Journal of Mechanical Design

Technical Brief

Design Optimization of Hierarchically Decomposed Multilevel Systems Under Uncertainty

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This paper presents a methodology for design optimization of hierarchically decomposed systems under uncertainty. We propose an extended, probabilistic version of the deterministic analytical target cascading (ATC) formulation by treating uncertain quantities as random variables and posing probabilistic design constraints. A bottom-to-top coordination strategy is used for the ATC process. Given that first-order approximations may introduce unacceptably large errors, we use a technique based on the advanced mean value method to estimate uncertainty propagation through the multilevel hierarchy of elements that comprise the decomposed system. A simple yet illustrative hierarchical bilevel engine design problem is used to demonstrate the proposed methodology. The results confirm the applicability of the proposed probabilistic ATC formulation and the accuracy of the uncertainty propagation technique. [DOI: 10.1115/1.2168470]

1 Introduction

Design optimization of complex engineering systems can often be accomplished only by decomposition. The system is partitioned into subsystems, the subsystems are partitioned into components, the components into parts, and so on. The outcome of the decomposition process is a multilevel hierarchy of system-constituent elements. Hierarchical decomposition facilitates employing decentralized optimization approaches that aid systems engineers to identify interactions among elements at lower levels and to transfer this information to higher levels, and has, in fact, become standard design practice, as evidenced by the organizational structure of engineering companies [1].

In this paper we extend a deterministic methodology for optimal and consistent design of hierarchically decomposed multilevel systems to account for the presence of uncertainties. Our objective is to introduce the concept of uncertainty and model its propagation through the multilevel hierarchy. Our motivation stems from the fact that deterministic approaches assume that complete information of the problem is available and that design decisions can be implemented. These assumptions imply that optimization results are as good (and therefore useful) as the design and simulation/analysis models used to obtain them, and that they are meaningful only if they can be realized exactly. In reality, these assumptions do not hold. We are rarely in a position to (i) represent a physical system without using approximations, (ii) have complete knowledge on all of its parameters, or (iii) control the design variables with high accuracy.

To the best of our knowledge, no research work on addressing the presence of uncertainties in hierarchically decomposed multilevel systems has been reported in the literature. However, there is ongoing work to take uncertainties into consideration in multidisciplinary design optimization (MDO) [2–6]. Most of these references utilize a simple first-order Taylor expansion to calculate the mean and variance of the response in robust multidisciplinary design. A worst-case concept based on first-order sensitivity has been used to evaluate the performance range of a multidisciplinary system [7]. Although the calculation of the response mean and variance using first-order sensitivity may be adequate for robustness calculations, it does not provide accurate enough information to consider design feasibility under uncertainty. Reliability analysis using probabilistic distributions has been used in MDO [8,9]. It introduces an additional iteration loop, resulting in nested optimization problems that are computationally expensive.

It is important to differentiate our research work from that related to MDO. Multidisciplinary system design approaches are used when considering the system as consisting of multiple interacting disciplines. Several approaches for MDO have been developed during the 1990s, all of which have as an objective the coordination of the interacting disciplines during the design optimization process, mostly using single-level or bilevel formulations [10]. These approaches are nonhierarchical in the sense that the disciplines are not decomposed into multilevel hierarchies. Discipline outputs are inputs to other disciplines and vice versa. In hierarchically decomposed multilevel systems, outputs of lower-level elements are inputs to higher-level elements, but not vice versa. There is a clear hierarchical functional dependency among elements at different levels.

The main contributions of this paper are as follows. We present a probabilistic formulation for hierarchical, multilevel system optimization (in contrast to multidisciplinary design optimization

Contributed by the Design Engineering Division of ASME for publication in the Journal of Mechanical Design. Manuscript received July 7, 2004; final manuscript received May 2, 2005. Review conducted by Kemper Lewis.

under uncertainty) based on the extension of the deterministic analytical target cascading (ATC) methodology. Assuming that standard deviations of random variables are available only at the bottom level of the hierarchy, we use a bottom-to-top ATC coordination that requires uncertainty propagation. Since we have observed that large errors may be introduced when propagating uncertainties in multilevel hierarchies using first-order approximations, we propose a technique based on the advanced mean value (AMV) method to estimate standard deviations of propagated random variables accurately.

2 Optimal Design of Hierarchically Decomposed Multilevel Systems

Our formulation for hierarchical multilevel system optimization under uncertainty is based on the analytical target cascading methodology. In this section, we review the deterministic formulation of ATC and present its extension to account for uncertainties.

2.1 Deterministic Formulation. ATC is a mathematical methodology for translating ("cascading") overall system design targets to element specifications based on a hierarchical multilevel decomposition [11–13]. The objective is to assess interactions and identify possible trade-offs among elements early in the design development process and to determine specifications that yield consistent system design with minimized deviation from system-design targets. For an engineering corporation, ATC provides a means to dictate technical objectives to different design teams, knowing a priori that these goals can be achieved without conflicting with those of other teams. Consistent system design can then be accomplished with minimum communication overhead, i.e., maximum efficiency, avoiding costly iterations late in the process.

The ATC process is proven to be convergent when using a specific class of coordination strategies [14] and has been successfully applied to a variety of optimal design problems, e.g., [15–17]. We refer the reader to the above references for a detailed description of ATC. Here, we will briefly present the concept and the general mathematical formulation. In ATC, a minimum deviation optimization problem is formulated and solved for each element in the multilevel hierarchy that reflects the decomposed optimal system design problem, cf. Fig. 1. Therefore, responses of lower-level elements are inputs into higher-level elements.

The ATC process aims at minimizing the gap between what higher-level elements "want" and what lower-level elements "can do." If design variables are shared among some elements at the same level, their consistency is coordinated by their parent element at the level above.

The key assumption of the ATC methodology is that there is a

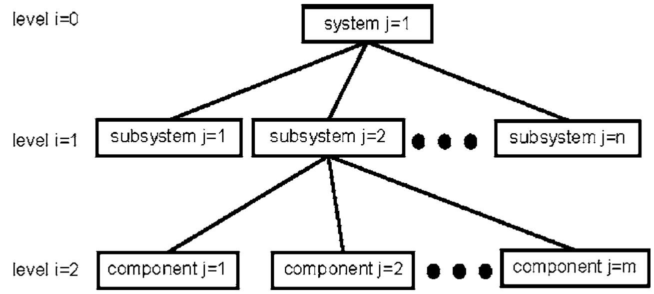

Fig. 1 Example of hierarchically decomposed multilevel system

functional dependency in the hierarchical, multilevel system decomposition. Assuming that element j at level i has n_{ij} children, this functional dependency is expressed as

$$\mathbf{r}_{ij} = \mathbf{f}_{ij}(\mathbf{r}_{(i+1)1}, \ldots, \mathbf{r}_{(i+1)n_{ij}}, \mathbf{x}_{ij}, \mathbf{y}_{ij}) \tag{1}$$

where $\mathbf{r}_{ij}$ are element's responses, $\mathbf{r}_{(i+1)1}, \ldots, \mathbf{r}_{(i+1)n_{ij}}$ denote children responses, $\mathbf{x}_{ij}$ represent local design variables, and $\mathbf{y}_{ij}$ denote local shared design variables (i.e., design variables that this element shares with other elements at the same level). The mathematical formulation of problem p_{ij} for element j at level i is

$$\begin{aligned}
&\text{min} && \|\mathbf{r}_{ij} - \mathbf{r}^u_{ij}\|_2^2 + \|\mathbf{y}_{ij} - \mathbf{y}^u_{ij}\|_2^2 + \epsilon^r_{ij} + \epsilon^y_{ij} \\
&\text{with respect to} && \mathbf{r}_{(i+1)1}, \ldots, \mathbf{r}_{(i+1)n_{ij}}, \mathbf{x}_{ij}, \mathbf{y}_{ij}, \mathbf{y}_{(i+1)1}, \ldots, \mathbf{y}_{(i+1)n_{ij}}, \epsilon^r_{ij}, \epsilon^y_{ij} \\
&\text{subject to} && \sum_{k=1}^{n_{ij}} \|\mathbf{r}_{(i+1)k} - \mathbf{r}^l_{(i+1)k}\|_2^2 \leq \epsilon^r_{ij} \\
& && \sum_{k=1}^{n_{ij}} \|\mathbf{y}_{(i+1)k} - \mathbf{y}^l_{(i+1)k}\|_2^2 \leq \epsilon^y_{ij} \\
& && \mathbf{g}_{ij}(\mathbf{r}_{ij}, \mathbf{x}_{ij}, \mathbf{y}_{ij}) \leq \mathbf{0} \\
& && \mathbf{h}_{ij}(\mathbf{r}_{ij}, \mathbf{x}_{ij}, \mathbf{y}_{ij}) = \mathbf{0}
\end{aligned} \tag{2}$$

where coordinating variables for the shared design variables of the children are denoted by $\mathbf{y}_{(i+1)1}, \ldots, \mathbf{y}_{(i+1)n_{ij}}$, local design inequality and equality constraints are represented by $\mathbf{g}_{ij}$ and $\mathbf{h}_{ij}$, respectively, and tolerance optimization variables ϵ^r and ϵ^y are introduced to coordinate responses and shared variables, respectively. Superscripts u (l) are used to denote response and shared variable

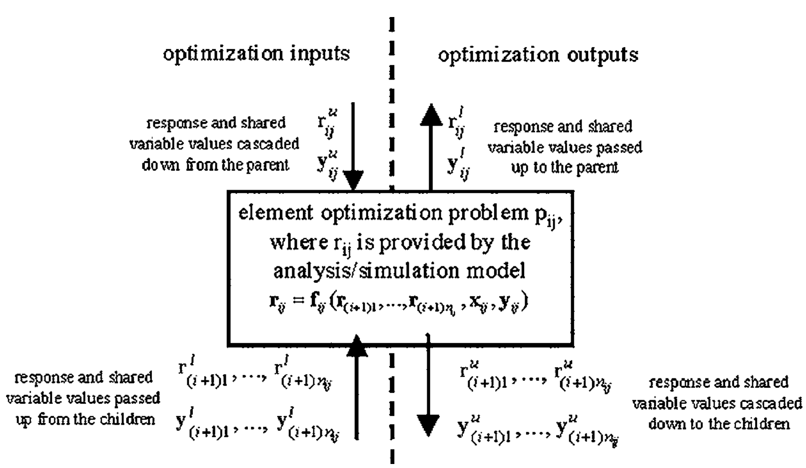

Fig. 2 ATC information flow at element *j* of level *i*

values that have been obtained at the parent (children) problem(s) and have been cascaded down (passed up) as design targets (consistency parameters), cf. Fig. 2. Note that although communication among levels, i.e., updating parameter values associated with the ATC process, is bidirectional, functional dependency is strictly hierarchical.

Assuming that all the element's problem parameters have been updated using the solutions obtained at the parent and children problems, Eq. (2) is solved to update the parameters of the parent and children problems. All the subproblems are solved iteratively according to an appropriate coordination strategy until all tolerance optimization variables cannot be reduced any further.

2.2 Probabilistic Formulation. In this section, the ATC formulation is extended to account for uncertainties. Uncertain quantities are represented by random variables and parameters (denoted by uppercase Latin symbols). We use the means of random design variables as optimization variables and assume that their standard deviation is known or has been estimated with sufficient accuracy. The objective and the constraints must be reformulated. We replace the objective function with its expectation, and we now require that the probability of violating a constraint under the presence of uncertainties is less than some prespecified probability of failure. The probabilistic formulation of problem (2) is

$$\begin{aligned}
\min \quad & \|E[\mathbf{R}_{ij}] - \mu^u_{\mathbf{R}_{ij}}\|_2^2 + \|\mu_{\mathbf{Y}_{ij}} - \mu^u_{\mathbf{Y}_{ij}}\|_2^2 + \epsilon^R_{ij} + \epsilon^Y_{ij} \\
\text{with respect to} \quad & \mu_{\mathbf{R}_{(i+1)1}}, \ldots, \mu_{\mathbf{R}_{(i+1)n_{ij}}}, \mu_{\mathbf{X}_{ij}}, \mu_{\mathbf{Y}_{ij}}, \mu_{\mathbf{Y}_{(i+1)1}}, \ldots, \mu_{\mathbf{Y}_{(i+1)n_{ij}}} \epsilon^R_{ij}, \epsilon^Y_{ij} \\
\text{subject to} \quad & \sum_{k=1}^{n_{ij}} \|\mu_{\mathbf{R}_{(i+1)k}} - E[\mathbf{R}_{ij}]^l\|_2^2 \leq \epsilon^R_{ij} \\
& \sum_{k=1}^{n_{ij}} \|\mu_{\mathbf{Y}_{(i+1)k}} - \mu^l_{\mathbf{Y}_{(i+1)k}}\|_2^2 \leq \epsilon^Y_{ij} \\
& P[\tilde{\mathbf{g}}_{ij}(\mathbf{R}_{ij}, \mathbf{X}_{ij}, \mathbf{Y}_{ij}) > 0] \leq \mathbf{P}_f
\end{aligned} \tag{3}$$

where $P[\cdot]$ denotes probability measure and $\mathbf{P}_f$ is a vector of prespecified probability of failure thresholds.

Note that the ATC-related consistency constraints in Eq. (3) are deterministic and that there are no probabilistic design equality constraints. Equality constraints do not make sense in a probabilistic formulation. It is meaningless to require that a function takes exactly a specific value under the presence of uncertainty, since the probability of a continuous random variable taking an exact value is zero. Therefore, we rewrite design equality constraints as inequality constraints and combine them with preexisting design inequality constraints into one vector function denoted by $\tilde{\mathbf{g}}$.

3 Propagation of Uncertainties

In a multilevel hierarchy, responses of lower-level elements are inputs to higher-level elements. This is an issue of outmost importance in design optimization of hierarchically decomposed systems under uncertainty, since the solution of probabilistic optimization problems requires variance information of the random optimization variables. Consider element j at level i. By solving Eq. (3), we obtain optimal values $\mu^*_{\mathbf{R}_{(i+1)1}}, \ldots, \mu^*_{\mathbf{R}_{(i+1)n_{ij}}}$, $\mu^*_{\mathbf{X}_{ij}}$, and $\mu^*_{\mathbf{Y}_{ij}}$. Using the functional dependency relation $\mathbf{R}_{ij} = \mathbf{f}_{ij}(\mathbf{R}_{(i+1)1}, \ldots, \mathbf{R}_{(i+1)n_{ij}}, \mathbf{X}_{ij}, \mathbf{Y}_{ij})$, we must now estimate the standard deviations of the responses $\mathbf{R}_{ij}$ since they constitute parameters of the parent probabilistic optimal design problem (estimation of means is not critical because they constitute optimization variables of the parent probabilistic optimal design problem; accurate estimates provide merely good initial guesses for the optimization algorithm). This needs to be done for all problems at all levels of the hierarchy. An efficient and accurate technique is therefore required for propagating uncertainties through the multilevel hierarchy. We assume that all element responses in the multilevel hierarchy are uncorrelated.

In an initial effort, a first-order Taylor expansion about the current mean design was used to estimate the mean and standard deviation of propagated random responses [18]. In further research, however, we found that while the mean values were estimated accurately, the standard deviations were unacceptably inaccurate. In this paper, we propose a technique based on the highly efficient and accurate advanced mean value (AMV) method [19].

3.1 Advanced Mean Value Method-Based Technique. The AMV method has been originally proposed as a computationally efficient method for generating the cumulative distribution function (CDF) of a response $R=f(\mathbf{X})$ that is a random variable [19]. It uses a simple correction to compensate for errors introduced by a utilized Taylor series approximation.

Based on the CDF definition, we have the following first-order relation between the CDF value of R at a particular value f_0 and the reliability index β

$$P[f \leq f_0] = P[g \leq 0] = \Phi(-\beta) \tag{4}$$

where $g(\mathbf{X})=f(\mathbf{X})-f_0$ and Φ is the standard normal cumulative distribution function. According to the AMV method, if the random variables $\mathbf{X}$ are uncorrelated and normally distributed with means $\mu_{\mathbf{X}}$ and standard deviations $\sigma_{\mathbf{X}}$, the most probable point (MPP) of failure (or design point) in the standard normal space can be computed by

$$\mathbf{U}^* = -\beta \Sigma_{\mathbf{X}} \frac{\nabla g_{\text{lin}}(\mu_{\mathbf{X}})}{|\nabla g_{\text{lin}}(\mu_{\mathbf{X}})|} = -\beta \Sigma_{\mathbf{X}} \frac{\nabla f(\mu_{\mathbf{X}})}{|\nabla f(\mu_{\mathbf{X}})|} \tag{5}$$

where $g_{\text{lin}}(\mathbf{X})$ is a linear approximation of $g(\mathbf{X})$ at $\mu_{\mathbf{X}}$ and $\Sigma_{\mathbf{X}}$ is a diagonal matrix, whose diagonal is the vector $\sigma_{\mathbf{X}}$. In the original space, the MPP coordinates are

$$\mathbf{X}^* = \Sigma_{\mathbf{x}} \mathbf{U}^* + \mu_{\mathbf{x}} \tag{6}$$

Note that for random variables that are not normally distributed, a nonlinear transformation is needed according to the Rackwitz-Fiessler method [20].

The AMV method corrects the CDF value of R in Eq. (4) with

$$P[f \leq f(\mathbf{X}^*)] = \Phi(-\beta) \tag{7}$$

by replacing the f_0 value corresponding to the reliability index β with $f(\mathbf{X}^*)$. The process of Eqs. (4)–(7) is repeated for a few (different) β values, so that a region of the CDF of R is constructed. The derivative of that CDF region provides the corre-

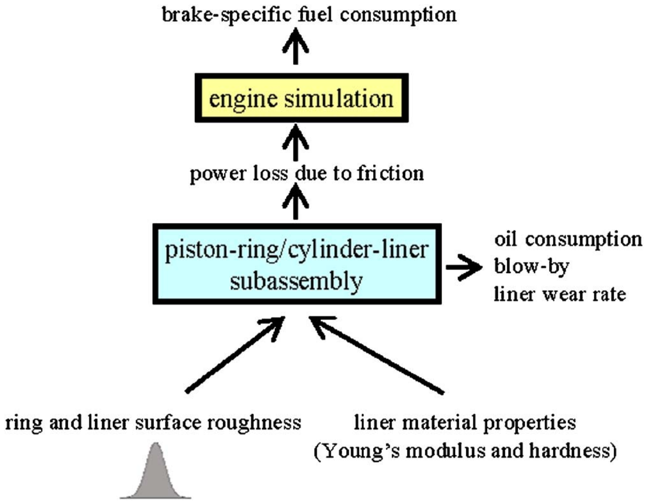

Fig. 3 Hierarchical bilevel system

sponding probability density function (PDF) value. The obtained CDF and PDF values are finally used to compute equivalent mean and standard deviation at the current design point [20].

This AMV-based technique is used to estimate the mean and standard deviation of each response for all the elements of the multilevel hierarchy according to the discussion in Sec. 3. The technique is computationally efficient since it requires only a single linearization of the performance function at the mean value and an additional function evaluation at each required CDF level. Reference 21 provides more details regarding the accuracy and efficiency of the AMV method on several applications.

3.2 Probabilistic ATC Coordination. Assuming that initial uncertainty information is available at the bottom (lowest) level of the hierarchy, our coordination strategy for propagating it during the ATC process can be summarized in the following steps:

1. Start at the bottom level of the hierarchy, where probability distribution on the input random variables and parameters is assumed known.
2. Solve the probabilistic design optimization problems for the level specified in step 1.
3. Use the approach described in Sec. 3.1 to obtain the equivalent means and standard deviations of the response variables that are inputs to the parent problems.
4. Using the information obtained at step 3, solve the parent problems.
5. Move your way to the top of the hierarchy.
6. Once you have reached the top-level problem, start moving toward the bottom using previous solutions to update parameters as shown in Fig. 2.
7. Keep iterating until all ϵ values in all problems have converged (i.e., are not changing anymore).

4 Example

The probabilistic formulation of the ATC process (Eq. (3)) is used to solve a simple yet illustrative simulation example. We consider a V6 gasoline engine as the system, which is "decomposed" into a subsystem that represents the piston-ring/cylinder-liner subassembly of a single cylinder. The system simulation predicts engine performance in terms of brake-specific fuel consumption. Although the engine has six cylinders, they are all designed to be identical. For this reason, we only consider one subsystem.

The associated bilevel hierarchy, shown in Fig. 3, includes the engine as a system at the top level and the piston-ring/cylinder-liner subassembly as a subsystem at the bottom level. The ring/liner subassembly simulation takes as inputs the surface roughness of the ring and the liner and the Young's modulus and hardness and computes power loss due to friction, oil consumption, blow-by, and liner wear rate. The root mean square (RMS) of asperity height is used to represent asperity roughness, which is assumed to be normally distributed. The engine simulation takes then as input the power loss and computes brake-specific fuel consumption of the engine. Commercial software packages were used to perform the simulations. A detailed description of the problem can be found in [18].

4.1 Problem Formulation. Because of the simplicity of the given problem structure, we will use here a simplified version of the notation introduced earlier. Since there are only two levels with only one element in each, we skip element indices and denote the upper-level element with subscript 0 and the lower-level element with subscript 1. We use second indices to denote the components of the design variable vector of the lower-level element optimization problem. The design problem is to find optimal mean values $\mu_{X_{11}}$ and $\mu_{X_{12}}$ for the piston-ring and cylinder-liner surface roughness random variables X_{11} and X_{12}, respectively, and optimal values for the deterministic design variables representing the material properties (Young's modulus x_{13} and hardness x_{14}) of the liner that yield minimized expected value of brake-specific fuel consumption R_0. The optimal design is subject to constraints on liner wear rate, oil consumption, and blow-by. The power loss due to friction R_1 links the two levels.

The top- and bottom-level ATC problems are formulated as

$$\begin{aligned} \min_{\mu_{R_1},\epsilon^R} \quad & (E[R_0]-T)^2+\epsilon^R \\ \text{subject to} \quad & (\mu_{R_1}-E[R_1]^l)^2 \leq \epsilon^R \\ \text{with} \quad & R_0=f_0(R_1) \end{aligned} \tag{8}$$

and

$$\begin{aligned} \min_{\mu_{X_{11}},\mu_{X_{12}},x_{13},x_{14}} \quad & (E[R_1]-\mu_{R_1}^u)^2 \\ \text{subject to} \quad & P[\text{liner wear rate} > 2.4\times10^{-12}\ \text{m}^3/\text{s}] \leq P_f \\ & P[\text{blow-by} > 4.25\times10^{-5}\ \text{kg/s}] \leq \text{P}_f \\ & P[\text{oil consumption} > 15.3\times10^{-3}\ \text{kg/hr}] \leq P_f \\ & P[X_{11}<1\ \mu\text{m}] \leq P_f, \quad P[X_{11}>10\ \mu\text{m}] \leq P_f \\ & P[X_{12}<1\ \mu\text{m}] \leq P_f, \quad P[X_{12}>10\ \mu\text{m}] \leq P_f \\ & 340\ \text{GPa} \geq x_{13} \geq 80\ \text{GPa} \\ & 240\ \text{BHV} \geq x_{14} \geq 150\ \text{BHV} \\ \text{with} \quad & R_1=f_1(X_{11},X_{12},x_{13},x_{14}) \end{aligned} \tag{9}$$

respectively. The standard deviation of the surface roughnesses was assumed to be 1.0 μm, and remained constant throughout the ATC process. The assigned probability of failure P_f was 0.13%, which corresponds to the target reliability index $\beta=3$. The fuel consumption target T was simply set to zero to achieve the best fuel economy possible.

Note that since the random variables are normally distributed, the associated linear probabilistic bound constraints are reformulated as deterministic. For example,

$$P[X_{11}<1\ \mu\text{m}] \leq P_f \Leftrightarrow P[X_{11}-1\ \mu\text{m}<0] \leq P_f \Leftrightarrow$$

$$\Phi\left(0-\frac{\mu_{X_{11}}-1\ \mu\text{m}}{\sigma_{X_{11}}}\right) \leq \Phi(-\beta) \Rightarrow -\frac{\mu_{X_{11}}-1\ \mu\text{m}}{\sigma_{X_{11}}} \leq -\beta \Leftrightarrow$$

$$\frac{\mu_{X_{11}}-1\ \mu\text{m}}{\sigma_{X_{11}}} \geq \beta \Leftrightarrow \mu_{X_{11}}-1\ \mu\text{m} \geq \beta\sigma_{X_{11}} \Leftrightarrow$$

Table 1 Optimal ring/liner subassembly design

Variable	Description	Value
$\mu_{X_{11}}$	Ring surface roughness (μm)	4.00
$\mu_{X_{12}}$	Liner surface roughness (μm)	6.15
x_{13}	Liner Young's modulus (GPa)	80
x_{14}	Liner hardness (BHV)	240

$$\mu_{X_{11}} \geqslant 1\ \mu\text{m} + \beta\sigma_{X_{11}} \Leftrightarrow \mu_{X_{11}} \geqslant 4\ \mu\text{m}$$

Similarly, the other three probabilistic bound constraints in Eq. (9) are reformulated as

$$\mu_{X_{11}} \leqslant 7\ \mu\text{m};\quad \mu_{X_{12}} \geqslant 4\ \mu\text{m};\quad \mu_{X_{12}} \leqslant 7\ \mu\text{m}$$

4.2 Results. It is desired to minimize power loss due to friction in order to optimize engine operation and, thus, maximize fuel economy. Therefore, it was anticipated that the bottom-level optimization problem would yield a design with as smooth surfaces (low surface roughnesses) as possible.

The probabilistic ATC process of solving Eqs. (9) and (8) iteratively converged after two iterations. The obtained optimal ring/liner subassembly design is shown in Table 1.The ring surface roughness optimal value is at its probabilistic lower minimum, while the liner's Young's modulus and hardness optimal values are at their deterministic lower and upper bounds, respectively. The liner surface roughness is not, however, at its lower bound because the problem is bounded by the oil consumption constraint. A certain degree of surface roughness is required to maintain an optimal oil film thickness in order to avoid excessive oil consumption. For this reason, the associated constraint is active and the surface roughness of the liner is an interior optimizing argument. Note that constraint activity in probabilistic design optimization indicates that the constraint's MPP lies on the target reliability circle as opposed to deterministic design optimization, where a constraint is active if removing it or moving its boundary affects the location of the optimum.

A Monte Carlo simulation was performed to assess the accuracy of the reliability analyses of the probabilistic constraints. One million samples were generated using the mean and standard deviation values of the design variables, and the constraints were evaluated using these samples to calculate the probability of failure. Results are summarized in Table 2. The obtained design is 0.03% less reliable than found for the active probabilistic constraint. This error is due to the first-order reliability approximation used in the probabilistic optimization problem.

Propagation of uncertainty was modeled using the approach described in Sec 3.1. Table 3 summarizes the estimated moments for the two responses of the bilevel hierarchy.Results obtained using the first-order approximation approach (linearization) are included to illustrate the large error that may be introduced. Specifically, it can be seen that the standard deviation estimate of the power loss (necessary for solving the top-level probabilistic optimization problem) is 0.0481 kW when using a first-order approximation. This value is 54.6% larger than the Monte Carlo simulation estimate of 0.0311 kW. Such large errors will be propagated during the ATC process and yield useless design results. Using the AMV-based approach, we obtained an estimate of 0.0309 kW, which is only 0.64% smaller than the Monte Carlo estimate.

Table 2 Reliability analysis results

Constraint	Active	P_f %	MCS P_f %
Liner wear rate	No	< 0.13	0
Blow-by	No	< 0.13	0
Oil consumption	Yes	0.13	0.16

Table 3 Estimated moments and errors relative to Monte Carlo simulation (MCS)

Response	Power loss (kW)	Fuel consumption (kg/kWhr)
μ_{lin}	0.3950	0.5341
μ_{AMV}	0.3922	0.5431
μ_{MCS}	0.3932	0.5432
ϵ_{lin} (%)	0.45	−0.01
ϵ_{AMV} (%)	−0.25	−0.01
σ_{lin}	0.0481	0.00757
σ_{AMV}	0.0309	0.00760
σ_{MCS}	0.0311	0.00759
ϵ_{lin} (%)	54.6	−0.25
ϵ_{AMV} (%)	−0.64	0.13

Using the AMV-based technique is advantageous because CDFs and PDFs can be generated with high efficiency. In our example, power loss (the subsystem response) is a highly nonlinear function of the subsystems inputs. In fact, its PDF is multimodal, as shown in Fig. 4(*a*), which depicts the PDF obtained using the AMV-based technique. Figure 4(*b*) depicts the frequency diagram gen-

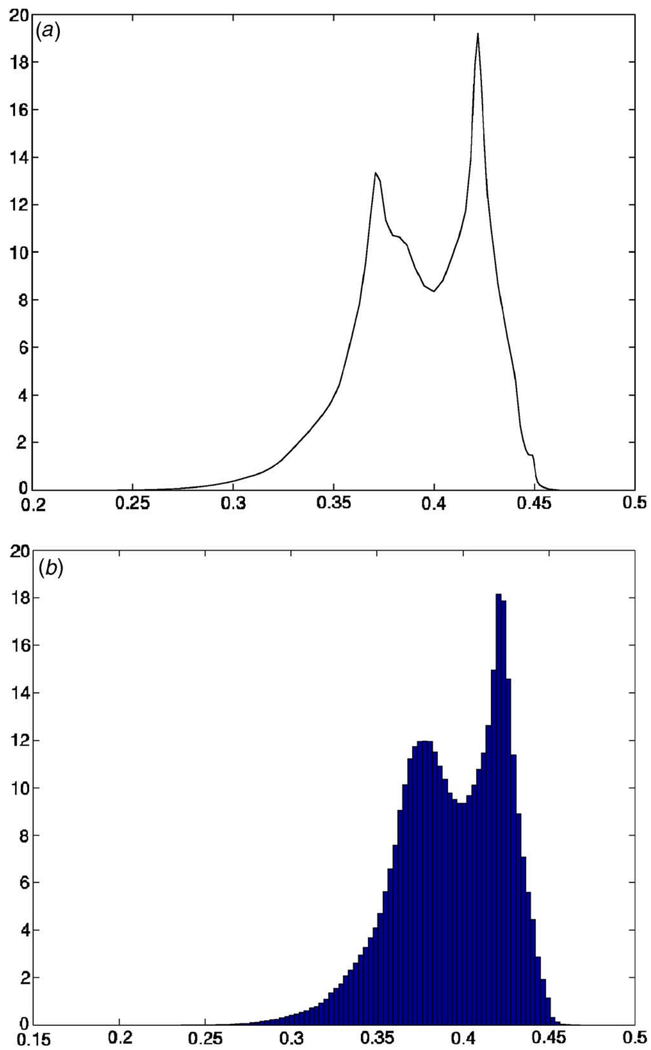

Fig. 4 Power loss due to friction (subsystem's response): (*a*) PDF obtained using the AMV-based technique and (*b*) frequency diagram obtained using Monte Carlo simulation

erated from a histogram that was obtained using Monte Carlo simulation with one million samples. The agreement is quite satisfactory and illustrates the usefulness of the AMV-based approach to propagate uncertainty for highly nonlinear functions. On the other hand, fuel consumption (the system's response) is almost a linear function of the power loss. In this case, the linearization approach is accurate.

5 Summary and Conclusions

We have presented a methodology for design optimization of hierarchically decomposed multilevel systems under uncertainty. We extended the deterministic formulation of analytical target cascading (ATC) by treating uncertain quantities as random variables and formulating probabilistically constrained optimization subproblems. Recognizing that first-order approximations may yield inaccurate estimates of standard deviations of propagated random variables, we proposed a technique that is based on the advanced mean value (AMV) method. This technique can be used to generate approximate CDFs and PDFs that yield sufficiently accurate estimations of means and standard deviations of propagated random variables.

A simple yet illustrative bilevel example was used to demonstrate the proposed methodology. The results showed that the probabilistic formulation of the ATC process can be applied successfully using a bottom-to-top coordination. The computationally efficient AMV-based technique for the required propagation of uncertainties produced standard deviation estimates that were much more accurate relative to the ones obtained using first-order approximations, ensuring thus the meaningfulness of the ATC results.

Acknowledgment

We would like to thank Kuei-Yuan Chan and Ashish Aleti for their assistance with computations. This research was partially supported by the Automotive Research Center, a U.S. Army Center of Excellence in Modeling and Simulation of Ground Vehicles at the University of Michigan, and by a Dual-Use Science and Technology Project cofunded by the U.S. Army and General Motors.

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