AN INTERACTIVE DESIGN PROCEDURE
FOR OPTIMIZATION OF
HELICAL COMPRESSION SPRINGS

by

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FOREWORD

This report describes an early effort in attempting to use monotonicity analysis in a computer-aided way. The work was conducted as a partial fulfillment of the precandidacy doctoral requirements (ME 990). In this respect the assistance of the committee members Prof. M. A. Zarrugh and R. C. Juvinall is gratefully appreciated.

The approach developed here is the global monotonicity analysis resulting to closed-form constraint-bound global optima. Dominance conditions are used to accommodate a variety of user-specified requirements. The optimization process is not iterative, except for an occasional one-dimensional non-linear equation solution.

The program developed is a special-purpose one that can be run inexpensively, thus parametric studies can be performed easily and efficiently.

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1. INTRODUCTION

This report describes the application of monotonicity analysis, a recent optimization technique [1] to optimal design of a helical compression spring. Without numerical iterative computation, this technique eliminates combinations of constraints which cannot be active at the optimum. When applicable, this procedure may lead directly to the global optimum.

The optimization problem is formulated to satisfy four different objectives: maximum reliability, maximum energy storage capacity, maximum natural frequency and minimum weight. Appropriate physical and practical constraints complete the model. The optimization problem is solved in closed-form by identifying and comparing combinations of possibly active constraints. Based on that, an interactive computer program is written in basic for a TRS-80 microcomputer to find the optimal dimensions of a helical compression spring, given the input data of a particular application.

Classical optimization methods have been used to optimize spring design. Mancini and Piziali [3] used geometric programming to maximize reliability by using the safety factors in yielding and fatigue as objectives. In their paper, combinations of active constraints were found by examining the dual constraints and determining combinations of positive dual variables. Papalambros [2] applied the monotonicity concept to posynomial geometric programming, establishing rules for constraint activity identification, and then applied the idea to spring design. Agrawal [4] also used geometric programming for a spring design problem. In his model yielding, surging and buckling were the only constraints. Batchler and Romel [5] used a direct search algorithm for minimum spring weight. A special feature of this algorithm was the creation of a reasonably symmetrical ridge along the boundary between feasible and infeasible regions. Thus allowing the pattern search to move along this boundary, of which the optimum is located. In a later paper Batchler [6] used a nested sequential
golden search method for the same problem with discrete con- 
method for their unconstrained spring design problem. Their 
objective was to maximize the delivering work and volume effici- 
cy (delivered work per unit volume) of the spring. The spring 
problem has been addressed also by Johnson [11], for a triple 
optimization problem, i.e., minimizing weight and length and 
maximizing natural frequency simultaneously. In his method of 
variational study, after setting the problem in a final formula- 
tion form, the problem is solved graphically. His method shares 
some ideas with monotonicity analysis though differing in the 
solution methodology.

2. MODEL DEVELOPMENT

The intended use of the spring influences the choice of the 
most important criterion which will constitute the objective of 
the optimization problem. In the present model four alternative 
criteria are examined. The designer can choose the one appearing 
appropriate for a particular application. These objectives are: 
maximum reliability, maximum energy storage capacity, maximum 
natural frequency and minimum weight. Appropriate physical and 
practical constraints complete the model. The physical constraints 
describe physical limitations such as strength, buckling, frequency, 
and limits on the number of coils and the spring index. The 
practical constraints describe size limitations. In this section, 
the analytical expressions for various objectives and constraints 
are derived in forms appropriate for use in the computer.

2.1 Objective Functions

Four different objectives are considered separately:

2.1.1 Maximizing Reliability

The objective function has been selected as the inverse of 
the safety factor for yielding or fatigue, whichever is critical 
(to maximize reliability, the reciprocals of the safety factors 
are minimized). Data shows that the ultimate strength of a 
spring material is a function of the wire diameter of the form
\[ S_u = A d^{A_1} \]  (1)

where \( A \) and \( A_1 \) are parameters given in Table 1. The ultimate shear strength \( S_{us} \), yield strength \( S_y \) and shear yield strength \( S_{sy} \) may be expressed in terms of the ultimate tensile strength \( S_u \) as follows:

\[ S_{us} = 0.8S_u \]  (2)

\[ S_{sy} = 0.577S_y \]  (3)

\[ S_y = 0.75S_u \]  (4)

The shear ultimate strength and shear yield strength are written in the following form for four materials: music wire, oil-tempered wire, chrome vanadium and chrome silicon:

\[ S_{us} = c_2 d^{A_1} \]  (5)

\[ S_{ys} = c_3 d^{A_1} \]  (6)

where the parameters \( c_2, A_2, c_3, A_1 \), are also given in Table 1.

The fatigue strength in shear is a function of life in cycles within the range of \( 10^3 \) to \( 10^6 \) cycles. The endurance limit (fatigue strength at \( 10^6 \) cycles) is:

\[ S_e = c_L c_S \frac{S_u}{2} \]  (7)

where \( c_L = 0.577 \) is a conversion factor from tensile to torsion strength since \( S_u \) is not known initially, \( c_S \) is also not known, although a conservative value can be chosen by knowing that \( c_S \) increases with \( S_u \) [8]. Thus for music wire, \( c_S = 0.577 \) [3] and for the other three materials, \( c_S = 0.70 \) will be used. However, a designer must also be aware of beneficial effects such as shot-peening and presetting on fatigue strength. At \( 10^3 \) cycles the fatigue strength is,

\[ S_3 = 0.9S_{us} \]  (8)
The fatigue strength as expressed by the S-N curve is

\[ S_{ns} = C_1 d^{-1} (NC)^{B_1} \]  \hspace{1cm} (9)

where \( C_1, A_1, B_1 \) are given in Table 1.

The fatigue diagram in Figure 2 is used to define both fatigue and yield failures. The safety factor is defined as the ratio of the distance from 0-0" to 0-0' for yielding and 0-0" to 0-0' for fatigue. For fatigue the safety factor is:

\[ SF_f = \frac{1}{T_a / S_{ns} + T_m / S_{us}} \]  \hspace{1cm} (10)

and for yielding the safety factor is:

\[ SF_y = \frac{S_{ys}}{(T_a + T_m)} \]  \hspace{1cm} (11)

It can be seen from Figure 2 that for

\[ \frac{T_a}{T_m} \geq \frac{S_{ns} (S_{ys} - S_{us})}{S_{us} (S_{ns} - S_{ys})} \]  \hspace{1cm} (12)

Fatigue will be critical and for the reversed inequality yielding will be critical. Maximizing the inverse of equations (10) and (11) gives the two objectives, where

\[ T_a = \frac{8 F_a D K_a}{\pi d^3} \]  \hspace{1cm} (13a) \hspace{1cm} (13b)

\[ T_m = \frac{8 F_m D K_m}{\pi d^3} \]

\[ K_s = \left( \frac{4c-1}{4c+4} \right) + \frac{.615}{C} \]  \hspace{1cm} (14)

\[ F_a = \frac{F_u - F_L}{2} \]  \hspace{1cm} (15a) \hspace{1cm} (15b)

\[ F_m = \frac{F_u + F_L}{2} \]

Note that the Wahl correction factor \( K \) can be approximated within 2.6% of the exact values [3], given in equation (14) by setting...
\[ K_s = \frac{1.6}{c^{1.14}} \text{ for } 5 \leq c \leq 20 \]  \hspace{2cm} (16)

Thus the general form of the objective functions are

\[ \text{Min} \left( \frac{1}{SF} \right) = \frac{T_a}{S_{ns}} + \frac{T_m}{S_{us}} \] \hspace{2cm} (Fatigue) \hspace{2cm} (17)

\[ \text{Min} \left( \frac{1}{SF} \right) = \frac{T_a + T_m}{S_{ys}} \] \hspace{2cm} (Yielding) \hspace{2cm} (18)

Substituting equations (13), (15), (16) in (17) and (18) yields the following:

\[ g_0^F = \text{Min} \left( \frac{1}{SF} \right) = 2.04 \frac{F_u - F_L}{C_1 (NC) B_1} + \frac{F_u + F_L}{C_2} c^{-86} d^{-(2+A_1)} \] \hspace{2cm} (19)

\[ g_0^Y = \text{Min} \left( \frac{1}{SF} \right) = \frac{4.07 (F_u)}{C_3} c^{-86} d^{-(2+A_1)} \] \hspace{2cm} (20)

The objective functions can be stated in a single expression as

\[ \text{Min } g_0 = K_{01} c^{-86} d^{-(2+A_1)} \] \hspace{2cm} (21)

where:

\[ K_{01}^F = 2.04 \frac{F_u - F_L}{C_1 (NC) B_1} + \frac{F_u + F_L}{C_2} \] \hspace{2cm} (Fatigue) \hspace{2cm} (22)

\[ K_{01}^Y = \frac{4.07 (F_u)}{C_3} \] \hspace{2cm} (Yielding) \hspace{2cm} (23)
<table>
<thead>
<tr>
<th></th>
<th>Music-Wire .004&quot; ≤ d ≤ .25&quot;</th>
<th>Oil-Tempered Wire .02&quot; ≤ d ≤ .5&quot;</th>
<th>Chrome Vanadium .32&quot; ≤ d ≤ .437</th>
<th>Chrome Silicon .063&quot; ≤ d ≤ .375</th>
</tr>
</thead>
<tbody>
<tr>
<td>A'</td>
<td>200,000</td>
<td>144,000</td>
<td>169,000</td>
<td>200,000</td>
</tr>
<tr>
<td>A</td>
<td>-.14</td>
<td>-.19</td>
<td>-.17</td>
<td>-.11</td>
</tr>
<tr>
<td>C</td>
<td>630,500</td>
<td>372,000</td>
<td>433,650</td>
<td>630,500</td>
</tr>
<tr>
<td>C₂</td>
<td>160,000</td>
<td>115,200</td>
<td>135,200</td>
<td>160,000</td>
</tr>
<tr>
<td>C₃</td>
<td>86,550</td>
<td>62,500</td>
<td>73,200</td>
<td>86,550</td>
</tr>
<tr>
<td>B</td>
<td>-.2137</td>
<td>-.1845</td>
<td>-.1840</td>
<td>-.2137</td>
</tr>
</tbody>
</table>

Table 1: Constants for material's strength

Figure 2: Fatigue Diagram
2.1.2 Maximizing Energy Storage Capacity

Quite frequently, in the selection and design of springs, the capacity of a spring to store energy is of major importance. Sometimes the designer is interested in absorbing shock and impact loads; at other times one is simply interested in storing the maximum energy in the smallest space. In the case of compression springs the strain energy per unit volume is [10]

\[ U = C_F \frac{t^2}{2G} \]  \hspace{1cm} (24)

where \( C_F = \frac{1}{2K_s^2} \) for compression spring [10]. Using equations (13) and (16). The objective function is:

\[ \text{Min} \frac{1}{u} = \text{Min} \, g_0^E = K_{01}^E \, C^{-2} d^4 \]  \hspace{1cm} (25)

where:

\[ K_{01}^E = (10.24G) \left( \frac{\pi}{12.8 \, F_u} \right)^2 \]  \hspace{1cm} (26)

2.1.3 Maximizing Natural Frequency

If the spring is used in a high speed mechanism and a condition of resonance exists with the forcing function, large amplitude longitudinal vibrations will result. This will appreciably affect the force exerted by the spring on the components of the mechanism; also, such internal vibrations will significantly increase the maximum shear stress in the spring, possibly resulting in a premature fatigue failure.

For a constrained end spring, the lowest (fundamental) natural frequency is [11]

\[ f_n = \frac{1}{2} \sqrt{\frac{K_s}{C \, w}} \]  \hspace{1cm} (27)
where:

\[
k_{\Delta} \text{ spring Gradient} = \frac{Gd^4}{8D^3N} = \frac{F_u - F_L}{\Delta} \quad (28)
\]

\[
w_{\Delta} \text{ weight of active coils} = \frac{\pi^2 d^2 D N \sigma}{4} \quad (29)
\]

for steels, equation (27) takes the form [11]

\[
f_n = \frac{14100d}{D^2 N} \quad (30)
\]

or:

\[
f_n = \left[ \frac{112800(F_u - F_L)}{G_{\Delta}} \right] C d^{-2} \quad (31)
\]

Thus the objective function becomes

\[
\text{Min} \frac{1}{f_n} = g_0^f = K_{01}^f C^{-1} d^2 \quad (32)
\]

where:

\[
K_{01}^f = \frac{G_{\Delta}}{112800(F_u - F_L)} \quad (33)
\]

2.1.4 Minimizing Weight

The weight of the spring including the end coils is given by

\[
W = \rho (N+Q) \frac{\pi^2 Dd^2}{4\cos \lambda} = W_1 C^{-2} d^4 + W_2 C d^3 \quad (34)
\]

where \( \cos \lambda \approx 1 \)

\[
W_1 = \frac{\rho \pi^2 G}{32k} \quad (35a) \quad W_2 = \frac{\rho \pi^2}{4} Q \quad (35b)
\]
2.2 Constraints

There are two types of constraints: Physical and space constraints.

The physical constraints are:

2.2.1 Strength (G)

This constraint is for the case where the objective is other than maximizing reliability. For a selected safety factor, the strength constraint is

\[
\frac{T_a}{S_{ns}} + \frac{T_m}{S_{us}} \cdot SF \leq 1 \quad \text{(Fatigue)} \quad (36)
\]

\[
\frac{SF}{S_{ys}} (T_a + T_m) \leq 1 \quad \text{(Yielding)} \quad (37)
\]

Application of the procedure used in 2.1.1. to these equation gives a similar result to (21):

\[
G = K_0 \cdot c^{0.86} \cdot d^{-(2+A_1)} \leq 1 \quad (38)
\]

where:

\[
K_0^F = (2.04) (SF) \left( \frac{F_u - F_L}{C_1 (NC)^{B_1}} + \frac{(F_u + F_L)}{C_2} \right) \quad \text{(Fatigue)} \quad (39)
\]

\[
K_0^Y = 4.07 (F_u) (SF)/C_3 \quad \text{(Yielding)} \quad (40)
\]

2.2.2. Surging (G1)

This constraint applies to all cases but maximizing natural frequency. In a high speed mechanism, the natural frequency of the spring, EQ. (27), should be appreciably greater than any significant fourier component of the forcing function acting on
the spring. The fundamental natural frequency is taken commonly 13 times the fundamental frequency of the applied force [4]. This requirement is expressed as follows:

\[ G_1 = K_1 C^{-1} d^2 \leq 1 \]  \hspace{1cm} (41)

where:

\[ K_1 = \frac{Gf \Delta}{112800(F_u - F_L)} \]  \hspace{1cm} (42)

2.2.3 Buckling (G2)

The critical length of spring is a function of boundary conditions and is normally presented graphically. An approximate equation to avoid buckling, conservatively assuming both ends hinged is [3,11]:

\[ L = N d (1 + A) \leq 11.5 k \left( \frac{D}{2} \right)^2 / F_u \]  \hspace{1cm} (43)

where

\[ N = \frac{Gd^4}{8D^3 N_k} \]  \hspace{1cm} (44)

substituting (44) in (43):

\[ G_2 = K_2 C^{-5} \leq 1 \]  \hspace{1cm} (45)

where:

\[ K_2 = \frac{GF_u (1 + A)}{22.3 k^2} \]  \hspace{1cm} (46)

2.2.4. Minimum Number of Coils (G3)

A lower limit on the number of coils must be imposed in order to obtain accuracy and control in manufacturing. A value of three is normally used as an absolute minimum, but a larger value can be selected to keep the pitch angle small (preferably 12°-15°). This requirement is expressed by:
\[ N \geq N_{\text{Min}} \]  \hspace{1cm} (47)

where:

\[ N = \frac{Gd^4}{8D^3k} \]  \hspace{1cm} (48)

substituting (48) in (47) and normalizing results to the constraint

\[ G3 = k_3c^3d^{-1} \leq 1 \]  \hspace{1cm} (49)

where:

\[ K_3 = \frac{8k N_{\text{Min}}}{G} \]  \hspace{1cm} (50)

2.2.5. Spring Index (G4, G5)

The spring index (defined by \( c = \frac{D}{d} \)) must be bounded by a maximum number, \( I_u \), and a minimum number, \( I_L \). Experience indicates that extreme values are \([3]\):

\[ I_L \geq 4 \]  \hspace{1cm} (51a)  \hspace{1cm} \[ I_u \leq 20 \]  \hspace{1cm} (51b)

the corresponding constraints are:

\[ G4 = K_4c \leq 1 \]  \hspace{1cm} (52)

\[ G5 = K_5c^{-1} \leq 1 \]  \hspace{1cm} (53)

where:

\[ K_4 = \frac{1}{I_u} \]  \hspace{1cm} (54a)  \hspace{1cm} \[ K_5 = I_L \]  \hspace{1cm} (54b)

The space constraints are the following:

2.2.6. Pocket Length (G6)

When space is limited, the length of a spring, in its machine setting, must be below an upper limit. The maximum length \( L_m \) at maximum force \( F_u \) is:

\[ L + Qd \leq L_m \]  \hspace{1cm} (55)

substituting (43), (44) and normalizing gives
\[ G6 = k_6d^2c^{-3} + L_6d \leq 1 \]  \hspace{1cm} (56)

where

\[ K_6 = \frac{G(1+A)}{8KL_m} \]  \hspace{1cm} (57a)

\[ L_6 = \frac{Q}{L_m} \]  \hspace{1cm} (57b)

2.2.7. Maximum Allowable Outside Diameter, OD.

Maximum allowable outside diameter, OD:

\[ D + d \leq OD \]

or in normalized form:

\[ G7 = K_7(C_d + d) \leq 1 \]  \hspace{1cm} (58)

where:

\[ K_7 = \frac{1}{OD} \]  \hspace{1cm} (59)

2.2.8. Minimum Allowable Inside Diameter (G8)

Minimum allowable inside diameter, ID:

\[ D - d \geq ID \]  \hspace{1cm} (60)

or in normalized form:

\[ G8 = c^{-1} + K_8c^{-1}d^{-1} \leq 1 \]  \hspace{1cm} (61)

where:

\[ K_8 = ID \]  \hspace{1cm} (62)

2.2.9. Upper and Lower Limits on Wire Diameter (G9, G10)

As discussed in Section 2.1.1, the strength of the spring material is a function of wire diameter. This relation was found for the given limits (Table 1) of wire diameter for different materials, viz.

\[ G9 = K_9d^{-1} \leq 1 \]  \hspace{1cm} (63)

\[ G10 = K_{10}d \leq 1 \]  \hspace{1cm} (64)
2.2.10. Clash Allowance (G11)

Stresses are limited by the spring solid condition; therefore, if the maximum working load approaches the spring solid condition, it is only necessary to provide sufficient clash allowance (i.e. the difference in spring length between maximum load and spring solid positions) to allow for any possible combination of tolerance stack-up, differential thermal expansion and wear of parts. The usual recommendation is to provide a clash allowance of approximately 10% of the total spring deflection at maximum working load, [9,12], i.e.

\[ L_2 - L_s \geq 0.1\Delta \]  
\[ \Delta = L_F - L_2 \]

where:

\[ L_F = \frac{F_L}{K} + L_1 = \frac{F_u}{K} + L_2 \]  
\[ \Delta = \frac{F_u}{K} \]  
\[ L_2 = [N(1+A) + Q]d \]  
\[ L_s = (N+Q)d \]

\[ L_2 - L_s = NAd \]

Constraint G11 in normalized form is:

\[ G11 = K_{11}C^3d^{-2} \leq 1 \]

where:

\[ K_{11} = \frac{0.8F_u}{AG} \]
3. Monotonicity Analysis

In many engineering design problems, the objective or constraint functions may be strictly increasing (decreasing) with respect to some or all of the design variables. Such a function is said to be monotonic with respect to one such variable. The function's monotonicity with respect to this variable is designated as positive (negative). The purpose of monotonicity analysis is to eliminate combinations of constraints which can not be simultaneously active at the optimum. An inequality constraint is said to be active at the optimum when the inequality must be satisfied as a strict equality, i.e. deleting such a constraint from the problem would change the optimal solution. A constraint is said to be inactive when its deletion from the problem would not change the solution. For a problem written in normalized form,

\[
\begin{align*}
\min g_0(x) \\
\text{sub to} \\
G_i(x) &\leq 1 \quad i=1,\ldots,n \\
x &> 0
\end{align*}
\]

monotonicity analysis gives the following two rules for constraint activity [1]:

(a) When the objective function is increasing (decreasing) with respect to one of the variables, there should be at least one constraint with negative (positive) monotonicity with respect to the same variable which is active at the optimum.

(b) If the objective is independent of a given variable, then, either all constraints containing that variable can be deleted or at least two constraints with opposite monotonicities with respect to that variable are active.
3.1. Problem 1: Maximizing Reliability

The problem in normalized form is summarized as follows:

\[
\min g_0 = K_0 C^{-0.86} d^{(2+ A_1)}
\]

Subject to:

\[
\begin{align*}
G_1 &= K_1 d^2 C^{-1} \leq 1 & \text{Surging} \\
G_2 &= K_2 C^{-5} \leq 1 & \text{Buckling} \\
G_3 &= K_3 C^3 d^{-1} \leq 1 & \text{Min. Coils} \\
G_4 &= K_4 C \leq 1 & \text{Max. Index} \\
G_5 &= K_5 C^{-1} \leq 1 & \text{Min. Index} \\
G_6 &= K_6 d^2 C^{-3} + L_6 d \leq 1 & \text{Pocket Length} \\
G_7 &= K_7 C d + k_7 d \leq 1 & \text{Outside Diameter} \\
G_8 &= C^{-1} + K_8 C^{-1} d^{-1} \leq 1 & \text{Inside Diameter} \\
G_9 &= K_9 d^{-1} \leq 1 & \text{Lower Limit on} \ d \\
G_{10} &= K_{10} d \leq 1 & \text{Upper Limit on} \ d \\
G_{11} &= K_{11} C^3 d^{-2} \leq 1 & \text{Clash Allowance}
\end{align*}
\]

Note that both \(G_2\) and \(G_5\) provide lower bounds on \(C\), so let

\[
K_{25} \triangleq \max\{K_5, K_2^{1/5}\} \tag{74}
\]

and replace \(G_2\) and \(G_5\) by the single constraint

\[
G_{25} = K_{25} C^{-1} \leq 1 \tag{75}
\]

Table 2 expresses monotonicity as positive (+), negative (−) or independent (0) signs showing whether each function increases, decreases or is independent with respect to a given variable.
<table>
<thead>
<tr>
<th>$g_0$</th>
<th>$G1$</th>
<th>$G25$</th>
<th>$G3$</th>
<th>$G4$</th>
<th>$G6$</th>
<th>$G7$</th>
<th>$G8$</th>
<th>$G9$</th>
<th>$G10$</th>
<th>$G11$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$d$</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 2: Monotonicities for Problem 1

From the monotonicity of $C$ and $d$ in the table, at least one of the following constraints must be active at the optimum:

For $C$: $G1$, $G25$, $G6$, $G8$
For $d$: $G1$, $G6$, $G7$, $G10$

CASE 1: If $G1$ is active, i.e. $G1=1$ then $C=K_1d^2$.
Elimination of $C$ from the model gives the resulting monotonicities in Table 3:

<table>
<thead>
<tr>
<th>$g_0$</th>
<th>$G25$</th>
<th>$G3$</th>
<th>$G4$</th>
<th>$G6$</th>
<th>$G7$</th>
<th>$G8$</th>
<th>$G9$</th>
<th>$G10$</th>
<th>$G11$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 3: Monotonicities for Case 1 of Problem 1

Closed form solutions for this case are found by solving the following systems of combinations of active constraints:

$$(G1,G3),(G1,G4),(G1,G7),(G1,G10),(G1,G11)$$

and then checking for feasibility.

CASE 2: If $G1$ is inactive, i.e. $G1<1$, then $G6=1$, i.e.

$$C = \frac{K_6d^2}{1-L_6d}^{1/3} \quad (76)$$

Substituting $C$ in the objective and constraints, the resulting monotonicities in Table 4:

<table>
<thead>
<tr>
<th>$g_0$</th>
<th>$G25$</th>
<th>$G3$</th>
<th>$G4$</th>
<th>$G6$</th>
<th>$G7$</th>
<th>$G8$</th>
<th>$G9$</th>
<th>$G10$</th>
<th>$G11$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 4: Monotonicities for Case 2 of Problem 1

Therefore, the following combinations of constraints should be checked:
(G6,G3), (G6,G4), (G6,G7), (G6,G10), (G6,G11)

CASE 3: If G1 and G6 are inactive and G7 is active, i.e.

\[ G_1 \leq 1, \ G_6 \leq 1, \ G_7 = 1 \]

Then:

\[ C = \frac{1 - K_d}{K_7 d} \]  

(77)

Substituting C in the objective and constraints other than G1 and G6 gives the resulting monotonicities in Table 5 as follows:

<table>
<thead>
<tr>
<th>( g_0 )</th>
<th>G25</th>
<th>G3</th>
<th>G4</th>
<th>G8</th>
<th>G9</th>
<th>G10</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 5: Monotonicities for Case 3 of Problem 1

The following combinations of constraints should be checked:

\( (G7,G25), (G7,G8), (G7,G10) \)

CASE 4: If G1, G6, G7 are inactive and G10 is active, i.e.:

\[ G_1 \leq 1, \ G_6 \leq 1, \ G_7 \leq 1, \ G_{10} = 1 \]

Then:

\[ d = \frac{1}{K_{10}} = \text{Const.} \]  

(78)

The resulting monotonicity table is:

<table>
<thead>
<tr>
<th>( g_0 )</th>
<th>G25</th>
<th>G3</th>
<th>G4</th>
<th>G8</th>
<th>G9</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Tables 6: Monotonicities for Case 4 of Problem 1.

The following combinations of active constraints should be solved and checked for feasibility:

\( (G10,G25), (G10,G8) \)

The global optimum is thus found by comparing the following combinations of active constraints:
(G1,G3)  (G1,G10)  (G6,G4)  (G6,G11)  (G7,G10)
(G1,G4)  (G1,G11)  (G6,G7)  (G7,G25)  (G10,G25)
(G1,G7)  (G6,G3)  (G6,G10)  (G7,G8)  (G10,G8)

In practice, the fifteen systems of equations are solved analytically, the results are checked for feasibility and then they are compared to determine the global optimum.

3.2. Problem 2: Maximizing Energy Storage Capacity

The problem in normalized form is

\[
\text{Min } g_0^E = K_{01} C^{-2} d^4
\]

Subject To:

\[
G = K_0 C \cdot 86 d^{-(2+A_1)} \leq 1
\]

Strength

\[
G_1 = K_1 d^2 C^{-1} \leq 1
\]

Surging

\[
G_{25} = K_{25} C^{-1} \leq 1
\]

Buckling & Min Index

\[
G_3 = K_3 C^3 d^{-1} \leq 1
\]

Min Coils

\[
G_4 = K_4 C \leq 1
\]

Max Index

\[
G_6 = K_6 d^2 C^{-3} + L_6 d \leq 1
\]

Pocket Length

\[
G_7 = K_7 C d + K_7 d \leq 1
\]

Outside Diameter

\[
G_8 = C^{-1} + K_8 C^{-1} d^{-1} \leq 1
\]

Inside Diameter

\[
G_9 = K_9 d^{-1} \leq 1
\]

Lower Limit on d

\[
G_{10} = K_{10} d \leq 1
\]

Upper Limit on d

\[
G_{11} = K_{11} C^3 d^{-2} \leq 1
\]

Clash Allowance
The monotonicity table for this model is:

<table>
<thead>
<tr>
<th>$g^E_0$</th>
<th>G</th>
<th>G1</th>
<th>G25</th>
<th>G3</th>
<th>G4</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
<th>G10</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 7: Monotonicities for Problem 2

At least one of the following constraints must be active at the optimum:

For C: G, G3, G4, G7, G11
For d: G, G3, G8, G9, G11

CASE 1: If G is active, i.e. $G \leq 1$ then:

$$d = K_0 C^{2+A1}$$ (79)

Substituting for $d$ in the model of Problem 2 gives the resulting monotonicities in Table 8:

<table>
<thead>
<tr>
<th>$g^E_0$</th>
<th>G1</th>
<th>G25</th>
<th>G3</th>
<th>G4</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
<th>G10</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 8: Monotonicities for Case 1 of Problem 2

The monotonicity of C for constraint G6 is not known. So the (G,G6) combination is also taken into account. Closed form solutions for this Case are found by solving the following combinations of active constraints:

(G,G3), (G,G4), (G,G6), (G,G7), (G,G10), (G,G11)

CASE 2: If G is inactive i.e. $G<1$, and G3 is active, G3=1 so that

$$d = K_3 C^3$$ (80)
<table>
<thead>
<tr>
<th>$g_E^0$</th>
<th>G1</th>
<th>G25</th>
<th>G4</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
<th>G10</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 9: Monotonicities for Case 2 of Problem 2

Closed form solutions for this case are found by solving the combinations

(G3,G25), (G3,G8), (G3,G9), (G3,G11)

CASE 3: If G and G3 are inactive and G4 active, i.e.

G<1  G3<1  G4≤1

then

$$C = \frac{1}{K_4} = \text{Const.} \quad (81)$$

The resulting monotonicity table is:

<table>
<thead>
<tr>
<th>$g_E^0$</th>
<th>G1</th>
<th>G25</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
<th>G10</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 10: Monotonicities for Case 3 of Problem 2

Closed form solutions for this case are found by solving the combinations

(G4,G8), (G4,G9), (G4,G11)

CASE 4: If G, G3, G4 are inactive, but G7 is active; i.e.

G<1  G3<1  G4<1  G7≤1

then

$$d = \frac{1}{K_7C+K_7} \quad (82)$$

The resulting monotonicity table is:
Table 11: Monotonicities for Case 5 of Problem 2

Closed form solutions for this case are found by solving the combinations:

(G7,G1), (G7,G9), (G7,G11)

CASE5: If G, G3, G4 and G7 are inactive but G11 is active:

\[ G < 1 \quad G3 < 1 \quad G4 < 1 \quad G7 < 1 \quad G11 \leq 1 \]

then

\[ d = K_{11}^5 \cdot 1.5 \]  \hspace{1cm} (83)

The resulting monotonicities are:

Table 12: Monotonicities for Case 5 of Problem 2

Closed form solutions for this case are found by solving:

The combinations:

(G11,G25), (G11,G8), (G11,G9)

Therefore; the global optimum is found for Problem 2, by comparing the solutions for the following combinations of active constraints

(G,G3) (G,G7) (G3,G25) (G3,G8) (G4,G7) (G4,G11) (G7,G11)
(G,G4) (G,G10) (G3,G4) (G3,G9) (G4,G8) (G7,G1) (G11,G25)
(G,G6) (G,G11) (G3,G7) (G3,G11) (G4,G9) (G7,G9) (G11,G8)
(G11,G9)

Note that all combinations have been considered. In practice, the twenty-two systems of equations are solved in closed form, the results are checked for feasibility and then compared to determine the global optimum.
3.3 Problem 3: Maximizing Natural Frequency

The mathematical model is the same as problem 2, without constraint G1 while the objective function now becomes

$$\min g_0^f = K_0^f C^{-1} d^2$$

(84)

the monotonicities of the objective with respect to C and d are the same as for the objective of Problem 2. Therefore; the global optimum is found for Problem 3, by comparing the solutions for the following combinations of active constraints:

(G,G3) (G,G11) (G3,G8) (G4,G8) (G7,G11)
(G,G4) (G3,G25) (G3,G9) (G4,G9) (G11,G25)
(G,G6) (G3,G4) (G3,G11) (G4,G11) (G11,G8)
(G,G7) (G3,G7) (G4,G7) (G7,G9) (G11,G9)
(G,G10)

3.4 Problem 4: Minimizing Total Weight

The constraints are the same as in Problem 2, but the objective function now becomes:

$$\min W = W_1 C^{-2} d^4 + W_2 C d^3$$

(85)

where $W_1$ and $W_2$ are constants. The monotonicity table in this case is:

<table>
<thead>
<tr>
<th>W</th>
<th>G</th>
<th>G1</th>
<th>G25</th>
<th>G3</th>
<th>G4</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
<th>G10</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>?</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 13: Monotonicities for Problem 4

The objective function is only monotonic wrt d, therefore; at least one of the following constraints must be active:

For d: G, G3, G8, G9, G11

CASE 1: If G is active, i.e. G ≤ 1

$$\therefore d = \left(\frac{1}{K_0^{2+A_1}}\right) \frac{.86}{C^{2+A_1}}$$

(86)
Substituting for d in the objective the resulting function is still non-monotonic with respect to C:

\[
W = \frac{-.56-2A_1}{2+A_1} C^{\frac{4.58+A_1}{2+A_1}} \quad (87)
\]

where:

\[
K' = K_0^{\frac{1}{2+A_1}} \quad (88)
\]

and

\[
A_1 = (-.14, -.19, -.17, -.11) \quad (89)
\]

for the four chosen materials.

The stationary point of the objective (87) is

\[
C^*_0 = \left( \frac{.56+2A_1}{W_2} \right) \frac{2+A_1}{5.14+3A_1} \quad (90)
\]

Then, lower and upper limits are found for C by substituting equation (86) for d in the constraints. For example,

\[
G1 = K_1d^2C^{-1} \leq 1 \quad (91)
\]

becomes

\[
C \geq C_1 \quad (92)
\]

where \(C_1\) is the solution for active \(G1\), i.e. \(G1=1\):

\[
C_1 = \left( K_1K' \right)^{\frac{2+A_1}{.28+A_1}} \quad (93)
\]

For constraint \(G6\), substitution for \(d\) results to a nonmonotonic function with respect to \(C\):

\[
G6 = \frac{-4.28-3A_1}{2+A_1} C^{\frac{.86}{2+A_1}} + \frac{L_6K'}{2+A_1} \leq 1 \quad (94)
\]
The stationary point for G6 is found to be:

\[
C_{+6} = \frac{\frac{2+A_1}{K_6 K' \left(4.28+3A_1\right)}}{\frac{5.14+3A_1}{0.86 L_6}}
\]

(95)

and

\[
C_{6L} \leq C \leq C_{6U}
\]

(96)

where \(C_{6L}\) and \(C_{6U}\) are lower and upper bounds on \(C\) which can be found by the Newton-Raphson Method for \(G6=1\). Note that, initial guess for \(C_{6L}\) should be less than \(C_{+6}\), and more than \(C_{+6}\) for \(C_{6U}\).

Therefore, the optimum solution for Case 1 is found by:

\[
CM = C_{\text{Min}} = \text{Max}\{C_1, C_{25}, C_{6L}, C_8, C_9\}
\]

(97)

\[
CN = C_{\text{Max}} = \text{Min}\{C_3, C_4, C_{6U}, C_7, C_{10}, C_{11}\}
\]

(98)

Note that:

1. If \(C_{\text{Min}} > C_{\text{Max}}\) Then No Feasible Solution
2. If \(C_{\text{Min}} = C_{\text{Max}}\) Then \(C^* = C_{\text{Min}} = C_{\text{Max}}\)
3. If \(C_{+} < C_{\text{Min}} < C_{\text{Max}}\) Then \(C^* = C_{\text{Min}}\)
4. If \(C_{\text{Min}} < C_{+} < C_{\text{Max}}\) Then \(C^* = C_{+}\)
5. If \(C_{\text{Min}} < C_{\text{Max}} < C_{+}\) Then \(C^* = C_{\text{Max}}\)

where \(C^*\) is the optimum solution for this case.

CASE 2: If \(G3\) is active, i.e. \(G3=1\), and \(G\) inactive, i.e. \(G<1\), then

\[
d = K_3 C^3
\]

(99)
The resulting monotonicity table is:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>G1</th>
<th>G25</th>
<th>G4</th>
<th>G5</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
<th>G10</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 14: Monotonicities for Case 2 of Problem 4

Close form solutions for this case are found by solving the combinations

\((G3,G25), (G3,G8), (G3,G9), (G3,G11)\)

**CASE 3:** If \(G\) and \(G3\) are inactive, but \(G8\) is active, i.e.

\[G < 1 \quad G3 < 1 \quad G8 \leq 1\]

then

\[C = \frac{d+k_8}{d}\]  \hspace{1cm} (100)

After substitution in the constraints, the resulting table is:

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>G1</th>
<th>G25</th>
<th>G4</th>
<th>G6</th>
<th>G7</th>
<th>G9</th>
<th>G10</th>
<th>G11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 15: Monotonicities for Case 3 of Problem 4

Closed form solutions for this case are found by solving. The combinations:

\((G8,G4), (G8,G9), (G8,G11)\)

**CASE 4:** If \(G, G3, G8\) are inactive, but \(G9\) is active, then

\[d = K_9\]  \hspace{1cm} (101)

again, the resulting objective is nonmonotonic wrt \(C\):

\[W = (W_1 K_9^4) C^{-2} + (W_2 K_9^3) C\]  \hspace{1cm} (102)
with stationary point at

\[ (C_+) g = \left( \frac{2w_{1}K_g}{W_2} \right)^{1/3} \]  \hspace{1cm} (103)

By substituting \( d \) in the constraints, lower and upper limits are found for \( C \), where:

\[ (CM)_g = (C_{Min})_g = \text{Max}\{C_1, C_{25}, C_6, C_8, C_{11}\}_g \]  \hspace{1cm} (104)

\[ (CN)_g = (C_{Max})_g = \text{Min}\{C_3, C_4, C_7, C_G\}_g \]  \hspace{1cm} (105)

Then, the same procedure as in case 1 should be followed.

CASE 5: If \( G, G_3, G_8, G_9 \) are inactive, but \( G_{11} \) is active, i.e. \( G_{11}=1 \) then

\[ d = K_{11}^{5} C^{1.5} \]  \hspace{1cm} (106)

The resulting monotonicity table is:

<table>
<thead>
<tr>
<th>W</th>
<th>G1</th>
<th>G25</th>
<th>G4</th>
<th>G6</th>
<th>G7</th>
<th>G10</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 16: Monotonicities for Case 5, Problem 4

Closed form solution for this case is found by solving the combination

\((G_{11}, G_{25})\)

Therefore, for problem 4, including Cases 1 and 4 the following systems of equations should be solved and compared for global optimality:

\((G_3, G_{25}), (G_3, G_8), (G_3, G_{11}), (G_8, G_4), (G_8, G_{11}), (G_{11}, G_{25})\)

Note that the following combinations have been considered already in case 4:

\((G_3, G_9), (G_8, G_9)\)
4. **COMPUTER SOLUTION**

The optimization procedure in the program is the following: for each objective and for a chosen material, the program solves all the sets of equations determined by monotonicity analysis. Solutions are then checked for feasibility. If feasible, the objective functions are calculated and compared to locate the global optimum. When the strength constraint G is active, the program calls a subroutine to determine whether fatigue or yielding is the governing criterion.

A flow chart and a program listing is in appendix C. The program is interactive, written in basic for a TRS-80 microcomputer. The amount of memory used for each run is approximately 16K.

5. **DISCUSSION**

Monotonicity analysis has proven useful in identifying which combinations of constraints might be active at an optimum. Since all local optima can be generated, as in this problem, finding the global optimum is guaranteed.

The tabulated results given in Appendix B were generated by 28 computer runs for the unique input data of Table 17. It is found that in the case of maximizing reliability, the results are in agreement with the data of Mancini [3]. In the case of maximizing natural frequency, i.e. K=3, and minimizing weight, i.e. K=4, the results are not always close or the same (Table 19). For example, in case 1, where an infeasible solution exists for the weight objective, the natural frequency objective does give optimum dimensions. In other cases, the results are close and even the same as in Case 6 (Table 19). The reason for the apparent discrepancy is that when maximizing natural frequency, only the active coils are taken into account, while when minimizing weight, the total weight (active plus inactive coils) is minimized. In terms of the solution procedure this causes the objective in the weight minimization case to be non-monotonic in C and results in a different procedure and constraint activity. From Table 19 it is seen that the best results among the seven cases are obtained
from Case 6, where the largest maximum is found for K=3 and 
the least minimum for K=4. The input data is the one of Table 
15.

As a final note, this program can be used for a variety of 
helical compression spring optimization problems having constraint 
limitations different from the ones originally assumed. For 
example when there is no limit on inside diameter (ID), we 
arbitrarily choose a very small false value such as ID=10⁻³. 
Or as another example in static applications, number of cycles 
to failure is chosen to be 10³ (lower limit on the parameter). 
An example from reference [12] is chosen to explain this further. 
The program is run for three different objectives namely, maximum 
reliability, maximum energy storage capacity and maximum natural 
frequency. The input data and output results are shown in Table 
21. Comparing the results with the one of [12], in case of maximum 
reliability the spring is bigger in size and with a high safety 
factor, i.e., 3.17. However, in other two cases it has smaller 
wire diameter and approximately half of the active coils of [12]. 
Although mean diameter is a little larger (about .05 inch), the 
spring is considerably smaller and therefore economical.
REFERENCES


Appendix A: Nomenclature and Definitions

\( q_0 \) = Objective function

\( G, G_m (m=1 \text{ to } 11) = \text{Constraints} \)

\[
K_{01}^F = 2.04 \left( \frac{F_u - F_L}{C_1 (NC) B_1} \right) + \frac{F_u + F_L}{C_2}
\]

\[
K_{01}^V = \frac{4.07 (F_u)}{C_3}
\]

\[
K_{01}^E = (10.24G) \left( \frac{n}{12.8 F_u} \right)^2
\]

\[
K_{01}^F = \frac{G i}{112800 (F_u - F_L)}
\]

\[
w_1 = \frac{\rho \pi^2 G}{32K}
\]

\[
w_2 = \frac{\rho^2}{4} Q
\]

\[
K_0 = K_{01}^F \cdot (SF)
\]

\[
K_0^V = K_{01}^V \cdot (SF)
\]

\[
K_1 = K_{01}^F \cdot (f)
\]

\[
K_2 = \frac{GF_u (1+A)}{22.3K^2}
\]

\[
K_3 = \frac{8PcN_{\text{min}}}{G}
\]

\[
K_4 = \frac{1}{F_u}
\]
\[ K_5 = \frac{I_L}{L} \]

\[ K_6 = \frac{G(1+A)}{8PcL_m} \]

\[ L_6 = \frac{Q}{L_m} \]

\[ K_7 = \frac{1}{OD} \]

\[ K_8 = ID \]

\[ K_{11} = \frac{0.8 \text{ Fu}}{AG} \]

Where:

- \( A \) = Dimensionless clearance constant; decimal percentage of wire diameter between adjacent coils under maximum force.
- \( C \) = Spring Index, i.e. \( C = \frac{D}{d} \), design variable
- \( C_L \) = Load factor
- \( C_S \) = Surface factor
- \( d \) = Wire diameter, in, design variable
- \( D \) = Average coil diameter, in
- \( f \) = Minimum allowable natural frequency, Hz
- \( F_n \) = Fundamental natural frequency, CPS
- \( \text{Fu} \) = Maximum force, lb
- \( \text{F_L} \) = Minimum force, lb
- \( F_a \) = Alternative force, lb
- \( F_m \) = Mean force, lb
- \( G \) = Shear modulus, psi
\[ I_u = \text{Maximum allowable spring index} \]
\[ ID = \text{Minimum allowable inside diameter, in} \]
\[ I_L = \text{Minimum allowable spring index} \]
\[ k = \text{spring rate, lb/in} \]
\[ K_S = \text{Wahl factor} \]
\[ L = \text{Length of spring (active), in} \]
\[ L_1 = \text{Spring length at maximum load, in} \]
\[ L_2 = \text{Spring length at minimum load, in} \]
\[ L_S = \text{Solid length, in} \]
\[ L_F = \text{Free Length, in} \]
\[ L_m = \text{Maximum spring length at maximum force, in} \]
\[ N = \text{Number of active coils} \]
\[ N_{\text{Min}} = \text{Minimum allowable number of coils} \]
\[ NC = \text{Number of cycles to failures} \]
\[ OD = \text{Maximum allowable outside diameter, in} \]
\[ Q = \text{Number of inactive coils} \]
\[ S_e = \text{Endurance limit, psi} \]
\[ S_{ns} = \text{Fatigue strength, psi} \]
\[ S_u = \text{Ultimate strength, psi} \]
\[ S_{us} = \text{Ultimate shear strength, psi} \]
\[ S_Y = \text{Yield strength, psi} \]
\[ S_{YS} = \text{Shear yield strength, psi} \]
\[ SF = \text{Safety factor} \]
\[ T_a = \text{Alternating shear stress, psi} \]
\[ T_m = \text{Average shear stress, psi} \]
\[ W = \text{Total weight of spring, i.e. active and inactive, lb} \]
\[ \rho = \text{Density, lb/\text{in}^3} \]
\[ \Delta = \text{Spring deflection, in} \]
\[ \lambda = \text{Pitch Angle} \]
### Appendix B: Program Results for Several Applications

<table>
<thead>
<tr>
<th>$F_U$</th>
<th>$F_L$</th>
<th>$\Delta$</th>
<th>$F$</th>
<th>$N_L$</th>
<th>$I_U$</th>
<th>$I_L$</th>
<th>OD</th>
<th>ID</th>
<th>$L_m$</th>
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<td>18</td>
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<td>.75</td>
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<td>10</td>
<td>.25</td>
<td>500</td>
<td>3</td>
<td>20</td>
<td>4</td>
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<td>.75</td>
<td>1.25</td>
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<td>.75</td>
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<td>4</td>
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<td>.75</td>
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<td>$10^6$</td>
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<td>.75</td>
<td>1.25</td>
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Table 17: Input data


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<tr>
<th>d</th>
<th>d</th>
<th>D</th>
<th>D</th>
<th>C</th>
<th>N</th>
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<th>SF</th>
<th>SF</th>
<th>Active Constraints</th>
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<td>.096</td>
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<td>.952</td>
<td>9.87</td>
<td>3</td>
<td>1.22</td>
<td>1.27</td>
<td>1.26 (1,3)</td>
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<tr>
<td>2</td>
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<td>.142</td>
<td>1.36</td>
<td>1.36</td>
<td>9.56</td>
<td>4.86</td>
<td>1.87</td>
<td>2.69</td>
<td>2.64 (6,7)</td>
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<tr>
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<td>.142</td>
<td>1.36</td>
<td>1.36</td>
<td>9.56</td>
<td>4.86</td>
<td>1.87</td>
<td>2.69</td>
<td>2.63 (6,7)</td>
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<tr>
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<td>.090</td>
<td>.917</td>
<td>.917</td>
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<td>3</td>
<td>1.1</td>
<td>1.52</td>
<td>1.51 (1,3)</td>
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<tr>
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<td>.073</td>
<td>.829</td>
<td>.829</td>
<td>11.3</td>
<td>3</td>
<td>1.7</td>
<td>.68</td>
<td>.67 (1,3)</td>
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<td>1.64</td>
<td>1.62</td>
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</tbody>
</table>

Infeasible

Table 18: Solutions for maximizing reliability, compared with Mancini [3]. Spring material: Music wire type of ends: Squared and Ground, i.e. Q=2.

---

B/1
<table>
<thead>
<tr>
<th>d</th>
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<td>K=3 K=4</td>
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<td>K=3 K=4</td>
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<td>K=3 K=4</td>
<td>K=3 K=3</td>
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<tr>
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<td>1.4 .85</td>
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<td>1.4 1.5</td>
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<td>1.4 .85</td>
<td>10.0 8.6</td>
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<tr>
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<td>1.1 .96</td>
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<td>9.6 9.6</td>
<td>3 3</td>
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<td>587 .02</td>
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<td>1.4 -</td>
<td>10.8 -</td>
<td>3 -</td>
<td>1.4 -</td>
<td>316 -</td>
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</tbody>
</table>

Table 19: Solutions for maximizing natural frequency (K=3) and minimizing weight (K=4) spring material: Music-wire type of ends: Squared & ground, i.e. Q=2, SF=1.5

<table>
<thead>
<tr>
<th>d</th>
<th>D</th>
<th>C</th>
<th>N</th>
<th>L_F</th>
<th>OBJ</th>
<th>Active Constraints</th>
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</thead>
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<td>.1</td>
<td>1.0</td>
<td>10</td>
<td>3</td>
<td>1.1</td>
<td>126</td>
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<td>.8</td>
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<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Infeasible</td>
</tr>
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<td>9.6</td>
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<td>.83</td>
<td>87</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Infeasible</td>
</tr>
</tbody>
</table>

Table 20: Solutions for maximizing energy storage (K=2) spring material: Chrome-Silicon type of ends: Plain, i.e. Q=.5 SF=1.2
Given [12]: Squared and ground compression spring to work in a
$D_H=1.5$ in. hole and exert $P_1=60$ lb at a length of
$L_1=2.5$ in. and $P_2=105$ lb at $L_2=2.0$ in.

Application: Static at room temperature.

Material: oil, tempered wire.

<table>
<thead>
<tr>
<th>$F_u$</th>
<th>$F_L$</th>
<th>$\Delta$</th>
<th>$f$</th>
<th>$N_L$</th>
<th>$I_u$</th>
<th>$I_L$</th>
<th>OD</th>
<th>ID</th>
<th>$L_m$</th>
<th>NC</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>60</td>
<td>.5</td>
<td>1E-3</td>
<td>3</td>
<td>20</td>
<td>4</td>
<td>1.5</td>
<td>1E-3</td>
<td>1E4</td>
<td>1E3</td>
<td>.4</td>
</tr>
</tbody>
</table>

Table 21: Input data and output results for example of Ref. [12].

Note: Safety factor = 1.5 for K=2 and 3
Appendix C: Flow Chart, List of Programs, Output Sample.

COMPUTER PROGRAM FLOW CHART

What is the objective?
K=1: Maximum Reliability
K=2: Maximum Energy Storage Capacity
K=3: Maximum Natural Frequency
K=4: Minimum Weight

What is the material of spring?
PP=1: Music Wire
PP=2: Oil-Tempered Wire
PP=3: Chrome-Silicon
PP=4: Chrome-Vanadium

INPUT DATA

K=1
K=2
K=3
K=4
Solve the Following Equations:
(1,3), (1,4), (1,7),... : I = 1-15
I = 1    I = 2    I = 3

Find Maximum Objective
I = 1-15

Max = 0?

Yes

No Feasible Solutions

No

Print Optimum Dimensions

End

\[
\frac{t_A}{t_M} > \ldots?
\]

No

Yielding:
Calc. Objective

Yes

Fatigue:
Calc. Objective

Return

Feasible Solutions?

No
For K=2: Solve the following equations:
\((G,3),(G,4),(G,6L),...\) \(I=1-23\)

For K=3: The same as K=2
Except \((7,1): I=1-22\)

I=1: Find Max \((C1,C25,C6L,C8,C9)\) and Min\((C3, C4,C60,C7,C10)\) and compare with stationary point for \(G=1\)

I=2: The same when \(G9=1\)

Solve the following Eqs:
\((3,25),(3,8),(4,8),... I=3-10\)

Check if fatigue or yielding?
CLSCLEAR

* OPTIMAL DESIGN OF A HELICAL COMPRESSION SPRING *
* BY: SHAPOUR AZARIN WINTER 1981 *

PRINT "THIS PROGRAM FINDS OPTIMUM DIMENSIONS OF A HELICAL COMPRESSION SPRING WITH FOUR DIFFERENT OBJECTIVES."
PRINT "ENTER 1 IF YOU WANT TO MAXIMIZE RELIABILITY; ENTER 2 IF YOU WANT TO MAXIMIZE ENERGY STORAGE; ENTER 3 IF YOU WANT TO MAXIMIZE NATURAL FREQUENCY; ENTER 4 IF YOU WANT TO MINIMIZE WEIGHT."
PRINT "WHAT IS YOUR OBJECTIVE?"; K
PRINT "FOR THE MATERIAL OF SPRING ENTER 1 FOR MUSIC WIRE ENTER 2 FOR OIL-TEMPERED WIRE ENTER 3 FOR CHROME VANADIUM ENTER 4 FOR CHROME-VANADIUM.";
INPUT "WHAT IS THE MATERIAL OF SPRINGS"; PP
INPUT "ENTER MAXIMUM SPRING FORCE;"; FU
INPUT "ENTER MINIMUM SPRING FORCE;"; FL
INPUT "ENTER SPRING DEFLECTION IN "; DE
INPUT "ENTER MINIMUM ALLOWABLE NATURAL FREQUENCY; AT LEAST 13 TIMES FREQUENCY OF APPLIED FORCE, HZ"; F1
INPUT "ENTER MINIMUM ALLOWABLE NUMBER OF ACTIVE COILS; INL >= 3"; IL
INPUT "ENTER MAXIMUM ALLOWABLE SPRING INDEX; IU <= 20"; IU
INPUT "ENTER MINIMUM ALLOWABLE OUTER DIAMETER; INCH"; OD
INPUT "ENTER MINIMUM ALLOWABLE INSIDE DIAMETER; INCH"; ID
INPUT "ENTER MAXIMUM ALLOWABLE SPRING LENGTH AT MAXIMUM FORCE, INCH"; LM
INPUT "ENTER NUMBER OF CYCLES TO FAILURE; E3 <= NC <= E6"; HC
INPUT "ENTER DIMENSIONLESS CLEARANCE CONSTANT; DECIMAL PERCENTAGE OF WIRE DIAMETER BETWEEN AN ADJACENT COILS UNDER MAXIMUM FORCE, A"

G = 11.5E6
B = C(25); D(25); OB(25); R(12)
P = 1080000
Ks = FU/FL
K1 = G*K1/(12800*Ks)
K2 = (G*KFU*(1+A)/(22.3*KSC(2)))*C,2
IF K2 > IL THEN K2 = K2 ELSE K2 = IL
K3 = 6*Ks*Ks/G
K4 = 1/IU
K5 = G*(1+A)/(6*Ks*K)
L6 = G/LH
K7 = 1/OD
K8 = -G*KU/(A*K)
IF PP = 1 THEN GOTO 166
IF PP = 2 THEN GOTO 167
IF PP = 3 THEN GOTO 168
IF PP = 4 THEN GOTO 169
C1 = 630500; A1 = -1.4; B1 = -2.137; C2 = 16E4; C3 = 86550; K7 = 0.04; KL = 4; GOTO 170
C1 = 3723; A1 = -1.17; B1 = -1.8455; C2 = 115200; C3 = 62500; K9 = 0.09; KL = 2; GOTO 170
C1 = 630500; A1 = -1.11; B1 = -2.137; C2 = 16E4; C3 = 86550; K9 = 0.063; KL = 2.667; GOTO 170
C1 = 433650; A1 = -1.17; B1 = -1.8455; C2 = 135200; C3 = 73200; K9 = 0.032; KL = 2.288; GOTO 170
IF K = 1 THEN GOTO 430
GOTO 960

' THIS SUBROUTINE CHECKS FEASIBILITY OF SOLUTIONS

LPFPRINT "**SUBROUTINE CHECKS FEASIBILITY OF SOLUTIONS**";
IF DI(I) <= 0 THEN LPFPRINT "D<0 FOR I="; I1R(I) = I1
IF CI(I) <= 0 THEN LPFPRINT "C<0 FOR I="; I2R(I) = I2
IF K < 3 AND (K1D(I)C2-C1(I))(0.05) THEN LPFPRINT "SURFACE CONSTRAINTS OF ITS VIOLATED FOR I="; I3R(I) = I3
IF K=1 THEN LPRINT "DESIGN OBJECTIVE:MAXIMIZING RELIABILITY";GOTO 730
IF K=2 THEN LPRINT "DESIGN OBJECTIVE:MAXIMIZING ENERGY STORAGE CAPACITY";GOTO 730
IF K=3 THEN LPRINT "DESIGN OBJECTIVE:MAXIMIZING NATURAL FREQUENCY";GOTO 730
IF K=4 THEN LPRINT "DESIGN OBJECTIVE:MINIMIZING WEIGHT"
730 IF PP=1 THEN LPRINT "SPRING MATERIAL IS MUSIC WIRE(1085)";GOTO 734
IF PP=2 THEN LPRINT "SPRING MATERIAL IS OIL-TEMPERED WIRE(AISI 1066)";GOTO 734
IF PP=3 THEN LPRINT "SPRING MATERIAL IS CHROME-SILICON(AISI 9254)";GOTO 734
IF PP=4 THEN LPRINT "SPRING MATERIAL IS CHROME-VANADIUM(AISI 6150)"
734 IF G=0.5 THEN LPRINT "TYPE OF ENDS:PLAIN END, NO. OF INACTIVE COILS=0.5";GOTO 741
IF G=1 THEN LPRINT "TYPE OF ENDS:SPLIT AND GROUND, NO. OF INACTIVE COILS=1";GOTO 741
IF G=2 THEN LPRINT "TYPE OF ENDS:SQUARED OR SQUARED AND GROUND, NO. OF INACTIVE COILS=2"
741 LPRINT "DATA:

1. MAX FORCE="FU","MIN FORCE="FL
2. DEFORMATION="DE","MIN NATURAL FREQ."="FI
3. OUTSIDE DIAM."="OD","INSIDE DIAM."="ID
4. MAX SPRING INDEX="IU","MIN SPRING INDEX="IL
5. MIN NO. OF ACTIVE COILS="NC","CLEARANCE="A
6. MAX LENGTH AT MAX FORCE="LN","NO. OF CYCLES TO FAILURE="NC
7. IF K>1 THEN LPRINT "SAFETY FACTOR="SF
8. IF K>3 THEN RETURN
9. IF MAX=0 THEN LPRINT "NO FEASIBLE SOLUTION FOR INPUT DATA"; END
10. LPRINT "OUTPUT DATA:"
11. HO=C(J)(K(J))#H=G#B(J)/(#S*K#(C(J)))/3)
12. FL=(K(K(A)+K(D(J)))/DEFFL)/DE/(FU-FL)
13. LPRINT "WIRE DIAM., IN=",D(J),"SPRING INDEX="C(J)
14. LPRINT "MAX COIL DIAM., IN=",MD,"NO. OF ACTIVE COILS="N
15. IF K>3 THEN RETURN
16. LPRINT "FREE LENGTH, IN="FL,"OPTIMUM VALUE OF OBJECTIVE="MAX
17. END
18. LF=1.6*C(I)*(I-0.14)
19. TH=4*(FU-FL)*C(I)*UF/(3.14*B(I)*C)
20. TH=4*(FU-FL)*C(I)*UF/(3.14*B(I)*C)
21. SHS=C10D1C1A1C1B1
22. SUS=C2D1C1A1
23. S1S=C1D1C1A1RETURN
VIOLATION *(TA/S)+TH/S-US-I/SF);RETURN
661 GOTO 911
670 IF K=2 THEN GSUB 1870:RETURN
680 IF K=3 THEN GSUB 2040:RETURN
690 IF K=4 THEN GSUB 2770:RETURN
700 IF K=1 THEN OBJ(1)=SYS/(TA+TK):RETURN
710 IF (TA+TM-SYS/SF);.05 THEN LPRINT "YIELD STRENGTH CONSTRAINT (G) IS VIOLATED FOR I=";"G VIOL
720 FOR Y=1 TO 12
730 IF C(Y)=1 THEN RETURN
740 NEXT
750 GOTO 670
760 INPUT "ENTER SAFETY FACTOR">=1,2 ;SF:GOTO 996
770 K=2.04*SF*{(FU-FL)/(C1*KCCB1)+(FU+FL)/C2):RETURN
780 K1=KOC(1/C2+1)
790 P1=(2+P1)/66
800 F2=KOC(-1/66):RETURN
810 IF C1<0 OR D1<0 THEN GOTO 978
820 GSUB 780:IF I1<>I AND (TA/TM)*(SHS/(SYS-SUS))/(SUS*(SHS-SYS)) THEN GOTO 973
830 IF I1=1 AND (TA/TM)*(SHS/(SYS-SUS))/(SUS*(SHS-SYS)) THEN GOTO 976
840 IF K<4 THEN GOTO 190 ELSE RETURN
850 IF I1<>I THEN K0=4.07*FU*SF/C3:I1=I:IF K<4 THEN GOTO 991 ELSE GOTO 2800
860 IF K<4 THEN GOTO 190 ELSE C(I)=-1
870 RETURN
880 IF I1=1 THEN GOTO 1030
890 IF I1=2 THEN GOTO 1055
900 IF I1=3 THEN GOTO 1063
910 IF I1=4 THEN GOTO 1322
920 IF I1=5 THEN GOTO 1330
930 IF I1=15 AND I2<1 THEN GOTO 1690
940 IF I1=16 THEN GOTO 1710
950 IF I1=17 THEN GOTO 1720
960 IF I1=18 THEN GOTO 1740
970 IF I1=15 AND I2=1 THEN GOTO 1920
980 IF K>3 THEN GOTO 2320 ELSE GOTO 1023
990 '*******************
1000 'SUBROUTINE FOR G3-1
1010 '*******************
1020 K(I)=K3*C(I):GOTO 974
1030 IF K>3 THEN RETURN
1040 '*******************
1050 'SUBROUTINE FOR G-1
1060 '*******************
1070 K(I)=K1*C(Y)/(1-P1);GOTO 974
1080 GSUB 970
1090 I=1:GOSUB 971
1100 C(I)=(K3/KP)*C(P1/(1-3*P1)):IF I=I THEN GOTO 1000
1110 GSUB 1000
1120 IF K>3 THEN RETURN
1130 GSUB 970
1140 I=2:GOSUB 971
1150 C(I)=1/K4
1160 IF I=I THEN GOTO 1020
1170 GSUB 1020:IF K>3 THEN RETURN ELSE GSUB 970
1180 I=3:GOSUB 971:IF I=I THEN GOTO 1070
1190 GSUB 1070:GOTO 1321
1200 '*******************
1210 'SUBROUTINE (G-6)
1220 '*******************
1230 Z=(4.24*3*A1)/6*P2C(-3)/(.86*L6))C(1.86/(5.14+3*A1))
1240 D(i)=.001
1250 Y=I+1
1140 IF D(I)<0 THEN GOTO 974
1145 IF ABS(Y1/Y2)>0.0001 THEN GOTO 1100 ELSE GOTO 1250
1153 D(I)=Z+1*GOTO 1100
1167 '***************
1170 'SUBROUTINE (G,7)
1171 '***************
30 D(I)=1
1190 X1=D(I)
1200 Y1=K7*(P2)*X1*(P1+1)+K7*X1-1
1210 Y2=(P1+1)*K7*P2*X1*(P1+K7
1220 D(I)=X1-Y1/Y2
1225 IF D(I)<0 THEN GOTO 974
1230 IF ABS(Y1/Y2)<0.0001 THEN GOTO 1250 ELSE GOTO 1190
1235 '***************
1240 'SUBROUTINE FOR CALC. C(I) WHEN G=1
1244 '***************
1250 C(I)=F2*D(I)*P1*GOTO 974
1259 '***************
1260 'SUBROUTINE (G,8)
1261 '***************
1270 D(I)=2
1280 X1=D(I)
1290 Y1=P2*X1*(P1+1)-X1-ID
1300 Y2=(P1+1)*P2*X1*(P1-1
1310 D(I)=X1-Y1/Y2
1315 IF D(I)<0 THEN GOTO 974
1320 IF ABS(Y1/Y2)<0.0001 THEN GOTO 1250 ELSE GOTO 1280
1321 GOSUB 970
1322 I=4:GOSUB 971:IF I=1 THEN GOTO 1153
1323 GOSUB 1153
1324 GOSUB 970
1329 I=5:GOSUB 971:IF I=1 THEN GOTO 1170
1331 GOSUB 1170
1334 I=6
1335 C(I)=K25:D(I)=K3*C(I)+3:IF K>3 THEN RETURN ELSE GOSUB 190
1336 I=7:C(I)=1/K4:D(I)=K3*C(I)+3:GOSUB 190
1339 GOTO 1550
1339 '***************
1400 'SUBROUTINE (3,7)
1401 '***************
1410 C(I)=7
1420 X1=C(I)
1430 Y1=K7*K3*X1*(4+K7*K3*X1)-3
1440 Y2=4*K7*K3*X1*(3+K7*K3*X1)*2
1450 C(I)=X1-Y1/Y2
1460 IF ABS(Y1/Y2)<0.0001 THEN RETURN ELSE GOTO 1420
1469 '***************
1470 'SUBROUTINE (3,8)
1471 '***************
1480 C(I)=7
1500 X1=C(I)
1510 Y1=K3*X1*(4-K3*X1)+3
1520 Y2=4*K3*X1*(3-K3*X1)*2
1530 C(I)=X1-Y1/Y2
1540 IF ABS(Y1/Y2)<0.0001 THEN RETURN ELSE GOTO 1500
1550 I=5:GOSUB 1400:D(I)=K3*C(I)+3:GOSUB 190
1559 I=9
1560 GOSUB 1480:D(I)=K3*C(I)+3:IF K>3 THEN RETURN ELSE GOSUB 190
1570 I=10
1600 D(I)=K9:C(I)=D(I)/K3*X(1/3)
1610 IF K>3 THEN RETURN ELSE GOSUB 190
1620 I=11:C(I)=1/K4:D(I)=1/(K7*C(I)+K7):GOSUB 190
1630 I=12
IF DI=0 THEN GOTO 974
IF ABS(Y1/Y2)>0.0001 THEN GOTO 1100 ELSE GOTO 1250
D(I)=Z-1.1 GOTO 1100

' SUBROUTINE (G+7)

D(I)=1
X1=D(I)
Y1=K7*K2*X1*C(P1+1)+K7*X1-1
Y2=-P1+1)*K7*K2*X1*C(P1+1)+K7
D(I)=X1-Y1/Y2
IF D(I)<0 THEN GOTO 974
IF ABS(Y1/Y2)<0.0001 THEN GOTO 1250 ELSE GOTO 1190

' SUBROUTINE FOR CALC. C(I) WHEN G=1

C(I)=P2*D(I)*C(P1+1) GOTO 974

' SUBROUTINE (G+8)

D(I)=1
X1=D(I)
Y1=P2*X1*C(P1+1)-X1-ID
Y2=P1+K7*K2*X1*C(P1+1)-1
D(I)=X1-Y1/Y2
IF D(I)<0 THEN GOTO 974
IF ABS(Y1/Y2)<0.0001 THEN GOTO 1250 ELSE GOTO 1280
GOSUB 970
I=I+1 GOSUB 971 IF I=1 THEN GOTO 1153
GOSUB 1153
GOSUB 970
: I=I+1 GOSUB 971 IF I=1 THEN GOTO 1170
GOSUB 1170
I=6
C(I)=K25*D(I)=K3*C(I)*C3: IF K>3 THEN RETURN ELSE GOSUB 190
I=I+1 IF K=1 THEN C(I)=K3*C(I)*C3: GOSUB 190
GOTO 1550

' SUBROUTINE (3+7)

C(I)=7
X1=C(I)
Y1=K7*K3*X1*C4+K7*K3*X1*C3-1
Y2=4*K7*K3*X1*C3+3*K7*K3*X1*C2
C(I)=X1-Y1/Y2
IF ABS(Y1/Y2)<0.0001 THEN RETURN ELSE GOTO 1420

' SUBROUTINE (3+8)

C(I)=7
X1=C(I)
Y1=K3*X1*C4-K3*X1*C3-ID
Y2=4*K3*X1*C3-3*K3*X1*C2
C(I)=X1-Y1/Y2
IF ABS(Y1/Y2)<0.0001 THEN RETURN ELSE GOTO 1500
I=5 GOSUB 1400: D(I)=K3*C(I)*C3: IF K>3 THEN RETURN ELSE GOSUB 190
I=40: I=9
GOSUB 1480: D(I)=K3*C(I)*C3: IF K>3 THEN RETURN ELSE GOSUB 190
I=10
D(I)=K9*C(I)=D(I)/K3*C(I)*C3
IF K>3 THEN RETURN ELSE GOSUB 190
I=I+1: C(I)=1/K4*D(I)=1/(K7*C(I)*K7) GOSUB 190
I=12
2180 'THIS ROUTINE FINDS CH
2181 '.............................................
2185 G=8
2190 CH=C(8)
2200 FOR K=9 TO L
2210 IF CH>C(K) THEN GOTO 2230
2230 NEXT
2239 '.............................................
2240 'THIS PART FINDS CH
2241 '.............................................
2250 IF YC=1 THEN U=12 ELSE U=13
2260 CH=C(U)
2270 FOR K=U+1 TO T
2271 IF CH<C(K) THEN GOTO 2290
2280 IF CH<C(N) THEN GOTO 2300
2290 CH=C(K)
2295 U=K
2300 NEXT
2310 GOTO 2120
2319 '.............................................
2320 'MINIMIZING WEIGHT <<<<<<<<
2321 '.............................................
2340 P=.2536*16=3.1415*2*G/(16*K)
2350 U2=P3.14152*0.4
2354 '.............................................
2355 'CALC. (C1,C25,C6L,C8,C9) WHEN G=1
2356 '.............................................
2359 GOSUB 970
2379 I=6: GOSUB 1690
2379 GOSUB 970
2380 I=10: GOSUB 971: GOSUB 971: GOSUB 970
2389 GOSUB 970
2390 I=11: GOSUB 971: GOSUB 1270
2399 GOSUB 970
2400 GOSUB 971: I=12: GOSUB 1250
2409 '.............................................
2410 'CALC. (C3,C4,C6U,C7,C10) WHEN G=1
2411 '.............................................
2420 I=13: GOSUB 970: GOSUB 971: GOSUB 970
2430 I=14: GOSUB 970: GOSUB 971: GOSUB 970
2440 I=15: GOSUB 970: GOSUB 971: GOSUB 1153
2450 I=16: GOSUB 970: GOSUB 971: GOSUB 1180
2460 I=17: D(I)=1/KL: GOSUB 970: GOSUB 971: GOSUB 1250
2469 '.............................................
2470 'FIND MAX. OF (C1,C25,C6L,C8,C9) MIN. OF (C3,C4,C6U,C7,C10) CH AND COMPARE WITH STATIONARY
2471 'POINT.
2480 L=12: T=17: GOSUB 2060: GOSUB 2180: IF C(I)=0 THEN OBJ(I)=0 ELSE GOSUB 2770
2499 '.............................................
2500 'THE SAME FOR (C1,C25,C6,C8) AND (C3,C4,C7,C9) AND STATIONARY POINT WHEN G=1
2501 '.............................................
2510 C(8)=K1*K9*2
2520 C(9)=K25
2530 C(10)=C(K6*K9*2)/(1-L6*K9)*C(1/3)
2540 C(11)=C(ID*K9)/K9
2541 YC=1
2550 C(12)=K9/K3*C(1/3)
2560 C(13)=1/K4
2570 R(14)=1-K7*K6: GOSUB 1250
2580
IF HI<0 THEN LPRINT "NO FEASIBLE SOLUTION FOR INPUT DATA"; END
LPRINT "OUTPUT DATA:"
GO SUB 752
LPRINT "FREE LENGTH: IN="FL,"MINIMUM WEIGHT: LB="MN
END

THIS ROUTINE CALC. WEIGHT

GO SUB I=1*2D(I)*C(I)**2*W2*C(I)*D(I)**3
RETURN

IF I=8 THEN GOTO 1690
IF I=9 THEN GOSUB 971:GOTO 1020
IF I=10 THEN GOSUB 971:GOTO 1070
IF I=11 THEN GOSUB 971:GOTO 1270
IF I=12 THEN GOSUB 971:GOTO 1250
IF I=13 THEN GOSUB 971:GOTO 1040
IF I=14 THEN GOSUB 971:GOTO 1060
IF I=15 AND I2<>1 THEN GOSUB 971:GOTO 1153
IF I=16 THEN GOSUB 971:GOTO 1180
IF I=17 THEN GOSUB 971:GOTO 1250
IF I=18 THEN GOTO 2070
IF I=15 AND I2=1 THEN GOTO 2575
IF I=3 THEN GOTO 2600
IF I=4 THEN GOTO 2610
I=12:D(I)=C(I)/K1*C(I)**2*D(I)**3:GOSUB 190
I=13:D(I)=C(I)/K2:GOSUB 190
I=14:D(I)=1/K1:GOSUB 190
I=15:D(I)=1/K1:GOSUB 190:RETURN
I=1
D(I)=C(I)/K3:D(I)=D(I)/K3*C(I)**3:GOSUB 190:IF K>3 THEN RETURN
I=14
I=1

NEWTON ROUTINE (7, 11)
D(I)=1
X(I)=D(I)
Y(I)=(X(I)/C(I))/(1/3)-0D/X1+1
Y(D(I)=C(I)/K2*D(I)**3:GOSUB 190
I=14
I=1

NEWTON (8, 11)
D(I)=1
X(I)=D(I)
Y(I)=(X(I)/C(I))/(1/3)-0D/X1+1
Y=2/(3*JPC(1/3)):X1C(-1/3)+0D/(X1C2)
D(I)=X1-Y1/Y2
IF ABS(Y1/Y2)> .0001 THEN GOTO 3150
C(I)=GD/D(I)-1*GOSUB 190
I=14
I=1

IF K<3 THEN GOTO 3430
IF K=4 THEN RETURN ELSE I=I+1:GOSUB 970:D(I)=1/K1:GOSUB 971:C(I)=C(I)*1/4:GOSUB 190
I=14
I=22
D(I)=C(I)**2/D(I)/E1+1:GOSUB 190:RETURN
MIN COIL CONSTRAINT(G3) IS VIOLATED FOR I = 2  G3 VIOLATION = 0.66416
OUTSIDE DIAMETER CONSTRAINT (G7) IS VIOLATED FOR I = 2  G7 VIOLATION = 1.39588
MIN COIL CONSTRAINT(G3) IS VIOLATED FOR I = 3  G3 VIOLATION = 0.963169
SURGING CONSTRAINT(G1) IS VIOLATED FOR I = 4  G1 VIOLATION = 46.7493
POCKET LENGTH CONSTRAINT(G6) IS VIOLATED FOR I = 4  G6 VIOLATION = 1535.76
SURGING CONSTRAINT(G1) IS VIOLATED FOR I = 5  G1 VIOLATION = 30.5414
OUTSIDE DIAMETER CONSTRAINT (G7) IS VIOLATED FOR I = 5  G7 VIOLATION = 1.24711
SURGING CONSTRAINT(G1) IS VIOLATED FOR I = 6  G1 VIOLATION = 98.1287
MIN COIL CONSTRAINT(G3) IS VIOLATED FOR I = 6  G3 VIOLATION = 0.467874
LOWER LIMIT ON D(G10) IS VIOLATED FOR I = 6  G10 VIOLATION = 0.334071
OUTSIDE DIAMETER CONSTRAINT (G7) IS VIOLATED FOR I = 6  G7 VIOLATION = 5.50387
SURGING CONSTRAINT(G1) IS VIOLATED FOR I = 7  G1 VIOLATION = 11.8448
Buckling Or Min Index Constraint(G25) Is Violated For I = 8  G25 VIOLATION = 216.341
POCKET LENGTH CONSTRAINT(G6) Is Violated For I = 8  G6 VIOLATION = 4706
SURGING CONSTRAINT(G1) Is Violated For I = 8  G1 VIOLATION = 0.5
CLASH ALLOWANCE(G11) Is Violated For I = 9  G11 VIOLATION = 0.954579
MIN COIL CONSTRAINT(G3) Is Violated For I = 9  G3 VIOLATION = 29.0417
MAX INDEX CONSTRAINT(G4) Is Violated For I = 9  G4 VIOLATION = 2.3187
OUTSIDE DIAMETER CONSTRAINT (G7) Is Violated For I = 9  G7 VIOLATION = 15.3435
MIN COIL CONSTRAINT(G3) Is Violated For I = 10  G3 VIOLATION = 0.100001
OUTSIDE DIAMETER CONSTRAINT (G7) Is Violated For I = 10  G7 VIOLATION = 2.54354
SURGING CONSTRAINT(G1) Is Violated For I = 11  G1 VIOLATION = 61.3739
Buckling Or Min Index Constraint(G25) Is Violated For I = 11  G25 VIOLATION = 0.746637
POCKET LENGTH CONSTRAINT(G6) Is Violated For I = 11  G6 VIOLATION = 2021.35
MIN COIL CONSTRAINT(G3) Is Violated For I = 12  G3 VIOLATION = 0.321734
OUTSIDE DIAMETER CONSTRAINT (G7) Is Violated For I = 12  G7 VIOLATION = 0.67248
SURGING CONSTRAINT(G1) Is Violated For I = 13  G1 VIOLATION = 279.271
MIN COIL CONSTRAINT(G3) Is Violated For I = 13  G3 VIOLATION = 8.22604
MAX INDEX CONSTRAINT(G4) Is Violated For I = 13  G4 VIOLATION = 1.22085
OUTSIDE DIAMETER CONSTRAINT (G7) Is Violated For I = 13  G7 VIOLATION = 23.574
LOWER LIMIT ON D(G10) Is Violated For I = 13  G10 VIOLATION = 1.20833
SURGING CONSTRAINT(G1) Is Violated For I = 14  G1 VIOLATION = 60.6273
POCKET LENGTH CONSTRAINT(G6) Is Violated For I = 14  G6 VIOLATION = 1982.49
OUTSIDE DIAMETER CONSTRAINT (G7) Is Violated For I = 14  G7 VIOLATION = 186657
SURGING CONSTRAINT(G1) Is Violated For I = 15  G1 VIOLATION = 62.3739
Buckling Or Min Index Constraint(G25) Is Violated For I = 15  G25 VIOLATION = 1.74664
POCKET LENGTH CONSTRAINT(G6) Is Violated For I = 15  G6 VIOLATION = 2057.95
DESIGN OBJECTIVE: MAXIMIZING RELIABILITY
SPRING MATERIAL IS MUSIC WIRE (AISI 1085)
TYPE OF ENDS: SQUARED OR SQUARED AND GROUND; NO. OF INACTIVE COILS = 2
MAX FORCE = 20
DEEFECTION = .25
OUTSIDE DIAM. = 1.5
MAX SPRING INDEX = 20
MIN NO OF ACTIVE COILS = 3
MAX LENGTH AT MAX FORCE = 1.25

MIN FORCE = 8
MIN NATURAL FREQ. = 500
INSIDE DIAM. = .75
MIN SPRING INDEX = 4
CLEARANCE = .4
NO OF CYCLES TO FAILURE = 1E+06

WIRE DIAM. IN = .0964225
MEAN COIL DIAM. IN = .952035
FREE LENGTH IN = 1.01449

SPRING INDEX = 9.07358
NO. OF ACTIVE COILS = 3
OPTIMUM VALUE OF OBJECTIVE = 1.63713