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MULTIOBJECTIVE OPTIMIZATION FOR INTEGRATED TOLERANCE ALLOCATION AND FIXTURE LAYOUT DESIGN IN MULTISTATION ASSEMBLY

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ABSTRACT

Cost and product quality are significant attributes in manufacturing processes, such as multistation assembly. We use multiobjective optimization for integrated tolerance allocation and fixture layout design to address their interaction and to quantify tradeoffs among cost, product quality, and assembly process robustness. Design decisions relate to product tolerances, assembly process tolerances, and fixture locating positions. A nested optimization strategy is adopted, and the proposed methodology is demonstrated using a vehicle side frame assembly example. The obtained results provide evidence for the existence of tradeoffs, based on which we can identify critical quality and budget requirements.

1 Introduction

Rapid changes in the market place have led designers and manufacturers to continuously develop new products to satisfy the demands of more exigent customers. Those changes have significantly affected producers of complex products such as automobiles and airplanes because their products have numerous components that have to be designed, fabricated and assembled to form the final product. For example, an automobile can have up to 10,000 components and the manufacturing processes can be quite complex. Therefore, to be competitive, designers and manufacturers have to perform those steps accurately to ensure final product quality and cost effectiveness.

The assembly of complex products is usually done following a sequential process performed at multiple stations. At each station different components are put together to form the final product. Figure 1 depicts an assembly example of an automobile body structure.

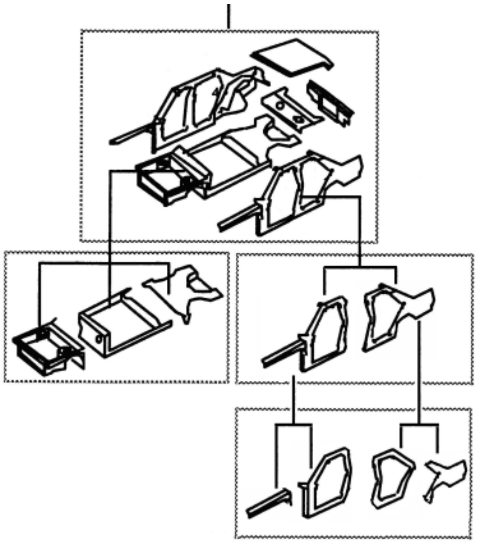

Figure 1. Schematic of an automobile body structure assembly [1]

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Depending on component characteristics, assembly processes can be classified into types I and II [2]. In Type-I assemblies, workpieces are assembled according to their prefabricated mating features. In Type-II assemblies, workpieces are positioned by fixtures. For both types of assemblies, variations of manufactured workpieces and manufacturing processes are propagated or accumulated station by station toward the final product. In Type-I assemblies, component or subassembly variation may cause interference problems, while in Type-II assemblies, propagated variation may increase or decrease variation associated with final assembly dimensions. This paper focuses on Type-II assembly processes.

Design evaluation of multistation assembly systems depends on a group of critical features, which are known as key characteristics. Thornton [3] defined key characteristics as *quantifiable features of a product or its assemblies, parts, or processes whose expected variation from target have an unacceptable impact on the cost, performance, or safety of the product*. In this work, dimensional key features for products are referred to as Key Product Characteristics (KPCs). Those usually correspond to measurement points that have a key role on product functionality and quality. Therefore, they are measured along and at the end of the process. The process features that control the position of the KPCs are known as the Key Control Characteristics (KCCs). They correspond to locators and clamps used to hold the parts in the process. Early and accurate evaluations of process variations and process configurations are crucial in determining the dimensional accuracy of KPCs; they in turn, affect the final dimensional quality of assembled products. Excessive dimensional variation of the KPCs and KCCs may cause assembly process difficulties at subsequent stations and/or final product quality concerns. For example, in an automotive body assembly process, variation may cause part fitting problems, water leakage and wind noise. Therefore, reducing dimensional variations in assembly process is important for final product quality improvement.

Process performance of assembly systems can be measured as the capability of a process to deliver final products of high quality at low cost. The capability of a process is defined by relating the variation to tolerance specifications. Stringent tolerance requires more expensive machines and tools. Thus, it is important to incorporate tolerances in quality-cost analysis.

The objective of this work is to conduct simultaneous fixture layout design and tolerance allocation to improve final product quality at minimum cost. To do so, it is necessary to: 1) use a rigid parts model of the multistation assembly process that connects KCCs and KPCs variation; 2) derive an index that relates fixture layout design with final product quality; 3) determine relations that use tolerances to link cost with product and process quality; and 4) formulate and solve a multiobjective design optimization problem that trades off cost, quality, and process robustness subject to process-related and geometric constraints.

This paper presents a method for integrated fixture layout design and tolerance allocation by solving the formulated problem using a nested optimization strategy. The remainder of the paper is organized as follows: In Section 2 we provide background on tolerance allocation and fixture layout design, and present the models we used in the integrated framework. In Section 3, we formulate the integrated design optimization problem for determining optimal fixture distribution, tolerance allocation, and dimensional quality. In Section 4, we use an example to demonstrate the proposed method, and introduce the critical quality and budget requirement concepts. Concluding remarks are provided in Section 5.

2 Tolerance Allocation and Fixture Layout Design

Traditionally, tolerance allocation and fixture layout optimization are two main activities in design of assembly systems that have been conducted separately.

Tolerance allocation is used to minimize cost and final product variations by optimally allocating tolerances of workpieces and fixtures. Allocation of tolerances has been addressed extensively, especially for rigid parts assembly processes [4]. For example, Ding *et al.* proposed a framework for process-oriented tolerance synthesis for rigid multistation assembly systems, where process tolerances were optimally allocated by solving a nonlinear constrained optimization problem [5]. To do so, they developed a a tolerance-variation model that relates pin-hole fixture tolerances into equivalent fixture variation.

Fixture layout optimization serves to improve process robustness to external variation by changing the fixture positions. In this area, Kim and Ding [6] presented a methodology for the optimal design of fixture layouts in multistation assembly processes, without considering tolerances and cost. Three key aspects of the multistation fixture layout design were addressed: a multistation variation propagation model, a quantitative measure of fixture design, and an effective and efficient optimization algorithm.

Few integrated design activities are reported in the literature for making decisions about product and process characteristics. For example, Zhong *et al.* [7] selected process parameters and conducted tolerance allocation studies for machining processes. Chen *et al.* [8, 9] developed an integrated framework of tolerance and maintenance design for multistation assembly process. This problem was formulated as an optimization one, where the objective was to reduce the overall average production cost in the long term, by including costs for tool fabrication, maintenance costs and the overall loss of quality.

The aforementioned research efforts focused on either tolerance allocation or fixture layout design. To the best of our knowledge, there is no general framework to analyze the interactions between tolerance allocation and fixture layout design qualitatively or quantitatively. In this article, we integrate these two design activities and study the relations among cost, final product quality, and process performance.

2.1 Variation Propagation Models

In multistation assembly processes, parts and processes variation are propagated station to station towards a subassembly or final product. This variation propagation process can be modeled to predict variation of final products. Several models have been proposed to study the variation propagation of dimensional features for rigid and compliant parts [2, 10, 11, 12, 13].

In this paper, we use the rigid parts state space model reported in [10]. This model determines the deviations of the parts $\mathbf{x}_k$ and the KPCs deviations $\mathbf{y}_k$ at station k (with $k = 1, 2, \ldots, N$) as

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k \text{ and} \tag{1}$$

$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{u}_k$ corresponds to the fixture deviations, $\mathbf{w}_k$ are the process disturbances and un-modeled factors and $\mathbf{v}_k$ is the measurement noise. State matrices $\mathbf{A}_k$, $\mathbf{B}_k$, and $\mathbf{C}_k$ depend on: fixture layout, assembly sequence, locators scheme and measurement in the multistation assembly system.

Due to the linear properties of the state space model, the deviations of the final product measurements (KPCs) can be represented as a linear combination of the deviations of the fixtures in all the stations, the incoming parts deviations $\mathbf{x}_0$, the external disturbances and the measurement noise as

$$\mathbf{y}_N = \sum_{k=1}^{N} \mathbf{C}_N\mathbf{\Phi}_{N,k}\mathbf{B}_k\mathbf{u}_k + \mathbf{C}_N\mathbf{\Phi}_{N,0}\mathbf{x}_0 + \sum_{k=1}^{N} \mathbf{C}_N\mathbf{\Phi}_{N,k}\mathbf{w}_k + \mathbf{v}_N, \tag{3}$$

where $\mathbf{\Phi}_{k,i} \equiv \mathbf{A}_{k-1}\mathbf{A}_{k-2}\ldots\mathbf{A}_i$ and $\mathbf{\Phi}_{i,i} \equiv \mathbf{I}$.

2.1.1 Sensitivity Index

Assuming there is no measurement and process noise, and no error from incoming parts, i.e., $\mathbf{w}_k = \mathbf{0}, \mathbf{v}_N = \mathbf{0}$, and $\mathbf{x}_0 = \mathbf{0}$, Eq. (3) can be simplified as

$$\hat{\mathbf{y}_\mathbf{N}} \equiv \mathbf{D}\mathbf{u} = \sum_{k=1}^{N} \mathbf{C}_N\mathbf{\Phi}_{N,k}\mathbf{B}_k\mathbf{u}_k, \tag{4}$$

where $\mathbf{D} \equiv [\mathbf{C}_N\mathbf{\Phi}_{N,1}\mathbf{B}_1, \ \mathbf{C}_N\mathbf{\Phi}_{N,2}\mathbf{B}_2, \ \ldots, \ \mathbf{C}_N\mathbf{B}_N]$, $\mathbf{u} \equiv [\mathbf{u}_1^T, \ \ldots, \ \mathbf{u}_N^T]^T$, and $\hat{\mathbf{y}_\mathbf{N}}$ is the fixture-induced product variation.

Following this simplification, Kim and Ding [6] define the sensitivity index SI for a rigid multistation assembly system as

$$SI \equiv \sup_{\mathbf{u}\neq 0} \frac{\mathbf{u}^T\mathbf{D}^T\mathbf{D}\mathbf{u}}{\mathbf{u}^T\mathbf{u}} = \lambda_{max}(\mathbf{D}^T\mathbf{D}). \tag{5}$$

The above definition is based on the E-optimality criterion (minimize the extreme eigenvalue of $(\mathbf{D}^T\mathbf{D})$), which is independent from tooling deviations (represented by $\mathbf{u}$). It should be noted that there exist alternative sensitivity index definitions for rigid multistation assembly systems. Frequently used criteria include D-optimality (minimize $\det(\mathbf{D}^T\mathbf{D})$) and A-optimality (minimize $\mathrm{tr}(\mathbf{D}^T\mathbf{D})$), where $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ are the trace and the determinant of a matrix, respectively. Each robustness criterion (sensitivity index) reflects only one aspect of the characteristics of the sensitivities to the final product attributes. For example, the A-optimality focuses on the sum of the sensitivities, while the E-optimality focuses on the maximum or minimum sensitivity.

2.2 Quality and Cost Models

Assuming that components, subassemblies and processes have capabilities (C_p) equal to one, then, the final product quality can be expressed as a function of the final product tolerances ($\mathbf{t_N}$) as presented by

$$q = \|\mathbf{t}_N\|_\infty. \tag{6}$$

The use of the infinity norm implies that the quality requirement is imposed on KPCs with relatively large variation values. This representation or evaluation is only one of many possible choices. Other valid measures such as the l_1-norm or the l_2-norm may also be used.

There are several types of cost, all related to pre-assembly, assembly, and post-assembly processes. After Taguchi popularized the idea of quality loss [14], cost models were developed using quality loss functions that account for customer satisfaction about products. Cost models for maintenance and life-cycle design are used to evaluate the time value of money for quality loss and product degradation, taking into consideration product volume, production cycle, and market requirements. In this work, we consider exclusively costs associated with product and process dimensional variations. Much research has been done in determining the impact of tolerances on cost [15, 16, 17, 18, 19, 20]. Since manufacturing cost is both site- and process-dependent, cost is usually calculated based on empirical relations. If data are not available, choosing an appropriate cost model depends on a comprehensive understanding of the specific manufacturing system.

Two models commonly used to relate cost and tolerances are based on reciprocal and exponential functions of the tolerances.

Those cost models are good alternatives, offering decent data fit and simple function structures. In this work, the reciprocal function of the tolerance (t) is chosen to represent cost-tolerance relations:

$$c(t) = \frac{\alpha}{t}. \tag{7}$$

For a tolerance design vector, the cost-tolerance model is

$$c(\mathbf{t}) = \sum_{i=1}^{n} \frac{\alpha_i}{t_i}, \tag{8}$$

where n is the dimension of the tolerance vector, and α_i is a constant fitted for each tolerance.

3 Problem Formulation

As shown in Figure 2, the considered design variables include product tolerances $\mathbf{t}$, process tolerances τ, and fixture locations $\mathbf{p}$. The system cost c depends only on tolerances $\mathbf{t}$ and τ. The sensitivity index SI, an evaluation of process robustness, changes with fixture locations $\mathbf{p}$. Dimensional tolerances, $\mathbf{t}$ and τ, originating from incoming parts and fixture elements on every station, are transferred along the production line, to the tolerances of final assembly representing quality q. In this paper, we will analyze cost-robustness and quality-robustness interactions.

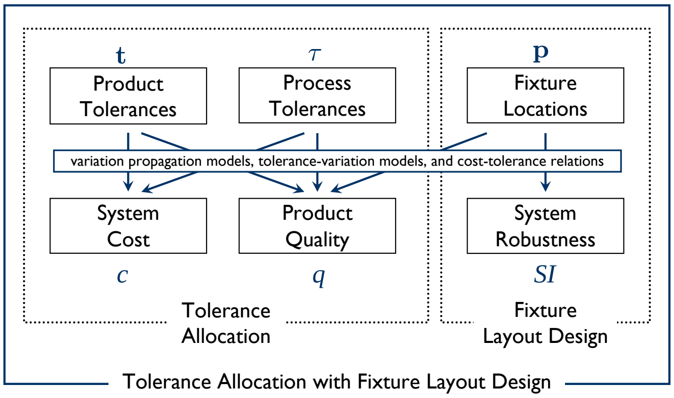

Figure 2. Relations among system inputs and major system attributes

3.1 Cost - Robustness Considerations

It is widely thought that an appropriate fixture layout design improves the robustness of a fixture system against environmental noise, reduces product variability, and leads to manufacturing cost reduction (e.g., see [6]). Here, we study cost-robustness relations to investigate on the existence of tradeoffs by formulating and solving the following two-objective optimization problem:

$$\begin{aligned} \min_{\mathbf{t},\tau,\mathbf{p}} \ & \{c(\mathbf{t},\tau), SI(\mathbf{p})\} \\ \text{s.t.} \ & q(\mathbf{t},\tau,\mathbf{p}) \leq q_s \\ & \mathbf{g}(\mathbf{t},\tau) \leq 0 \\ & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}, \end{aligned} \tag{9}$$

The design variables $\mathbf{t}$, τ and $\mathbf{p}$ determine the total cost$c$ and process robustness (SI). Those can be evaluated using variation propagation models. The constraint $q(\mathbf{t},\tau,\mathbf{p}) \leq q_s$ ensures that the final product quality will satisfy the quality requirement q_s. The term $\mathbf{g}(\mathbf{t},\tau) \leq 0$ corresponds to inequality constraints representing the lower and upper design bounds for $\mathbf{t}$ and τ. Finally, $\mathbf{g}(\mathbf{p}) \leq \mathbf{0}$ accounts for geometrical constraints on fixture locations, which are imposed by parts geometry.

It is important to note that we assume that changing fixture layout is not associated with any cost and that cost depends exclusively on product and process tolerances. Without lost of generality, in this paper the value of the tolerance-cost constants (α) are set all equal to one.

Problem (9) is difficult to solve due to the large number of design variables and the required computationally intensive simulations. A nested optimization strategy is thus adopted to improve efficiency. Specifically, Problem (9) can be rearranged and written as,

$$\begin{aligned} \min_{\mathbf{p}} \ & \{\min_{\mathbf{t},\tau}\{c(\mathbf{t},\tau) | q(\mathbf{t},\tau,\mathbf{p}) \leq q_s, \mathbf{g}(\mathbf{t},\tau) \leq 0\}, SI(\mathbf{p})\} \\ \text{s.t.} \ & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}. \end{aligned} \tag{10}$$

The optimization process is shown in Figure 3. The purpose of the outer loop is to determine values for fixture layout design variables $\mathbf{p}$. Given $\mathbf{p}$, the sensitivity matrices are obtained and used as parameters in variation propagation models for the inner loop optimization process. Given the variation propagation models and cost models, product and process tolerances, $\mathbf{t}$ and τ, are allocated to achieve minimum cost c while satisfying the quality requirement q_s.

3.2 Quality - Robustness Considerations

We will also study the relation between final product quality and process robustness. To accomplish this, quality is considered as an objective, and cost is now treated as a constraint. The two-

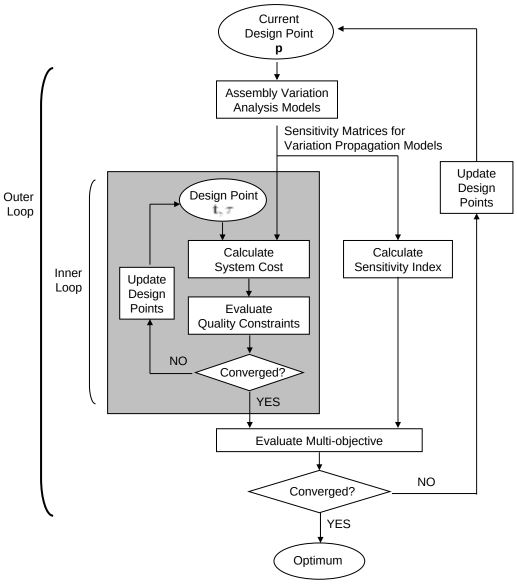

Figure 3. Nested optimization strategy for Problem (9)

objective problem is reformulated as

$$\begin{aligned} \min_{\mathbf{t},\tau,\mathbf{p}} \ & \{q(\mathbf{t},\tau,\mathbf{p}), SI(\mathbf{p})\} \\ \text{s.t.} \quad & c(\mathbf{t},\tau) \leq b \\ & \mathbf{g}(\mathbf{t},\tau) \leq 0 \\ & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}, \end{aligned} \tag{11}$$

Using the nested optimization strategy, Problem (11) is rewritten as

$$\begin{aligned} \min_{\mathbf{p}} \ & \{\min_{\mathbf{t},\tau}\{q(\mathbf{t},\tau,\mathbf{p}) | c(\mathbf{t},\tau) \leq b, \mathbf{g}(\mathbf{t},\tau) \leq 0\}, SI(\mathbf{p})\} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{p}) \leq \mathbf{0}. \end{aligned} \tag{12}$$

4 Example

We use a four-station assembly process of a sedan vehicle side frame as an example to illustrate the integrated framework for tolerance allocation and fixture layout design. The two-dimensional rigid body panel assembly model is shown in Figure 4.

The panel consists of four parts: front wheel house (Part 1), front passenger compartment (Part 2), rear passenger compartment (Part 3), and rear quarter panel (Part 4). For simplicity, the parts are modeled as quadrilaterals, with dimensions represented by the location of the vertices A, B, C, and D. The locations of measurement points (MP), where key product characteristics (KPCs) are evaluated, are marked with triangles in the figure.

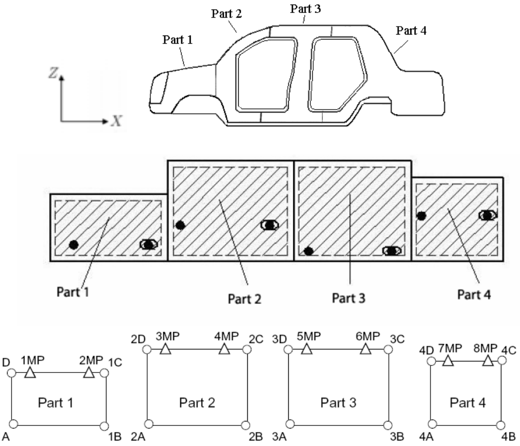

Figure 4. Example: Vehicle side frame model

The assembly sequence is presented in Figure 5. Starting at Level 4 (station I) parts 1 and 2 are assembled to form subassembly 1. Subassembly 1 and part 3 are assembled at station *II* to form subassembly 2. At station *III*, subassembly 2 and part 4 are joined together as the final assembly. Then measurement points on the final assembly are inspected at station *IV* (measurement station).

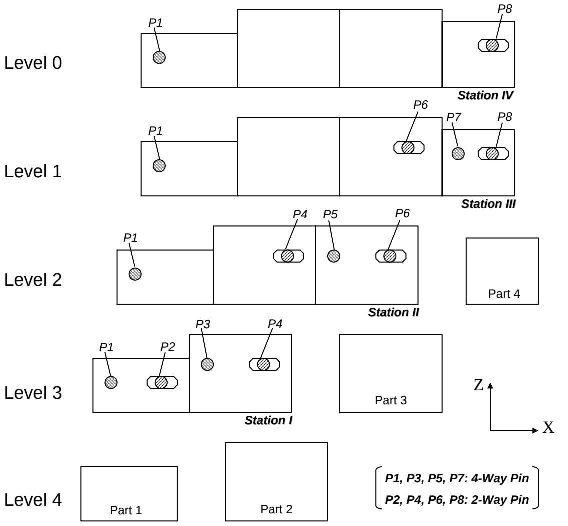

Figure 5. Fixture layout in the rigid multistation assembly system

A typical "3-2-1" fixture layout used in this type of assembly processes consists of two locating pins, P_{4way} and P_{2way}, and three net contact (*NC*) blocks (or clamps, or supports), NC_{1-3}

[6]. In this study for rigid multistation assembly processes, we focus on the possible variation of locating pins only; thus, $\{P_{4way}, P_{2way}\}$ is used as a simplified representation of a “3-2-1” fixture layout. Locating pins for parts, subassemblies, and final products for the case analyzed are shown on Figure 5. The datum scheme (set of fixtures used to hold parts and subassemblies on each station) is the following

$$\begin{aligned}\{\{P_1,P_2\},\{P_3,P_4\}\}_I &\rightarrow \{\{P_1,P_4\},\{P_5,P_6\}\}_{II}\\ &\rightarrow \{\{P_1,P_6\},\{P_7,P_8\}\}_{III}\\ &\rightarrow \{\{P_1,P_8\}\}_{IV}.\end{aligned}$$

Considering only the assembly stations (stations I, II and II), there are 12 tolerance design variables for fixtures, and 16 position design variables for the fixtures (X-Z position of each fixture).

4.1 Cost - Robustness Results

Problem (9) is solved for $q_s = 2$mm using the nested optimization strategy. A gradient-based optimization algorithm (the Matlab implementation of Sequential Quadratic Programming) is used to solve the tolerance allocation problem in the inner loop. For the outer loop, the Neighborhood Cultivation Genetic Algorithm (NCGA) of the iSIGHT software package [21] is used to generate the Pareto set. The NCGA number of function evaluations is set to 10,000.

As can be seen in Figure 6 there exists a cost-robustness tradeoff. Specifically, a 12-percent decrease in robustness (sensitivity index) can result in an 18-percent increase in system cost.

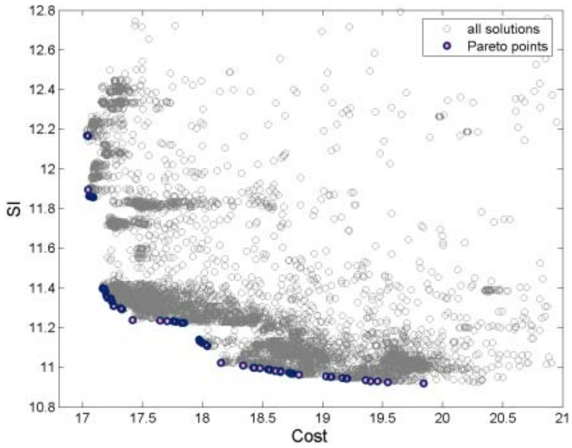

Figure 6. Tradeoff between cost and robustness (E-optimality SI) for $q_s = 2$mm

The tradeoff between the system cost and the sensitivity index does not exist only for $q_s = 2$mm. Problem (9) is solved for different quality requirements: $q_s = 1.3$mm, 1.5mm, 1.8 mm, and 2mm. The results are seen in Figure 7. Again, there are tradeoffs for the system cost and the sensitivity index.

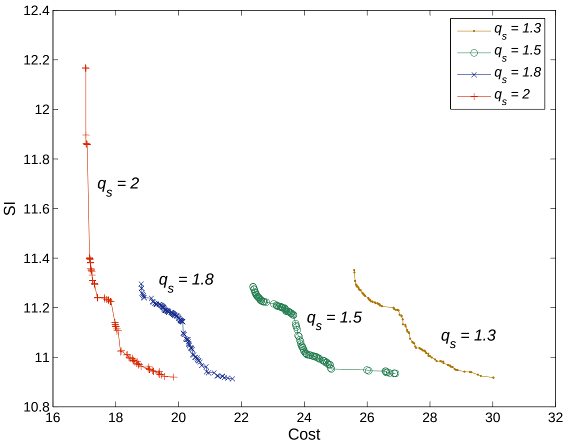

Figure 7. Tradeoffs between cost and robustness (E-optimality SI) for $q_s = 1.3$mm to 2mm

Fixture layout does not only determine the sensitivity index; it also changes the parameters of the variation propagation models in the tolerance allocation problem, and therefore affects cost. Thus, depending on the quality requirement constraint, there may exist a tradeoff between robustness and cost.

In fact, the tradeoff between system cost and robustness does not exist for all quality requirements. Problem (9) is solved for several values of q_s to address its effect on cost-robustness tradeoff. The results can be seen in Figure 8. It is observed that the tradeoff becomes less and less significant with increasing quality requirement values. At $q_s = 10$mm, there is only one solution.

At $q_s = 10$mm, all 12 of the tolerance design variables reach the upper bound of 2mm. Therefore, the minimum cost for the system is $c = 12 \times \frac{1}{t} = 12 \times \frac{1}{2(\text{mm})} = 6$. The goal is then to find a fixture layout that provides the minimum sensitivity index. Thus, at $q_s = 10$mm, there is only one solution that ensures both minimum system cost and minimum sensitivity index.

Based on this analysis, we introduce the *critical quality requirement* q_c defined as the final product quality evaluated at the optimal fixture layout, with all the tolerance variables at their upper design bounds. Problem (9), solved at $q_s = q_c$, has only one solution. At this solution, all tolerances reach the upper design bounds. According to the cost-tolerance relations, the cost is the minimum for that assembly system. Additionally, the sensitivity index is the lowest for that assembly system.

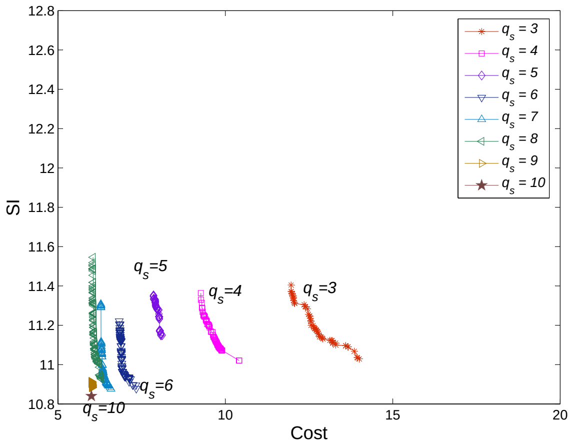

Figure 8. Relation between cost and SI for $q_s = 3$mm to 10mm

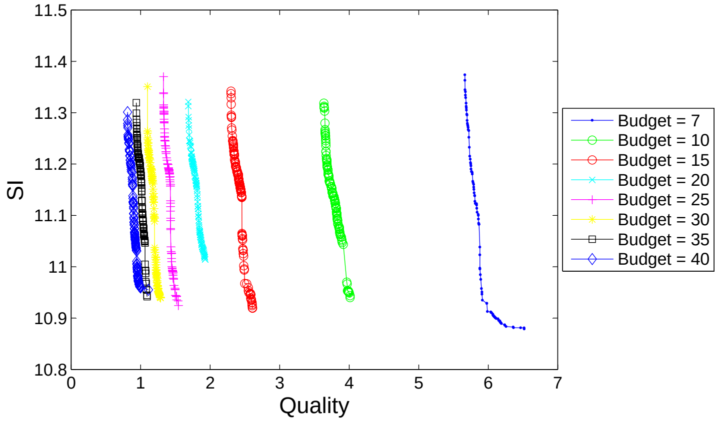

Figure 9. Relation between quality and SI for the rigid system

If the quality requirement is less than the critical quality requirement, a choice must be made between cost and robustness along the Pareto curve. Otherwise, the solution at the critical quality requirement should be chosen, ensuring both the minimum cost and the minimum SI. Then the design goal for the multistation assembly system becomes to decrease the critical quality requirement.

In the example, the critical quality requirement q_c is much larger than the expected product quality, which is always less than 2mm. There are two ways to decrease q_c for an assembly system. One way is to change station characteristics, such as the assembly sequence. The other way is to decrease the upper design bounds, for both product and process tolerance variables. For example, when the upper bound of the tolerance design is changed from 2mm to 1.5mm, the critical quality requirement q_c changes from approximately 10mm to 7mm.

4.2 Quality - Robustness Results

Problem (11) is solved for budgets $b = 7, 10, 15, 20, 25, 30, 35,$ and 40. Once again, SQP is used as the inner loop optimizer and NCGA (with $10,000$ function evaluations) as the outr loop optimizer. The results are illustrated in Figure 9. It may be difficult to see due to the plot scale, but there exists a tradeoff between product quality and robustness. For example, at $b = 7$, a five percent decrease in robustness can result in a thirteen percent increase in final product quality.

Another observation from Figure 9 is that the tradeoff between the final product quality and the sensitivity index diminishes as the budget increases. The critical budget requirement b_c can be defined as the required cost of an optimal fixture layout when all tolerance variables are at their lower bound values. If the budget is less than the critical budget requirement, a choice has to be made between quality and SI along the Pareto curve. The design goal for the multistation assembly system then becomes to decrease the critical budget requirement. This can be realized by changing the station characteristics or increasing the lower design bounds, for both product and process tolerance variables.

5 Concluding Remarks

A framework is proposed to consider tolerance allocation and fixture layout design simultaneously using multiobjective optimization formulations. A nested optimization strategy was used to solve the formulated multiobjective problems that considered final product quality, assembly process robustness, and cost. Tradeoffs between cost and robustness and between quality and robustness were identified and quantified under the assumptions that changing fixture layouts does not incur costs and that the considered system cost depends only on allocated product and process tolerances.

We found that the existence of cost-robustness and quality-robustness tradeoffs depend on the value of the active quality and budget constraints, respectively. Based on our findings, we defined critical quality and budget requirements as the values where the tradeoffs cease to exist. If current quality or budget requirements are far from their critical values one has to choose a design from the Pareto set. Alternatively, one can change the design bound values for the product and process tolerance variables in order to change critical requirement values and therefore obtain a single design solution.

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