x_{148}/ 2$

$x_{153} = [(p)_{181} \cos (p_3)\, (x_1 - x_2)(x_7/2)]/x_{157}$

$x_{154} = [(p)_{181} \sin (p_3)]/x_{156}$

$x_{155} = x_{153} + x_{154}$

$x_{156} = x_6\, x_7$

$x_{157} = (1/12)\, x_6\, (x_7)^3$

$x_{158} = [(p)_{182} \cos (p_3)\, (x_1 - x_2)(x_7/2])/x_{157}$

$x_{159} = [(p)_{182} \sin (p_3)]/x_{156}$

$x_{160} = x_{158} + x_{159}$

$x_{161} = x_{160}/2$

$x_{162} = x_{160}/2$

$x_{163} = x_{70} / [x_{164} \cos (x_{205})]$

$x_{164} = x_8 \, x_9$

$x_{165} = [(\pi^2) \, p_{50} \, x_{166}]/ (x_3{}^2)$

$x_{166} = (1/12) \, x_8 \, (x_9)^3$

$x_{167} = x_{67}/(2 \, x_{164} \cos (x_{66}))$

$x_{168} = x_{67}/(2 \, x_{164} \cos (x_{66}))$

$x_{169} = (p)_{181}/x_{218}$

$x_{170} = (p)_{182}/x_{218}$

$x_{171} = ((p)_{181} \, x_{10})(x_{11}/2)/x_{219}$

$x_{172} = ((p)_{182} \, x_{10})(x_{11}/2)/x_{219}$

$x_{173} = [(x_{172}{}^2) + 3 \, (x_{170}{}^2)]^{1/2}$

$x_{174} = x_{173}/2$

$x_{175} = x_{173}/2$

$x_{176} = x_{70}/[x_{220} \cos (x_{205})]$

$x_{177} = x_{67}/[x_{220} \cos (x_{66})]$

$x_{178} = (x_{70} \, x_{12})(x_{13}/2)/[x_{221} \cos (x_{205})]$

$x_{179} = (x_{67} \, x_{12})(x_{13}/2)/[x_{221} \cos (x_{66})]$

$x_{180} = [(x_{179}{}^2) + 3 \, (x_{177}{}^2)]^{1/2}$

$x_{181} = x_{180}/2$

$x_{182} = x_{180}/2$

$x_{183} = x_{70}/[x_{222} \cos (x_{205})]$

$x_{184} = x_{67}/[x_{222} \cos (x_{66})]$

$x_{185} = (x_{70}\ x_{14})(x_{15}/2)/[x_{223} \cos (x_{205})]$

$x_{186} = (x_{67}\ x_{14})(x_{15}/2)/[x_{223} \cos (x_{66})]$

$x_{187} = [(x_{186}^2) + 3\ (x_{184}^2)]^{1/2}$

$x_{188} = x_{187}/2$

$x_{189} = x_{187}/2$

$x_{190} = x_{72} / [(\pi/4)\ (x_{31})^2]$

$x_{191} = x_{190} / 2$

$x_{192} = x_{190} / 2$

$x_{193} = x_{73} / [(\pi/4)\ (x_{38})^2]$

$x_{194} = x_{193} / 2$

$x_{195} = x_{193} / 2$

$x_{196} = 2$

$x_{197} = 4(\text{cosec}\ (x_{123}))$

$x_{198} = [x_{129} - x_{130}]\ x_{41}\ p_{155}\ p_{156}$

$x_{199} = x_{131} / x_{132}$

$x_{200} = x_{226} + x_{227}$

$x_{201} = x_{230} + x_{231}$

$x_{202} = (\pi/4)(x_{29}^2)$

$x_{203} = (\pi/4)(x_{36}^2)$

$x_{204} = (x_{34}/2 + x_{32})\ x_{110}\ p_{152}$

$x_{205} = \sin\text{-}1\{(x_4 - x_2 \cos (x_{60}))/x_3\}$

$x_{206} = 0.9\ (x_{34})$

$x_{207} = 0.9\ (x_{34}/2)$

$x_{208} = (1/12)(x_{19})(x_{20})^3$

$x_{209} = [p_{15}\ p_{17}\ p_{144}]\ (x_{17} - x_{16})$

$x_{210} = x_{92} + (x_{93} - x_{92})\ (x_{16}/(-x_{207} + x_{18})) + x_{94}$

$x_{211} = [x_{208} / (x_{21}\ x_{22})]^{1/2}$

$x_{212} = (\pi/64)(x_{31}{}^4)$

$x_{213} = (\pi/64)(x_{38}{}^4)$

$x_{214} = (\pi/64)(x_{47}{}^4)$

$x_{215} = (\pi/64)(x_{49}{}^4)$

$x_{216} = x_{80}$

$x_{217} = x_{80}$

$x_{218} = (\pi/4)(x_{11}{}^2)$

$x_{219} = (\pi/64)(x_{11}{}^4)$

$x_{220} = (\pi/4)(x_{13}{}^2)$

$x_{221} = (\pi/64)(x_{13}{}^4)$

$x_{222} = (\pi/4)(x_{15}{}^2)$

$x_{223} = (\pi/64)(x_{15}{}^4)$

$x_{224} = x_{233}/\ (x_{139} - x_{141})$

$x_{225} = x_{235}/\ (x_{140} - x_{142})$

$x_{226} = \{p_8 + p_9 + 2(x_{202}{}^2 p_{10} + x_{203}{}^2 p_{11}) + 20\ (p_{12} x_{202} + p_{13} x_{203}\}[(145\ x_{79}\ x_{77})/\ (1000\ x_{78})]$

$x_{227} = \{p_8 + p_9\}[(145\ x_{79}\ x_{77})/\ (1000\ x_{78})]$

$x_{228}$ =$\{p_8 + p_9 + 2(x_{202}^2 p_{10} + x_{203}^2 p_{11}) + 20\ (p_{12}x_{202} + p_{13}x_{203}\}[(145\ x_{80}\ x_{77})/\ (1000\ x_{78})]$

$x_{229} = \{p_8 + p_9\}[(145\ x_{80}\ x_{77})/\ (1000\ x_{78})]$

$x_{230} = 2(x_{196}\ p_{179}\ x_{202})\{x_{77}/x_{78}\}$

$x_{231} = 2(x_{197}\ x_{204}\ x_{203})\{x_{77}/x_{78}\}$

$x_{232} = (x_{226} + x_{230})(x_{139}/x_{77})$

$x_{233} = (x_{228} + x_{230})(x_{139}/x_{77})$

$x_{234} = (x_{227} + x_{231})(x_{140}/x_{77})$

$x_{235} = (x_{229} + x_{231})(x_{140}/x_{77})$

**Design Criteria**

Required front braking force:

1) $x_{54} = p_1$

Required rear braking force:

2) $x_{55} = p_2$

Pedal travel at maximum foot load:

3) $x_{56} \leq p_{47}$

Pedal travel at regular foot load:

4) $x_{57} \leq p_{51}$

Minimum booster ratio:

5) $x_{199} \geq p_{184}$

Maximum booster ratio:

6) $x_{199} \leq p_{185}$

Partial Failure Deceleration:

7) $x_{55} / (p_{22} /g) \geq p_{53}$

Pedal travel at partial failure front line

8) $x_{58} \leq p_{52}$

Pedal travel at partial failure rear line

9) $x_{59} \leq p_{52}$

Minimum angle requirements of $(\phi_1)_{min}$

10) $x_{60} \geq p_{186}$

Maximum angle requirements of $(\phi_1)_{min}$

11) $x_{60} \leq p_{187}$

Pedal should always be angled up to 203.2 mm (or 8") of pedal travel

12) $x_1 \;\; x_5 \geq p_{188}$

Minimum deceleration in the event of a booster failure

13) $[x_{54} + x_{55}]/ [x_{199} (p_{22} /g)] \geq 2.94$

Minimum deceleration with parking brake

14) $x_{86} / (p_{22} /g) \geq 2.94$

Holding the car on a grade ($30^o$) with parking brake

15) $x_{86} \geq p_{68} [p_{22} \sin (p_{31}) - p_{21} p_{22} \cos (p_{31})]$

Maximum length of the cross rod

16) $x_{210} \leq x_{93} + x_{94}$

Maximum bending stress in the pull rod

17) $x_{88} \leq [p_{69} / p_{70}]$

Maximum compressive stress in the cross rod

18) $x_{89} \leq [p_{71} / p_{72}]$

Euler Buckling limit

19) $x_{86}\, p_{17}\, p_{144}\, (x_{17}/x_{16}) \leq x_{90}$

Johnson Buckling limit

20) $x_{86}\, p_{17}\, p_{144}\, (x_{17}/x_{16}) \leq x_{91}$

Limit on the width of the pad

21) $[x_{23} - (p_{20} + 2\, x_{24})]/2.0 \geq 2\, x_{28}$

Limits on the outer diameter of the disc with respect to the wheel

22) $(0.5\, x_{23}) / p_{142} \geq 0.25$

Limits on the outer diameter of the disc with respect to the wheel

23) $(0.5\, x_{23}) / p_{142} \leq 0.4$

Limits on the volume of the disc

24) $x_{95} \leq p_{56}$

Limits on temperature due to sudden stop

25) $x_{101} \leq p_{58}$

Limits on temperature due to repeated braking

26) $x_{102} \le p_{59}$

Limits on temperature due continued braking

27) $x_{103} \le p_{60}$

Stress limits on the disc (radial)

28) $x_{104} \le p_{73}/p_{74}$

Stress limits on the disc (longitudinal)

29) $x_{105} \le p_{73}/p_{74}$

Limits on disc pad length

30) $2\ x_{27} \le [x_{23} - (p_{20} + 2\ x_{24})]/2.0$

Stress limits on the disc pads

31) $x_{106} \le p_{157}/p_{158}$

Stress limits on the push rod

32) $x_{107} \le p_{75}/p_{76}$

Buckling limit on the push rod

33) $x_{74} \le x_{108}$

Drums:

Limits on the outer diameter of the drum with respect to the wheel

34) $(x_{34} + 2\, x_{32}) / (2\, p_{142}) \geq 0.25$

Limits on the outer diameter of the drum with respect to the wheel

35) $(x_{34} + 2\, x_{32}) / (2\, p_{142}) \leq 0.4$

Limits on the volume of the drum

36) $x_{109} \leq p_{57}$

Lower limit on the value of $h_d$

37) $x_{206} \geq (0.75)\, x_{34}$

Upper limit on the value of $h_d$

38) $x_{206} \leq (0.85)\, x_{34}$

Radius of the axis of the lining

39) $x_{35} = 0.9\, (x_{34} / 2)$

Limits on temperature due to sudden stop

40) $x_{115} \leq p_{58}$

Limits on temperature due to repeated braking

41) $x_{116} \leq p_{59}$

Limits on temperature due continued braking

42) $x_{117} \leq p_{60}$

Stress limits on the drum (radial)

43) $x_{118} \leq p_{77}/p_{78}$

Stress limits on the drum (longitudinal)

44) $x_{119} \leq p_{77}/p_{78}$

Stress limits on the push rod

45) $x_{120} \leq p_{75}/p_{76}$

Buckling limit on the push rod

46) $x_{75} \leq x_{121}$

Vacuum Booster:

Limits on reaction piston diameter

47) $x_{42} \geq x_{39}$

Limits on reaction piston diameter

48) $x_{42} \leq 1.5\, x_{39}$

Limits on booster push rod diameter

49) $2\, x_{45} \leq x_{39}$

Limits on booster push rod diameter

50) $2\, x_{39} \leq x_{44}$

Volume limits on the booster

51) $(\pi/4)\, x_{40}^2\, x_{128} \leq p_{62}$

Limits on maximum effective vacuum

52) $x_{41} \geq 6.9$

Limits on maximum effective vacuum

53) $x_{41} \leq 13.8$

Maximum stress limits in the thin section of the booster

54) $x_{133} \leq (p_{159}/p_{160})$

Fatigue limit on the booster push rod

55) $(p_{92}\ p_{93}) / [\ p_{92}\ x_{136} + \ p_{93}\ x_{135}] \ \geq 3$

Buckling limit on the booster push rod

56) $x_{80}\ x_{132} \leq p_{161}\ x_{137}$

Master Cylinder:

Limits on volume of the fluid in the master cylinder (primary side)

57) $[x_{51} + x_{50}]\ x_{139} - x_{50}\ x_{141} \leq p_{65}$

Limits on volume of the fluid in the master cylinder (secondary side)

58) $[x_{53} + x_{52}]\ x_{140} - x_{52}\ x_{142} \leq p_{66}$

Manufacturing requirements

59) $x_{51} = x_{53}$

Manufacturing requirements

60) $x_{50} = x_{52}$

Limits on the length of the master cylinder primary piston rod

61) $x_{50} \geq k_{mr} (x_{143})$

Limits on the length of the master cylinder secondary piston rod

62) $x_{52} \geq k_{mr} (x_{144})$

Limits on the clearance in the primary piston side

63) $x_{51} \geq (x_{224})$

Limits on the clearance in the secondary piston side

64) $x_{53} \geq (x_{225})$

Limits on line pressure in the primary piston side

65) $x_{71} / x_{139} \leq p_{67}$

Limits on line pressure in the secondary piston side

66) $x_{71} / x_{140} \leq p_{67}$

Buckling limit on the master cylinder primary piston rod

67) $x_{71} \leq (x_{145} / p_{164})$

Buckling limit on the master cylinder secondary piston rod

68) $x_{71} \leq (x_{146} / p_{167})$

Normal stress limits on the master cylinder primary piston rod

69) $x_{216} \leq (p_{162} / p_{163})$

Normal stress limits on the master cylinder secondary piston rod

70) $x_{217} \leq (p_{165} / p_{166})$

Fatigue limit on the master cylinder primary piston rod

71) $(p_{100}\, p_{101}) / [\, p_{101}\, x_{149} + p_{100}\, x_{151}] \geq 3$

Fatigue limit on the master cylinder secondary piston rod

72) $(p_{100}\, p_{101}) / [\, p_{101}\, x_{150} + p_{100}\, x_{152}] \geq 3$

Pedal Linkage:

Member 1

Limit on normal stress in member 1

73) $x_{155} \leq (p_{168} / p_{169})$

Fatigue limit in member 1

74) $(p_{108}\, p_{109}) / [p_{109}\, x_{161} + p_{108}\, x_{162}] \geq 3$

Member 2

Limit on normal stress in member 2

75) $x_{163} \leq (p_{170}/p_{171})$

Buckling limit in member 2

76) $x_{97} \leq (x_{165}/p_{172})$

Fatigue limit in member 2

77) $(p_{116}\ \ p_{117}) / [p_{116}\, x_{168} + \ p_{117}\, x_{167}] \geq 3$

Pin 1:

Combined stress limits on pin 1

78) $x_{173} \leq (\ p_{173}/p_{174}\ )$

Fatigue stress limits on pin 1

79) $(p_{124}\ \ p_{125}) / [p_{124}\, x_{175} + \ p_{125}\, x_{174}] \geq 3$

Pin 2:

Combined stress limits on pin 2

80) $x_{180} \leq (\ p_{175}/p_{176})$

Fatigue stress limits on pin 2

81) $(p_{132} \; p_{133}) / [p_{133} \, x_{181} + \; p_{132} \, x_{182}] \geq 3$

Pin 3:

Combined stress limits on pin 3

82) $x_{187} \leq ( p_{177}/p_{178} )$

Fatigue stress limits on pin 3

83) $(p_{140} \; p_{141}) / [p_{140} \, x_{189} + \; p_{141} \, x_{188}] \geq 3$

Limit on diameter of pin 1

84) $x_{11} \leq 0.8 \, x_7$

Limit on length of pin 1

85) $x_{10} = 1.1 \, x_6$

Limit on diameter of pin 2

86) $x_{13} \leq 0.8 \, x_7$

Limit on diameter of pin 2

87) $x_{13} \leq 0.8 \, x_9$

Limit on length of pin 2

88) $x_{12} = 1.1 (x_6 + x_8)$

Limit on length of pin 3

89) $x_{14} = 1.1 x_8$

Limit on diameter of pin 3

90) $x_{15} \leq 0.8 x_9$

## CLOSURE

The description of the brake system design problem along with the mathematical model has been presented in this report. The main aim for developing such a model was to use it as an example to illustrate different ways of partitioning a general design problem of considerable size. This model is based on numerous assumptions. Thus any results obtained using this model would be very approximate in nature. Detailed validation of this model is necessary before it can be used be used as tool for even preliminary design of vacuum assisted hydraulic braking system.

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Typical Values of Parameters

| Parameter | Value | Parameter | Value | Parameter | Value | Parameter | Value |
|---|---|---|---|---|---|---|---|
| p1 | 9673.4 N | p44 | 207 MPa | p87 | 1.0 | p130 | 0.5 |
| p2 | 1934.5 N | p45 | 150 $^oF$ | p88 | 1.0 | p131 | 427.392 MPa |
| p3 | 15 $^o$ | p46 | 0.7 | p89 | 1.0 | p132 | 96.80MPa |
| p4 | 15.25 mm | p47 | 165.1 mm | p90 | 0.5 | p133 | 848 MPa |
| p5 | 0.254 mm | p48 | 207 MPa | p91 | 427.392 MPa | p134 | 0.755 |
| p6 | 0.254 mm | p49 | 207 MPa | p92 | 161.33 MPa | p135 | 0.6 |
| p7 | $12x10^{-6}$ /$^oC$ | p50 | 207 MPa | p93 | 848 MPa | p136 | 1.0 |
| p8 | 0.1 $in^3$ | p51 | 101.6 mm | p94 | 0.755 | p137 | 1.0 |
| p9 | 0.08 $in^3$ | p52 | 152.4 mm | p95 | 1.0 | p138 | 0.5 |
| p10 | 0.013 in | p53 | 2.943 $m/s^2$ | p96 | 1.0 | p139 | 427.392 MPa |
| p11 | 0.0 in | p54 | 2.943 $m/s^2$ | p97 | 1.0 | p140 | 96.80MPa |
| p12 | 0.0009 in | p55 | 848 MPa | p98 | 0.5 | p141 | 848 MPa |
| p13 | 0.0 in | p56 | $1.5x10^6$ $mm^3$ | p99 | 427.392 MPa | p142 | 300.0 mm |
| p14 | 0.6 | p57 | $1.25x10^6$ mm3 | p100 | 161.33 MPa | p143 | 2.0 (for N) |
| p15 | 18 N | p58 | 650 $^oF$ | p101 | 848 MPa | p144 | 40.0 |
| p16 | 0.9 | p59 | 700 $^oF$ | p102 | 0.755 | p145 | 207 GPa |
| p17 | 0.9 | p60 | 700 $^oF$ | p103 | 0.6 | p146 | 1.0 |
| p18 | 0.9 | p61 | 203.2 mm | p104 | 1.0 | p147 | 0.45 KJ/Kg K |
| p19 | 127 mm | p62 | TBD 1 | p105 | 1.0 | p148 | 0.8 |
| p20 | 150 mm | p63 | 6.9 $N/cm^2$ | p106 | 0.5 | p149 | 100 $^oF$ |
| p21 | 0.02 | p64 | 13.8 $N/cm^2$ | p107 | 427.392 MPa | p150 | 150.0 mm |
| p22 | 18689 N | p65 | 66500 $mm^3$ | p108 | 96.80MPa | p151 | 0.45 KJ/Kg K |
| p23 | 1.20 $Kg/m^3$ | p66 | 66500 $mm^3$ | p109 | 848 MPa | p152 | $12x10^{-6}$ /$^oC$ |
| p24 | 1.04 | p67 | 1500 psi | p110 | 0.755 | p153 | 10 mm** |
| p25 | 2 $m^2$ | p68 | 2 | p111 | 1.0 | p154 | 6.045 secs |
| p26 | 36.88 m/s | p69 | 648 MPa | p112 | 1.0 | p155 | 0.9 |
| p27 | 6.1 $m/s^2$ | p70 | 2 | p113 | 1.0 | p156 | 0.9 |
| p28 | 7850 $Kg/m^3$ | p71 | 648 MPa | p114 | 0.5 | p157 | 125 MPa |
| p29 | 10 | p72 | 2 | p115 | 427.392 MPa | p158 | 2 |
| p30 | 370000 $Nm/hKm^2$ | p73 | 648 MPa | p116 | 161.33 MPa | p159 | 648 MPa |
| p31 | 0.025 h | p74 | 2 | p117 | 848 MPa | p160 | 2 |
| p32 | 30$^o$ | p75 | 648 MPa | p118 | 0.755 | p161 | 2 |
| p33 | 10$^o$ | p76 | 2 | p119 | 0.6 | p162 | 648 MPa |
| p34 | 8000 m | p77 | 207 MPa | p120 | 1.0 | p163 | 2 |
| p35 | 33.2 Km/h | p78 | 2 | p121 | 1.0 | p164 | 2 |
| p36 | 207 MPa | p79 | 0.755 | p122 | 0.5 | p165 | 648 MPa |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p_{37}$ | 0.292 | $p_{80}$ | 1.0 | $p_{123}$ | 427.392 MPa | $p_{166}$ | 2 |
| $p_{38}$ | 207 MPa | $p_{81}$ | 1.0 | $p_{124}$ | 96.80MPa | $p_{167}$ | 2 |
| $p_{39}$ | 150 $^{o}$F | $p_{82}$ | 1.0 | $p_{125}$ | 848 MPa | $p_{168}$ | 648 MPa |
| $p_{40}$ | 7200 Kg/m$^3$ | $p_{83}$ | 0.5 | $p_{126}$ | 0.755 | $p_{169}$ | 2 |
| $p_{41}$ | = $p_{30}$ | $p_{84}$ | 427.392 MPa | $p_{127}$ | 0.6 | $p_{170}$ | 648 MPa |
| $p_{42}$ | 100 MPa | $p_{85}$ | 161.33 MPa | $p_{128}$ | 1.0 | $p_{171}$ | 2 |
| $p_{43}$ | 0.211 | $p_{86}$ | 0.755 | $p_{129}$ | 1.0 | $p_{172}$ | 2 |
| $p_{173}$ | 648 MPa | $p_{176}$ | 2 | $p_{179}$ | 0.0 mm | | |
| $p_{174}$ | 2 | $p_{177}$ | 648 MPa | $p_{180}$ | TBD | | |
| $p_{175}$ | 648 MPa | $p_{178}$ | 2 | $p_{181}$ | TBD | | |

Typical Bounds On the Variables

| Variable | l.bound | u.bound | Variable | l.bound | u.bound | Variable | l.bound | u.bound |
|---|---|---|---|---|---|---|---|---|
| $x_1$ | 203.2 | 304.8 | $x_{19}$ | 5.0 | 20.0 | $x_{37}$ | 7.5 | 15.0 |
| $x_2$ | 63.5 | 127.0 | $x_{20}$ | 5.0 | 30.0 | $x_{38}$ | 5.0 | 60.0 |
| $x_3$ | 76.2 | 177.8 | $x_{21}$ | 5.0 | 20.0 | $x_{39}$ | 10 | 60 |
| $x_4$ | 63.5 | 127.0 | $x_{22}$ | 5.0 | 30.0 | $x_{40}$ | 50 | 250 |
| $x_5$ | 30.0 | 40.0 | $x_{23}$ | 160.0 | 240.0 | $x_{41}$ | 6.9 | 13.8 |
| $x_6$ | 19.05 | 38.1 | $x_{24}$ | 5.0 | 30.0 | $x_{42}$ | 10 | 60 |
| $x_7$ | 6.35 | 25.4 | $x_{25}$ | 20.0 | 40.0 | $x_{43}$ | 10 | 40 |
| $x_8$ | 19.05 | 38.1 | $x_{26}$ | 10.0 | 30.0 | $x_{44}$ | 10 | 150 |
| $x_9$ | 6.35 | 25.4 | $x_{27}$ | 5.0 | 90.0 | $x_{45}$ | 5 | 50 |
| $x_{10}$ | 19.05 | 44.45 | $x_{28}$ | 5.0 | 90.0 | $x_{46}$ | 10 | 50 |
| $x_{11}$ | 5.08 | 19.05 | $x_{29}$ | 10.0 | 75 | $x_{47}$ | 5 | 40 |
| $x_{12}$ | 19.05 | 44.45 | $x_{30}$ | 7.5 | 15.0 | $x_{48}$ | 10 | 50 |
| $x_{13}$ | 6.35 | 19.05 | $x_{31}$ | 5.0 | 60.0 | $x_{49}$ | 5 | 40 |
| $x_{14}$ | 19.05 | 44.45 | $x_{32}$ | 5.0 | 15.0 | $x_{50}$ | 10 | 100 |
| $x_{15}$ | 6.35 | 19.05 | $x_{33}$ | 30 | 60.0 | $x_{51}$ | 10 | 100 |
| $x_{16}$ | 0.0 | 30.6 | $x_{34}$ | 160.0 | 240.0 | $x_{52}$ | 10 | 100 |
| $x_{17}$ | 0.0 | 102.0 | $x_{35}$ | 68 | 102.0 | $x_{53}$ | 10 | 100 |
| $x_{18}$ | 0.0 | 183.6 | $x_{36}$ | 10.0 | 75 | | | |