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Optimization and integration of ground vehicle systems

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This article deals with the optimal design of ground vehicles and their subsystems, with particular reference to 'active' safety and comfort. A review of state-of-the-art optimization methods for solving vehicle system design problems, including the integration of electronic controls, is given, thus further encouraging the use of such methods as standard tools for automotive engineers. Particular attention is devoted to the class of methods pertaining to complex system design optimization, as well as approaches for the optimal design of complex systems under uncertainty. Some examples of design optimizations are given in the fields of vehicle system dynamics, powertrain/internal combustion engine design, active safety and ride comfort, vehicle system design and lightweight structures, advanced automotive electronics, and smart vehicles.

Keywords: Ground vehicles; Vehicle design; Optimization; Active safety; Ride comfort; Smart vehicles

1. Introduction

In the extant literature, the design process has been described in many ways [60, 66]. Here, we assume that this process is composed of four stages, namely *conceptual design* (determines the principle of a design solution), *preliminary design* (verifies whether the conceptual design complies with physical laws), *embodiment design* (the design is physically instantiated), and *detail design* (refines the results of the previous stage). We assume that, after the conceptual design stage, design optimization can be performed on the basis of the methods presented in this article. In addition, we assume that suitable mathematical models are available for establishing the relationships (in general complex) between *design variables* and *performance indices* [75, 79]. Performance indices can be computed by means of the *objective functions.* Given the validated model, the designer is often charged with the task of finding 'one design solution' by changing a number of model parameters. The methods presented in this article allow one to achieve (or to approach as much as possible) the values of the performance indices required by the designer. The problem that arises in such a context is that often the designer

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is not aware of the possible performances of the system under consideration and not able to start the optimization process by setting the numerical values of performance indices. So, the designer's activity undergoes iterations between an analysis stage, in which the relationships between parameters, performance indices, and inherent design limitations are assessed, and a synthesis stage, in which the possible and desired performances are obtained by a proper parameter setting. This iteration between analysis of the problem and synthesis of a solution is common to many methods of design theory [60, 79].

Optimisation methods may not be employed in practice because of the inherent complexity of many practical problems, which make a mathematical formulation based on optimization theory impossible or ineffective. Increases in modelling and computational capabilities are making optimization more attractive, so that new design solutions will be obtained on the basis of a rigorous mathematical formulation as well as on the designers' *ex tempore* inventions.

A very important issue of modern optimization is that the designer is made aware of why and how a solution set is optimal, *i.e.* an insight into the design problem is provided during the optimization process [77].

After a brief presentation of complex system design methods, we present examples of optimal designs performed in the field of vehicle system dynamics, powertrain/internal combustion engine design, active safety and ride comfort, vehicle system design and lightweight structures, advanced automotive electronics, and smart vehicles.

2. Basics

In this section, some basic concepts of optimization theory are reviewed. The interested reader may refer to refs. [3, 66, 75, 79, 80] for an in-depth documentation on these topics. It is assumed that an accurate (validated) mathematical model of the system under optimization is available.

2.1 *Formulation of the optimization problem*

The majority of engineering problems involve a constrained optimization formulation, that is, the task of minimising (or maximising) a vector of objective functions (performance indices) subject to different types of constraints. Objective functions and constraints must be quantifiably expressed as functions (*f, g*) of the design variables *x*.

Find

$$\begin{aligned} &\min_{x\in R^n} f(x) \\ &h_j(x) = 0, \quad j = 1, \dots, m1 \\ &g_j(x) \le 0, \quad j = 1, \dots, m2 \\ &x \in X \end{aligned} \tag{1}$$

where f is the objective function vector, x the vector of the design variables, X the definition domain of x, and h_j, and g_j the equality and inequality constraints, respectively. The set X can represent certain ranges of real values or certain types, such as integer or 'standard' values, which are very often used in design specifications [75, 79]. Should a maximisation problem occur (*i.e.* max $f(x)$), the vector function $f(x)$ could be re-written as $-f(x)$ or $1/f(x)$ and formulation (1) could be still used [3].

Several methods exist for converting the multi-objective formulation into one with a scalar objective function that can be solved with the usual single objective optimization methods, typically using some sort of weighting function. The shortcoming of such scalarisation is that the designer must include subjective information and preferences *a priori*. In addition, some optimal solutions can be unattainable for non-convex problems [see refs. 75, 79].

The most common and rigorous approach is to define (and then select) those (and only those) solutions, as *efficient solutions* which improve at least one objective function and worsen at least one of the others. Under this assumption, the set of efficient solutions constitutes the Pareto-optimal set [66, 71, 74]. The disadvantage of this approach is that generating Pareto-optimal sets is generally computationally expensive.

The Pareto-optimal solutions are defined mathematically as follows. Given the minimisation problem (1) with k objective functions and n design variables, a solution x_i is Pareto-optimal if there is no solution x_j such that

$$\begin{cases} f_m\left(x_j\right) \leqslant f_m\left(x_i\right) & m = 1, 2, 3, \ldots, k \\ \exists l : f_l\left(x_j\right) < f_l\left(x_i\right) \end{cases} \tag{2}$$

This definition can be used to find the Pareto-optimal solutions directly [68].

2.2 *The solution methods*

The problem in equation (1) is formally a mathematical programming problem [3, 75, 79, 80]. The functions f and g in equation (1) can be expressed with algebraic equations or computer simulations. If the functions f and g are all linear, the problem is a linear programming one. Otherwise, the problem is a nonlinear programming one. Discrete programming refers to problems where all the design variables have only discrete values. A large class of design problems comprises mixed-discrete/continuous design variables values [80]. These types of problems are usually a challenge for the standard methods of nonlinear programming. In table 1, some of the methods for solving optimization problems are presented and compared [60]. This topic is further discussed in the following subsections.

Table 1. Methods for solving optimization problems.

	Computational efficiency					
	Simple problem	Complex problem	Accuracy		Discrete design variables values allowed	Continuous objective functions
Exhaustive	−	−−	++ (Discrete design variables values)	− (Continuous design variables values)	Yes	No
Uniformly distributed sequences	+	−	−		Yes	No
Evolutionary algorithms	+	+	+		Yes	No
Penalty methods (SUMT)	+	−	+		No	Yes
Sequential quadratic programming	+	+	+		No	Yes

Note: −, poor; −−, very bad; +, good; ++, very good.

2.2.1 Nonlinear programming. Nonlinear programming methods have been extensively studied in the literature. The reader may refer to [3, 75, 79, 80] for a detailed discussion on this topic.

2.2.2 Search based on uniformly distributed sequences. Within the large set of methods to find optimal solutions, the *exhaustive search method* [68] is the simplest, but it requires a huge amount of computational effort. Orthogonal arrays and low-discrepancy sequences are used to reduce as much as possible the number of combinations of design variables used to explore the design variable domain [90]. In multi-dimensional spaces (more than three and up to 30 or 40), a low-discrepancy sequence can be used to define a set of design variables combinations to be exploited to sample the objective functions domain. Low-discrepancy sequences differ from pseudo-random sequences in that the points are more evenly distributed in the feasible space. For a detailed description of this topic, see [68, 90].

2.2.3 Evolutionary algorithms. Two types of evolutionary algorithms are considered here, namely genetic algorithms (GAs) and simulated annealing.

GAs are suitable for finding the minimum of a function (or of a set of functions) by performing a semi-stochastic search [8, 12, 34, 39, 82, 87]. The design variable vector referring to a particular design solution is typically encoded into binary or real strings (chromosomes). GAs are based on an elitist reproduction strategy, where the strongest members of the population (design solutions) are selected for reproduction and are given the opportunity to strengthen the chromosomal (*i.e.* genes, namely design variables) makeup of the next generation. Unlike many other search techniques, GAs consider multiple design solutions (a population) at each iteration.

Some comments on GAs are as follows.

- GAs work on function evaluations alone and do not require function derivatives;
- GAs proceed from several points in the design variable domain (population); consequently, the method has a better probability of locating the global minimum;
- GAs allow design variable spaces consisting of a mix of continuous and discrete variables;
- GAs use probabilistic transition rules, not deterministic rules;
- GAs can be easily implemented on parallel computers;
- Computational costs are high, and good performance often requires tuning of the algorithm for the specific problem at hand.

A ranking method can be easily included to grade the population in terms of Pareto-optimality and construct a procedure able to identify the entire Pareto-optimal set and not, as it occurs for standard algorithms, one single Pareto-optimal solution for each run [8, 13, 34, 87].

Simulated annealing is a heuristic search method for obtaining good solutions to difficult optimization problem. Kirkpatrick *et al.*, Pardalos *et al.* [50, 80] showed how a model of simulating the annealing process in solids as proposed by Metropolis *et al.* [67] could be used for optimization problems. Annealing is the process that refers to finding a low-energy state of a solid by initially melting the substance and then lowering the temperature slowly. At high temperatures, the molecules of a liquid move freely with respect to each other; if the liquid is cooled slowly, the thermal mobility is lost and the atoms are able to line themselves up and form a pure crystal structure that is completely ordered in all directions. This crystal structure corresponds to the state of minimum energy of the system.

In 1953, Metropolis *et al.* [67] introduced a simple algorithm to simulate annealing process by considering a collection of atoms at a given temperature. At each iteration, an atom is given a small random displacement and the resulting change in energy (δ) of the system is calculated. If the change in energy δ is < 0, the resulting change is accepted. If the change $\delta > 0$, the change is accepted with probability $\exp[\delta/(K_b T)]$, where T is the temperature and K_b is the Boltzmann constant. If a large number of iterations are carried out at each temperature, the system attains thermal equilibrium at each temperature. At thermal equilibrium, the probability distribution of the system follows the Boltzmann distribution; the probability of the system being in a state i at temperature T is $\exp(-E_i)/(K_b\ T\ Z)$, where E_i is the energy of the state i and Z is the partition function required for normalisation. The aforementioned statement expresses the idea that a system in thermal equilibrium at temperature T has its energy probabilistically distributed among all different energy state E. Even at low temperature, there is a chance of system being in high energy state; therefore, there is a corresponding chance for the system to get out of a local minimum energy state in favour of finding a better global one.

Kirkpatrick *et al.* [50] applied the concept proposed by Metropolis *et al.* [67] to solve combinatorial optimization problems. To apply Metropolis algorithm for other than thermodynamic process, one must provide the following:

(a) A description of possible system configurations.
(b) A generation of the random changes in the configurations.
(c) An objective function E, whose minimisation is the goal.
(d) A control parameter T and an annealing schedule.

2.2.4 Global approximation. Optimisation of very complex systems may require a prohibitive simulation effort, even with the most efficient techniques. In this case, the relationship between design variables and objective functions can be *globally approximated* by means of a simpler, approximate mathematical model (*e.g.* an artificial neural network or a piecewise quadratic function or other approximation). This approximate or surrogate model or pure numerical model is defined on the basis of a limited number of simulations (*i.e.* computational experiments) performed by means of the originally validated physical model of the system under consideration.

Typically, the simulation time for the pure numerical model is a small fraction (1/10 to 1/10,000) of the simulation time needed by the original model [65, 69, 72].

The accuracy in the approximation of the outputs depends on the approximation model employed and the computational effort expended to build the model. The first attempts using neural networks for global approximation were reported in ref. [35]. A comparison of different approximation methods is given in ref. [23] and some results are given in table 2.

Table 2. Approximation methods.

	Approximation domain	Evaluation accuracy: Simple model	Evaluation accuracy: Complex model	Tuning effort
Linear interpolation	Local	+	−	++
Quadratic interpolation	Local	+	−	++
Neural networks (Radial basis function)	Local/global	++	+	+
Neural networks (Multi layer perceptron)	Global	++	++	−
Statistical approximation	Global	++	+	−

Note: −, bad; +, good; ++, very good.

2.2.5 Optimal design of multi-level systems. Every system is analysed at the level of complexity that corresponds to the interest of the designer. For this reason, we can identify hierarchical levels in the system definition; the system can be divided into subsystems that can be further subdivided. Analytical target cascading (ATC) is a methodology for optimal design of hierarchical multi-level systems where outputs of lower-level elements are the inputs to higher-level elements (figure 1).

The objective is to identify the element interactions early in the design process and to determine specifications that yield consistent system design with minimised deviation from design targets. The process formulates and solves a minimum deviation optimization problem for each element of the multi-level system. By solving the element optimization problems sequentially in an iterative manner, ATC aims at minimising discrepancies between the optimal design variable values desired at elements higher in the hierarchy and response values that elements at lower levels can actually deliver [49, 52, 73, 74, 79]. In addition, if design variables are shared among some elements at the same level, their required common final optimal value is coordinated by their parent element at the level above.

2.3 *Stochastic multi-objective optimization*

Recently, many authors have introduced procedures that combine design optimization techniques with reliability-based design methods [5, 31, 53, 57, 86, 93]. The approach allows the designer to minimize the objective function while accounting for the influence of stochastic variations of some parameters and/or variables. Typically, this involves trading (deterministic) optimality for robustness, whether in the value of the objective (robust design) or in the satisfaction of the constraints (reliable design).

In the presence of random variables and parameters, responses, such as objective and constraint value calculations, are themselves random variables, whose expected values and/or variances must be computed when solving probabilistic optimization problems. Evaluation of probability distributions of responses can be achieved using Monte Carlo simulation, but this is a very expensive process. Hence, various approximation schemes must be used in practical situations.

A mean-value first-order second-moment approach (first-order Taylor expansion about the current design) can be adopted to estimate the mean and standard deviation of the objective functions.

When a trained artificial neural network is used to compute model responses (see section 2.2), an accurate estimate of the variance of the objective functions can be obtained with no additional computational cost [24].

Uncertainty can also come from considering market behaviour when launching new products. For example, developing a new vehicle design in a dual-use context (commercial

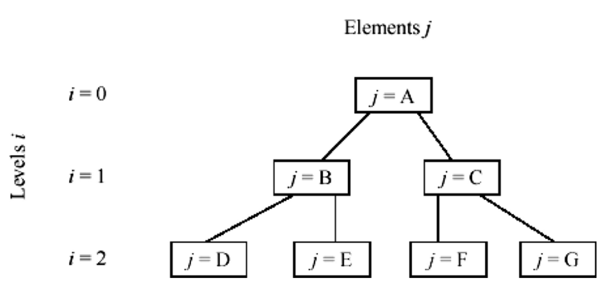

Figure 1. Example of hierarchical multi-level system decomposition.

enterprise and government military uses) is affected by uncertainty in market penetration and cannibalization [10].

2.4 *Analysis of the Pareto-optimal set*

After the optimal solutions have been computed (Pareto-optimal set), the designer can make a choice and select from this set the preferred solution featuring the desired compromise among objective functions.

This process can be preceded and even accelerated by special analyses allowing the designer to have an insight into the physical phenomena being tried to analyse. In other words, there are some analyses by which the designer may understand the reason why the considered optimal solution requires such a design variable combination. In particular, these analyses may show:

– the relationship between two objective functions (within the Pareto-optimal set),
– the relationship between design variables (within the Pareto-optimal set),
– the relationship between design variables and objective functions (within the Pareto-optimal set).

These different relationships are shown in figure 2.

The Spearman rank-order correlation coefficient [33, 66, 77] can be used to assess the previously introduced relationships. This rank-order correlation coefficient has proved to be robust in presence of nonlinearity or outlayers.

2.5 *Available commercial software for engineering design optimization*

A list of commercial software suitable for solving design optimization problems related to vehicles is given subsequently.

The list is by no means exhaustive. The aim is, instead, to give to the reader an idea of the wide set of software solutions available on the market nowadays.

(1) ADAMS/Insight [44] is the optimization tool designed for use within the ADAMS multibody dynamics simulation environment. The software can use design of experiments techniques in order to define a global approximation of the objective functions (polynomial fit).

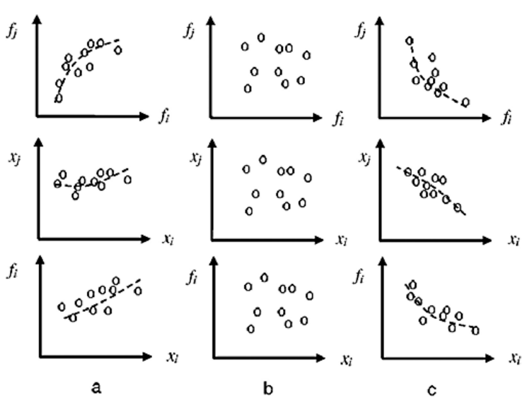

Figure 2. Relationship between Pareto-optimal objective functions (f_i–f_j), between Pareto-optimal design variables (x_i–x_j), and between objective functions and design variables (f_i–x_i). The values can be either directly correlated (column (a)), uncorrelated (column (b)), or indirectly correlated (column (c)).

The optimizer includes several standard nonlinear programming algorithms and allows a multi-objective formulation of the design problem. Some analysis tools can be employed to identify the design variables that most affect the system design.

(2) FRONTIER [41] has a strong emphasis on multi-objective formulations. A design of experiments module can be used in the preliminary design phase (factorial, Sobol, etc.). Linear and nonlinear response surfaces for data modelling and approximation can be used (polynomial fitting, neural networks, etc.). Advanced visualization tools are included.

(3) GENESIS [46] is a structural analysis and optimization software. It integrates various finite element analysis and optimization capabilities. It can also be used as a more general purpose optimizer.

(4) iSIGHT [47] is a general-purpose optimization tool with a large number of alternative algorithms for optimization and building surrogate model functions. It includes ability to solve mixed-discrete problems and to generate Pareto sets. It can also be used to perform probabilistic optimization.

(5) MATLAB Optimization Toolbox [43] is a general-purpose collection of functions for optimization. The Toolbox includes nonlinear programming and multi-objective minimization algorithms (for example, goal attainment). This tool can be integrated with different tools useful in building surrogate model functions.

(6) OPTIMUS [45] is a general-purpose optimization tool. The software can explore the design space using design of experiments; it includes several standard nonlinear programming algorithms and state-of-the-art evolutionary algorithms. Special algorithms are available for solving problems including a mixture of continuous and discrete variables. The variation of a design around its optimal value can be evaluated (stochastic variations) with extended six-sigma methods.

(7) OPTISTRUCT [42] is a finite element-based software for both structural analysis and design optimization. The integrated state-of-the art gradient-based optimization methods allow component size and shape optimization. OptiStruct includes morphing technology to prepare finite element meshes for optimization. Size optimization defines ideal component parameters such as material type, cross section dimensions, and thicknesses. Design of experiments capabilities and stochastic studies are included in the HyperStudy tool, which can be directly interfaced to OptiStruct.

(8) VisualDOC [46] is a general-purpose optimization tool. The software can perform linear, nonlinear, constrained, and unconstrained as well as integer, discrete, and mixed-discrete optimization. Gradient-based, non-gradient-based, and response surface approximate optimization algorithms are available. In addition, a design of experiments module and probabilistic analysis and design capabilities are included.

3. Optimal design of vehicles and subsystems

3.1 *Vehicle system dynamics*

There are a number of papers addressing the preliminary design of ground vehicle suspension systems (active and/or passive). The first attempts to design optimally a suspension system appeared in refs. [11, 38, 40, 48, 59, 92] and in a broader sense in refs. [76, 85]. Referring to the tuning of gains of an actively suspended vehicle, analytical formulae seem to have been derived only for simple system models (results reported in refs. [1, 40, 91]), establishing the well-known ‘sky-hook’ control strategy. Further studies, using quarter car models, on the optimization of the ride comfort and active safety of road vehicles are presented in refs. [27, 29, 30]. Both passively and actively suspended vehicles have been considered. Optimization of vehicle design variables is often performed using multi-objective programming (MOP) (see section 2.2). Simple analytical formulae for the estimation of ride comfort and active safety of vehicles are derived, and these formulae are used in conjunction with MOP to find the optimal suspension design variables symbolically, ensuring the best compromise among comfort, road holding, and working space. In refs. [26, 27], the design variables, *i.e.* suspension stiffness and damping, are considered as stochastic variables and the vehicle body mass and tyre radial stiffness as stochastic parameters in order to derive the optimal trade-off solutions in a stochastic framework (see section 2.3).

A basic study on the optimization of the handling behaviour and active safety of road vehicles is presented in ref. [70]. That article demonstrates how the front and rear tyre cornering stiffnesses of a vehicle can be tuned in order to obtain a preferred compromise among the many conflicting performance indices that describe the handling behaviour and active safety. The Pareto-optimal set has been derived in analytical form (Figure 3).

Similar studies in optimizing vehicle-handling behaviour over a range of manoeuvres but involving a much greater number of design variables are reported in refs. [25, 33, 64, 70, 71, 81, 83, 84, 89]. See section 3.4 for further details.

In order to design a car effectively, one is often required to tune the entire set of design variables related to tyres, aerodynamics, and chassis characteristics (stiffness, damping, and kinematics of the suspension system). Such tuning presents good opportunities for effective use of optimization methods.

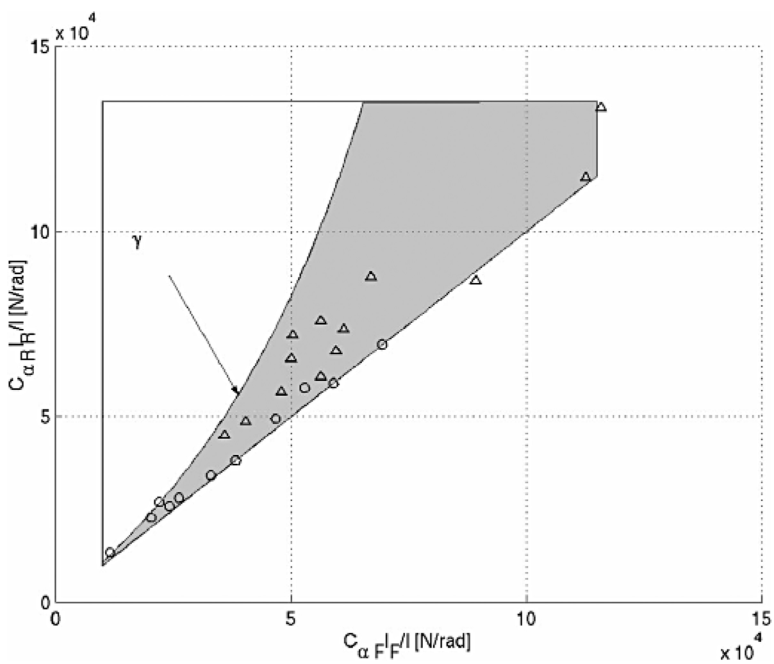

Figure 3. Pareto-optimal set (grey area) featuring the optimal handling performances of a road vehicle in the design variables domain $C_{\alpha F}/l_F$ and $C_{\alpha F}/l_F$ (from [66, 70]), where $C_{\alpha F/R}$ is the front/rear cornering stiffness and $l_{F/R}$ the respective distances from the centre gravity of the front/rear axles. Experimental data denoted by ◯ were derived from the literature and data denoted by Δ were available to the authors and were measured by tyre manufacturers. Adapted from [70].

3.2 *Powertrain/internal combustion engine design*

Hybrid-electric propulsion systems have been optimized with respect to fuel economy, ride, and mobility targets in ref. [51] by applying a target-cascading approach. A method for combined design optimization and power management of a hybrid propulsion system integrated within a complete vehicle system was proposed in ref. [19]. A sequential quadratic programming (SQP) algorithm with a DOE-based multi-start technique was used to enhance the chances of achieving the global optimum (see section 2.2).

The design of the synchroniser and selector mechanism of a vehicle gearbox are discussed in refs. [31, 32]. The main aim there is to improve shifting ability during a reference shift action. Fifty-eight variables of the system model were thus tuned. The approach proposed in [31] addresses both the optimization of nine system performance indices and the reduction of the sensitivity (variance) of the performance indices to stochastic perturbations (see section 2.3). The variances are computed by means of a procedure based on the global approximation of the objective functions (see section 2.2.4).

The use of ATC (see section 2.2.5) in vehicle transmission design was reported in [4]. ATC was applied to the simulation-based design of a CVT implemented in a mid-sized truck. The overall system was decomposed into three levels of increasing modelling detail, leading to three interconnected multi-criteria optimization problems. The study highlighted two main issues. First, the development of the coordination strategy and appropriate models appeared to be an intricate task that benefits from extensive knowledge of the overall problem. Secondly, the values of the arbitrary weights defined at all levels had a significant impact on the results.

3.3 *Internal Combustion engine design*

A probabilistic formulation of the ATC process has been used in refs. [7, 53, 93] to solve an internal combustion engine design problem. An engine is considered at the top level system, which is then decomposed into subsystems representing the piston-ring/cylinder-liner subassembly of the cylinders. The system model predicts engine performance in terms of brake-specific fuel consumption. The ring/liner subassembly simulation takes the surface roughness of the ring and the liner (assumed to be normally distributed) and the Young's modulus and hardness and computes power loss caused by friction. The engine simulation then takes the power loss as inputs and computes brake-specific fuel consumption of the engine.

The link between manufacturing process and product performance is studied in [56] in order to construct quantifiable criteria for the introduction of new engine technologies and processes. Cost associated with a new process must be balanced against increases in engine performance. A predictive engine simulation model is used to quantify performance gains due to the new surface finish obtained by means of abrasive flow machining for finishing the inner surfaces of intake manifolds for two engines. Subsequently, economic cost-benefit analysis is used to evaluate manufacturing decisions on the basis of their impact on the firm's profitability.

3.4 *Active safety and ride comfort*

In [22], an MOP approach using GAs is employed for the design of a multi-link suspension on the basis of performance indices related to vehicle handling and stability (see section 2.2.3).

In [71], a procedure is presented for the integrated design (tuning) of tyres and suspensions of a racing car. Proper objective functions are defined after a subjective–objective correlation analysis. Different driving situations (steady state, J-turn, lane-change, power on–off while steering, braking on a bend, passing over a kerb while steering) are optimized with respect to 18 design variables related to the suspension system and the tyre characteristics. A global approximation model (see section 2.2) was used together with a search method based on low-discrepancy sequences (see section 2.2.2). The same approach has been followed in refs. [20, 81, 84] and in [25] for the problem of the chassis design of a production car. In [33], the computation of the Pareto-optimal set is performed for a similar chassis design problem by using GAs (see section 2.2.3).

The design of heavy vehicles for good dynamic performance in a variety of scenarios or manoeuvres (from standard test manoeuvres to extreme emergency manoeuvres) is discussed in [6]. The problem is formulated as a multi-criteria, multi-scenario design problem with the goal of finding an optimal vehicle design, which can improve the vehicle dynamic performance in all the considered scenarios (lane change, pulse steer, ramp steer, induced roll-over) simultaneously. A modified Monte Carlo optimization technique is used to find the optimal designs.

In refs. [49, 52, 53], ATC has been used to optimise ride quality and handling performances by considering suspension, tyre, and spring analysis models. Potential incompatibilities among targets and constraints throughout the entire system can be uncovered and the trade-offs involved in achieving system targets under different design scenarios can be quantified.

The problem of choosing a single solution within a set of Pareto-optimal alternatives is addressed in [55]. Two methods, the k-optimality approach and the more general $k\varepsilon$-optimality method, were considered. These two methods theoretically justify and mathematically define the designer's tendency to choose solutions that are 'in the middle' of the Pareto-optimal set. The methods were successfully applied to the optimization of the tyre/suspension system of a car.

An optimization method for the improvement of the dynamic behaviour of railway vehicles is introduced in [63]. A method based on MOP was applied to find the best trade-off between conflicting performance indices, standard deviation of body acceleration *vs.* standard deviation of the secondary stroke, with respect to variables associated with the secondary suspension.

In [37], a similar MOP approach was used to optimize wheel and rail profiles to improve lateral stability, wear of components, and curving behaviour of rail vehicles.

3.5 *Vehicle system design and lightweight structures*

The optimal layout of structural components is discussed in many papers [3, 28, 36, 54, 58, 62]. Given a design domain and boundary conditions, the problem is to find the best structure (light and/or stiff) that can carry the loads. The problem of the definition of the layout of the entire vehicle is addressed in recent papers [36]. Real-coded GAs have been used to solve the multi-objective optimization problem employing an elitist search (see section 2.2.3).

An efficient way to find good lightweight designs for vehicle structures is to use different tools for structural optimization [2, 18, 21, 78, 88]. Structural optimization includes several approaches, *e.g.* material [2], size, shape, and topology optimization, where size and shape optimization are used for fixed topologies and for improvements of existing structures. Topology optimization, however, is a more general optimization method that can be used in the early conceptual design phase of vehicle development, where simplified models often are enough to analyse the structural behaviour. An application of this approach is shown in ref. [20]. Using topology optimization methods, the most efficient structures can be found in a relatively early stage of the vehicle design process.

The automotive body structure design problem has been considered in refs. [15–17, 52] by applying a sharing penalty vector method in order to define the product platform, *i.e.* to choose which components to share and to design the product family with minimal individual variation from the ideal individually optimized designs.

The ATC process has been applied also to the optimal design of complete vehicle systems. Novel technologies, such as a hybrid-electric propulsion system, in-hub motors, and variable height suspensions, were considered in [51]. Emphasis was given to fuel economy, ride, and mobility characteristics (see section 2.2.5).

In [61], a method was presented for the conceptual design of railway passenger vehicles. The aim was to define the layout of vehicles (*i.e.* length, number of wheelsets, etc.) in order to obtain the lowest possible life cycle cost. GAs were used to find the numerical solution to the problem (see section 2.2.3).

Several decision-making models related to the acquisition process of vehicles that address a range of conflicting concerns, such as customer needs, life cycle costs, new technology and design, budget allocation, competitive bidding, and the ability of contractors to meet market targets, are discussed in [9].

3.6 *Integration of vehicle electronic controls*

A basic study on active suspensions optimization and integration is given in refs. [1, 30, 94]. As discussed previously, simple analytical formulae for the estimation of ride comfort and active safety of vehicles are derived and used to get the optimal controller gains that offer the best compromise among comfort, road holding, and packaging. In refs. [26, 27], this problem is reformulated considering vehicle body mass and tyre radial stiffness as stochastic parameters in order to derive the optimal trade-off solutions in a stochastic framework (see section 2.3).

Research in several application areas has unveiled a coupling between the plant (design) and control optimization problems. Because of this coupling, optimizing a system's plant and control sequentially does not guarantee system optimality. The results have been demonstrated using a simple combined passive/active car suspension case study in [14].

A method to improve ride and handling of an automobile fitted with four active suspensions (sky-hook dampers and active anti-roll bars), active four-wheel steering, and traction control (controlled differential) has been proposed in [64]. The designer is asked to iterate between an innovation stage and a trade-off stage in order to assess the effectiveness of candidate design issues for the synthesis of new control schemes.

3.7 *Comprehensive optimization example*

A simple example is presented in order to show how, with respect to an engineering problem, very general results can be obtained by applying correctly the optimization procedures introduced previously.

The problem is to select the stiffness and damping of a passively suspended vehicle. The vehicle behaviour, while running on a randomly profiled road, is described by means of the well-known quarter car vehicle model (figure 4) and the objective is to minimize discomfort (the standard deviation of the acceleration of the body of the vehicle, $\sigma_{\ddot{x}_2}$), road holding (the standard deviation of the force acting between the road and the wheel, σ_{F_z}), and working space (the standard deviation of the relative displacement between the wheel and the vehicle body, $\sigma_{x_2-x_1}$).

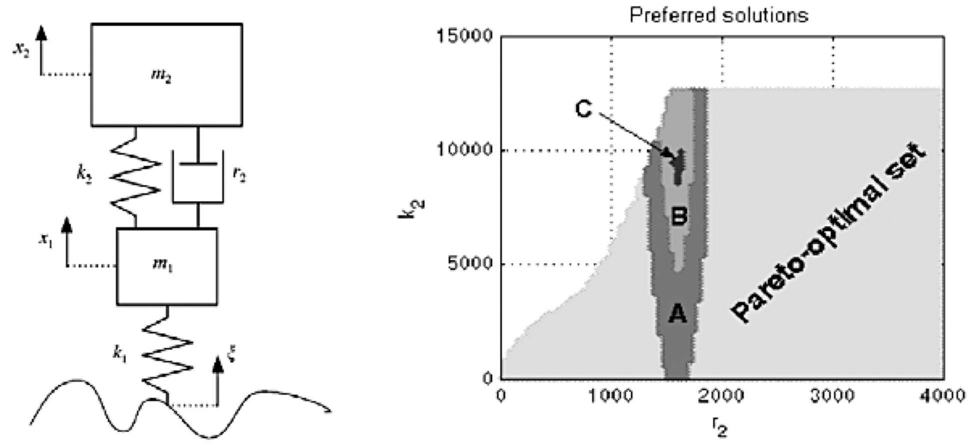

Figure 4. Pareto-optimal set (filled area) and $k\varepsilon$-optimal solutions [55] for the vehicle passive suspension optimization problem in the design variables space (suspension damping r_2 (N s/m), suspension stiffness k_2 (N/m)), and vehicle data reported in ref. [29].

Being (figure 4)

$$q = \frac{m_1}{m_2}, \quad K_x = k_2 \frac{(1+q)^2}{k_1 q}, \quad R_x = r_2 \sqrt{\frac{(1+q)^3}{k_1 m_2 q}}$$

the analytical expressions of the Pareto-optimal set into the (non-dimensional) design variables domain (K_x, R_x) for the three combinations of two objective functions (boundaries of the Pareto-optimal set for the problem with three objective functions, namely $\sigma_{\ddot{x}_2}, \sigma_{F_z}, \sigma_{x_2-x_1}$) have been derived in refs. [27, 29] and they read

$$\begin{aligned}
\text{optimal } (\sigma_{\ddot{x}_2}, \sigma_{F_z}) &\longrightarrow R_x = \sqrt{(1+q)K_x - qK_x^2} \quad \text{with} \quad 0 \le K_x \le 1 \\
\text{optimal } (\sigma_{\ddot{x}_2}, \sigma_{x_2-x_1}) &\longrightarrow R_x \ge 0,\ K_x = 0 \\
\text{optimal } (\sigma_{F_z}, \sigma_{x_2-x_1}) &\longrightarrow R_x \ge 1,\ K_x = 1.
\end{aligned}$$

By inspection of these expressions, the designer can get a direct insight into the suspension design problem. The Pareto-optimal set into the design variable domain (suspension stiffness k_2 and suspension damping r_2) for the $\sigma_{\ddot{x}_2}, \sigma_{F_z}, \sigma_{x_2-x_1}$ problems is the filled area in figure 4 (reference vehicle data are reported in [29]). The solutions belonging to the Pareto-optimal set are equally desirable from the designer point of view.

For the selection of the final suspension design solution, a $k\varepsilon$-optimality approach, extensively described in [55], has been followed. The results are the design solutions within the subset of the Pareto-optimal set named C (darker area) in figure 4. The result seems consistent with the solution selected by very skilled suspensions specialists after an expensive amount of trials.

The described approach has shown to be very effective when applied to complex design problems too.

4. Conclusion

This article presented a brief outline of optimal design methods and their applications to vehicle dynamics problems. Successful optimization requires availability of appropriate analysis models and knowledge of the capabilities and limitations of the mathematical optimization techniques. Most of the modern software available for commercial use are capable of supporting such efforts effectively. However, an increased degree of familiarity is required before more complex studies are undertaken. The inclusion of uncertainty in the problem formulations is a relative recent development in the available optimization tools. Although solution robustness is desirable along with optimality, probabilistic optimal design formulations are substantially more computationally intensive than deterministic ones. Similarly, performing full Pareto-optimality studies can be very expensive.

Included in the end of this article is a large (even though still partial) list of references that can be used to get a deeper insight into the subject and its potential practical implementations. These include problems in vehicle system dynamics, powertrain and internal combustion engine design, active safety and ride comfort, vehicle system design, lightweight structures, and controls.

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