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MANUFACTURING INVESTMENT AND ALLOCATION IN PRODUCT LINE DESIGN DECISION-MAKING

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ABSTRACT

An important aspect of product development is design for manufacturability (DFM) analysis that aims to incorporate manufacturing requirements into early product decision-making. Existing methods in DFM seldom quantify explicitly the tradeoffs between revenues and costs generated by making design choices that may be desirable in the market but costly to manufacture. This paper builds upon previous work coordinating models for engineering design and marketing product line decision-making by incorporating quantitative models of manufacturing investment and production allocation. The result is a methodology that considers engineering design decisions quantitatively in the context of manufacturing and market consequences in order to resolve tradeoffs, not only among performance objectives, but also between market preferences and manufacturing cost.

Keywords: preference coordination, analytical target cascading, design optimization, marketing, decision-based design, design for manufacturing, design for production

1. INTRODUCTION

The new era of globalization has influenced both product portfolio variety and the architecture of the manufacturing systems producing these products. Product designers are interested in reducing the cost of their products while offering product characteristics demanded by a heterogeneous market. Tools such as Quality Function Deployment (QFD) have helped designers organize thinking about the relationship between design decisions and stakeholder preferences, and research in Design for Manufacturing (DFM) has offered practical methods for improving designs with respect to manufacturing considerations (Herrmann *et al*., 2004). However, few of these methods incorporate quantitative models for making tradeoffs between the revenue and cost consequences of design changes that are less costly to manufacture but also less desirable in the marketplace. For example, Taylor *et al*. (1994) discuss how the strategy of “design to fit an existing environment (DFEE)” can significantly reduce costs by adapting new designs to better fit the capacity and capability constraints of existing manufacturing equipment; however, the issue of how much to compromise the desirable features of a new design to improve accommodation on existing equipment, particularly when design compromises have market consequences, remains an open question.

Recent design research has taken interest in coordinating traditional normative performance-based engineering design decision models with models of business objectives. In particular, decision-based design (DBD) research has focused on utilizing the framework of decision theory to examine design decisions under uncertainty with respect to a single objective function, called designer’s utility (Hazelrigg, 1988). This designer’s utility function is typically implemented in terms of the producer’s downstream business objectives, such as profit or market share, and consequently, research on understanding and utilizing models that predict the effects of product characteristics on these firm-level objectives has become critical to defining DBD problem statements fully (Wassenaar and Chen, 2003).

An array of methods has emerged, both within and outside the DBD label, to consider quantitatively the link between technical decisions and business objectives (for example, Gu *et al*., 2002; Georgiopoulos, 2003; Michalek *et al*., 2005a). Most of these methods address the design of a single product; however, two methods in particular address decision making for lines of products, a more useful scope for consideration of manufacturing investment and production allocation. Li and

Azarm (2002) proposed a two-stage method that involves generating a set of designs that approximates the Pareto surface and selecting candidate designs from this set to compose the product line. This method is suited for products with characteristics that are monotonically preferred by the entire consumer population, so that a common Pareto set is defined for all potential users; however, extension to product characteristics that have different ideal values throughout the population, such as the example examined in this paper, is not obvious. Michalek *et al.* (2005b) proposed an alternative method for product line design using analytical target cascading (Kim, 2001) to coordinate a product planning subproblem with a set of engineering design subproblems. In this formulation the product planning subproblem sets target product characteristics for the line based on a heterogeneous model of consumer preferences, and each engineering design subproblem attempts to achieve target product characteristics for one product in the line subject to engineering constraints. This second method is adopted here and extended to include: (1) the setting of production volume targets to be achieved for each product in the line subject to capacity constraints that depend on the allocation of purchased equipment; and (2) the setting of cost budget targets in the planning subproblem, to be achieved by the purchase of equipment and production of the design. The result is a methodology that considers engineering design decisions quantitatively in order to resolve tradeoffs not only among performance objectives, but also between market preferences and manufacturability.

Research in concurrent engineering has aimed to move the product development process from a sequential approach, as shown in Figure 1, toward a concurrent process where the goals and preferences of interrelated disciplines are negotiated iteratively, as shown in Figure 2. The proposed methodology can be viewed as an approach to facilitating communication in concurrent engineering at design stages where parametric models can be called upon to predict results of design decisions. Herrmann *et al.* (2004) discuss: "As industries have grown in size and complexity, marketing, design, and manufacturing departments have evolved into separate organizations, each with [its] own specialized knowledge. While this makes the streamlined creation of complex products possible, it has also increased the knowledge and communication barriers between these areas. … In practice, engineered systems are usually too complex to truly consider all issues simultaneously. More commonly, concurrent engineering (and DFM) is accomplished through an iterative 'spiral' design process … in which marketing experts, designers, manufacturing engineers, and other personnel jump back and forth between identification of customer needs, design of the product, and assessment of manufacturing issues." It is this iterative coordination process that the proposed ATC methodology aims to automate using rigorously defined coordination of mathematical models from each discipline. It is also the hope that the existence of such a structure for coordination and optimization may drive the development of appropriate models in cases where models from some disciplines have not yet been built. Previous work has shown that the ATC coordination process produces optimal solutions from a firm's perspective that are superior to those produced through a sequential approach (Michalek *et al.*, 2005a).

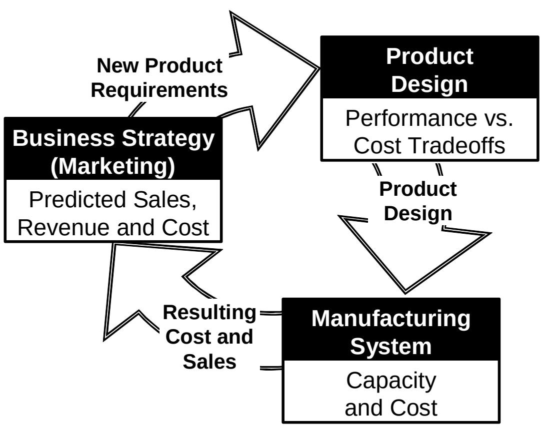

Figure 1: A sequential product development process

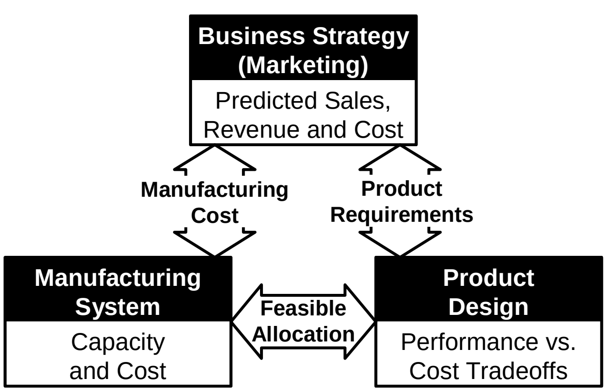

Figure 2: A concurrent engineering process

2. METHODOLOGY

In the proposed methodology, shown in Figure 2, decision models from design, business, and manufacturing are coordinated with one another to make tradeoffs with respect to a firm-level objective and reach a consistent solution that is optimal for the firm. The proposed methodology for coordinating design, business, and manufacturing decisions is built on models developed by Michalek *et al.* (2005b) using analytical target cascading (ATC) for coordination of the disciplines. ATC is a mathematical optimization technique for decomposing a system into a hierarchy of subsystems and coordinating optimization of each subsystem in such a way as to achieve the optimal solution of the overall system. In this case, the system is the firm, and design and manufacturing are viewed as subsystems. Decomposition of a system into a hierarchy of subsystems can be advantageous because it assists in model management and it reduces practical difficulties associated with problem dimensionality, since the models of each subsystem typically have fewer variables and constraints than the combined full system model. ATC was applied first to engineering systems (Kim *et al.*, 2003), and has since been applied in the field of architecture (Choudhary, 2004) and to the coordination of marketing and engineering design decisions in an enterprise context (Michalek *et al.*, 2005a). ATC achieves joint solutions by a procedure involving the setting of targets at each level of the hierarchy for the subsystems at the level below in order to achieve targets passed by supersystems above. This procedure is iterated at each level of the hierarchy until convergence. It was proven by Michelena *et al.* (2003) and later clarified by Michalek and Papalambros (2005) that separately solving subsystems in the ATC hierarchy using certain coordination strategies will produce a solution arbitrarily close to the solution obtained when the full non-decomposed system is solved all together. In the present case, this means that marketing planning, engineering design, and manufacturing models can be solved separately and coordinated, resulting in solutions that are optimal from the firm's perspective. The previous product line decision model (Michalek *et al*., 2005b), upon which the current formulation is built, accounts for marketing and engineering design but does not include manufacturing decisions. In the proposed formulation, previous models for marketing and engineering design are used whenever possible, and Sriraman *et al.* (2002) is referenced for development of the manufacturing components of the model.

In the proposed formulation the ATC hierarchy contains a marketing planning subproblem, a manufacturing subproblem, and one design subproblem for each product $j = \{1, 2, \ldots, J\}$ in the product line. The task of the marketing planning subproblem is to set the price for each product in the line along with targets for each product's characteristics, production volume, and cost so that predicted profit is maximized over a fixed time period. Here, the term "product characteristics" refers to quantitative aspects of the final product observed by the customer resulting from the detailed engineering design decisions. Profit is predicted as a function of cost and demand, where demand depends on the characteristics and prices of the products. Without information from the manufacturing and design subproblems, marketing would set low cost, high production volume, and desirable product characteristics to maximize profit; however, ATC coordination with the other subproblems will ensure that these targets are mutually realizable at the solution. Target costs and product characteristics passed to the product design subproblems are achieved as closely as possible by manipulating the design of each product. Likewise, production volume targets are achieved by allocating design of the product's components to available machines while ensuring that each component can only be made on machines capable of manufacturing the component design.

Life-cycle and dynamic manufacturing issues are not considered in the model. Instead, it is assumed that a set of candidate machine types are available for purchase, and the manufacturing subproblem manages decisions of how many machines of each machine type will be purchased to match the cost targets set by marketing while simultaneously providing sufficient machine capacity for producing the components designed in each engineering design subproblem. The production volume achievable for each product depends on the amount of machine time available for that product, so linking variables are included to coordinate machine time requests and allotments between the engineering design subproblems and the manufacturing subproblem. In the following sections, the mathematical formulation of each subproblem will be described in detail.

2.1 Marketing Planning Subproblem

In the marketing subproblem, shown in Figure 3, decision variables include price p_j, target production volume V_j^{M}, target unit material cost c_j^{M}, and a vector of target product characteristics $\mathbf{z}_j^{\mathrm{M}}$ set for each product j in the product line, as well as a target for manufacturing cost C^{M}, and a coordinating linking variable T_{jm}^{M}, which will be discussed later. These variables are manipulated in order to maximize profit Π. In the ATC framework, the target values set in the marketing planning subproblem are manipulated independently of the values achieved by the manufacturing (C^{P}) and engineering design (V_j^{E}, c_j^{E}, $\mathbf{z}_j^{\mathrm{E}}$) subproblems, but the objective function contains terms to minimize deviation between each target value and achieved value, where deviation is measured using the square of the l_2 norm $\| \|_2^2$, as shown in Figure 3. In this way the details of the design, cost, and capacity allocation are handled outside of the marketing subproblem, but they are coordinated with marketing targets for these values, which are driven by the profit objective. As detailed in Michalek and Papalambros (2005), each deviation term contains a weighting coefficient vector to specify the importance of maintaining consistency between targets and responses relative to the importance of maximizing profit, and sufficiently large weights are required to achieve a solution with acceptably small inconsistency tolerances; however, this detail is removed in the figure for clarity.

Predicted profit of the product line depends on the selling price of each product, the costs incurred, and the demand for each product. While price and cost targets are variables in the marketing subproblem formulation, demand is a function of the characteristics and prices of the products. This functional relationship is taken from Michalek *et al.* (2005b), who use discrete choice econometric models fit to consumer choice data collected through a conjoint survey to predict demand. A brief description of this model is presented here, and interested readers may consult the references for full detail.

The discrete choice demand model is a random utility model, which assigns a scalar utility value to each alternative in a choice set and models individual choice as a process of selecting the alternative with the highest associated utility value. Utility itself is not observed directly; however, aspects of the choice situation, such as the characteristics of the product, can be used to infer statistical patterns of choice through observation. Specifically, the utility u_{ij} of a product j to an individual i consists partly of a deterministic term v_{ij}, based on observable, measurable aspects of the choice scenario, and partly of a stochastic, unobservable error term ε_{ij}, so that $u_{ij} = v_{ij} + \varepsilon_{ij}$. Utility is used to describe probabilistic choice, so that the probability P_{ij} of individual i choosing product j from a set of options is equal to the probability that $u_{ij} > u_{ij'}$ for all alternatives $j' \neq j$ in the set, so that $P_{ij} = \Pr[v_{ij} + \varepsilon_{ij} > \{v_{ij'} + \varepsilon_{ij'}\}_{\forall j' \neq j}]$.

The observable component of utility v_{ij} is a function of the measured aspects of the choice situation. In the homogeneous case, only aspects of the product j are measured, not aspects of the consumer i, so $v_{ij} = v_j = f(\mathbf{z}_j, p_j)$. This function can take different forms, and here it is a spline interpolation of the main-effects model of the discretized product characteristics and price. Procedurally, $\mathbf{z}_j$ and p_j are compiled into a single vector of "attributes" with elements indexed by ζ; the domain of each attribute is discretized into a finite number of levels Ω_ζ indexed $\omega = 1, 2, \ldots, \Omega_\zeta$; a binary dummy variable $\delta_{j\zeta\omega}$ is defined such that $\delta_{j\zeta\omega} = 1$ if product j has attribute ζ at level ω, and $\delta_{j\zeta\omega} = 0$ otherwise; and finally $v_j = \Sigma_\zeta \Sigma_\omega (\beta_{\zeta\omega} \delta_{j\zeta\omega})$, where $\beta_{\zeta\omega}$ is the "part-worth" or component of utility associated with attribute ζ at level ω. The values for the $\beta_{\zeta\omega}$ coefficients in this main-effects model are determined by conducting a choice-based conjoint survey generated using experimental design techniques where the levels of each attribute are systematically varied to reduce biases in estimating the model using a small number of survey questions (experiments). Each respondent is shown product profiles in a series of choice sets and asked to choose one from each set. The resulting data are used to estimate the best fit values of $\beta_{\zeta\omega}$ using classical maximum likelihood techniques or Bayesian methods.

In order to account for heterogeneity of preferences in the consumer population, the $\beta_{\zeta\omega}$ coefficients may be assumed to vary across the population. In the present model the $\beta_{\zeta\omega}$ coefficients of individuals are distributed following a mixture of multivariate normal distributions, and the parameters defining the mixing components are fit to the data using Bayesian Markov Chain Monte Carlo (MCMC) techniques. Finally, $\beta_{i\zeta\omega}$ coefficients are drawn for a random set of individuals i from the mixture distribution, and a natural cubic spline function $\Psi_{i\zeta}$ is fit through the $\beta_{i\zeta\omega}$ coefficients at levels $\omega = 1, 2, \ldots, \Omega_\zeta$ for each attribute ζ to interpolate β values of intermediate attribute levels for that individual. Using these splines, $v_{ij} = \Psi_{i0}(p_j) + \Sigma_\zeta \Psi_{i\zeta}(\mathbf{z}_{j\zeta})$, where price p is indexed as attribute $\zeta = 0$. Now, with the spline-interpolated function for v_{ij} estimated using survey data, it is possible to calculate the observable component of utility v_{ij} for a product j with any given product characteristics $\mathbf{z}_j$ and price p_j.

The form of P_{ij} with respect to v_{ij} depends on assumptions about the distribution of the unobserved error term ε_{ij}. The two common assumptions are: (1) Take ε_{ij} to be normally distributed, resulting in the probit model, which requires multidimensional integration to evaluate, or (2) take ε_{ij} to follow the double exponential distribution, resulting in the logit model, which produces nearly indistinguishable results from the probit model and results in a simple closed form solution:

$$P_{ij} = \frac{\exp(v_{ij})}{\sum_{j'} \exp(v_{ij'})}. \qquad (1)$$

This logit form is preferable for optimization, because it is quick and precise to evaluate. Finally, the demand q_j for a product j can be calculated by evaluating the average P_{ij} across a number of individuals $i = 1, 2, \ldots, I$ and multiplying by the size of the represented population S. This model of product demand is summarized in Figure 3, and greater depth regarding

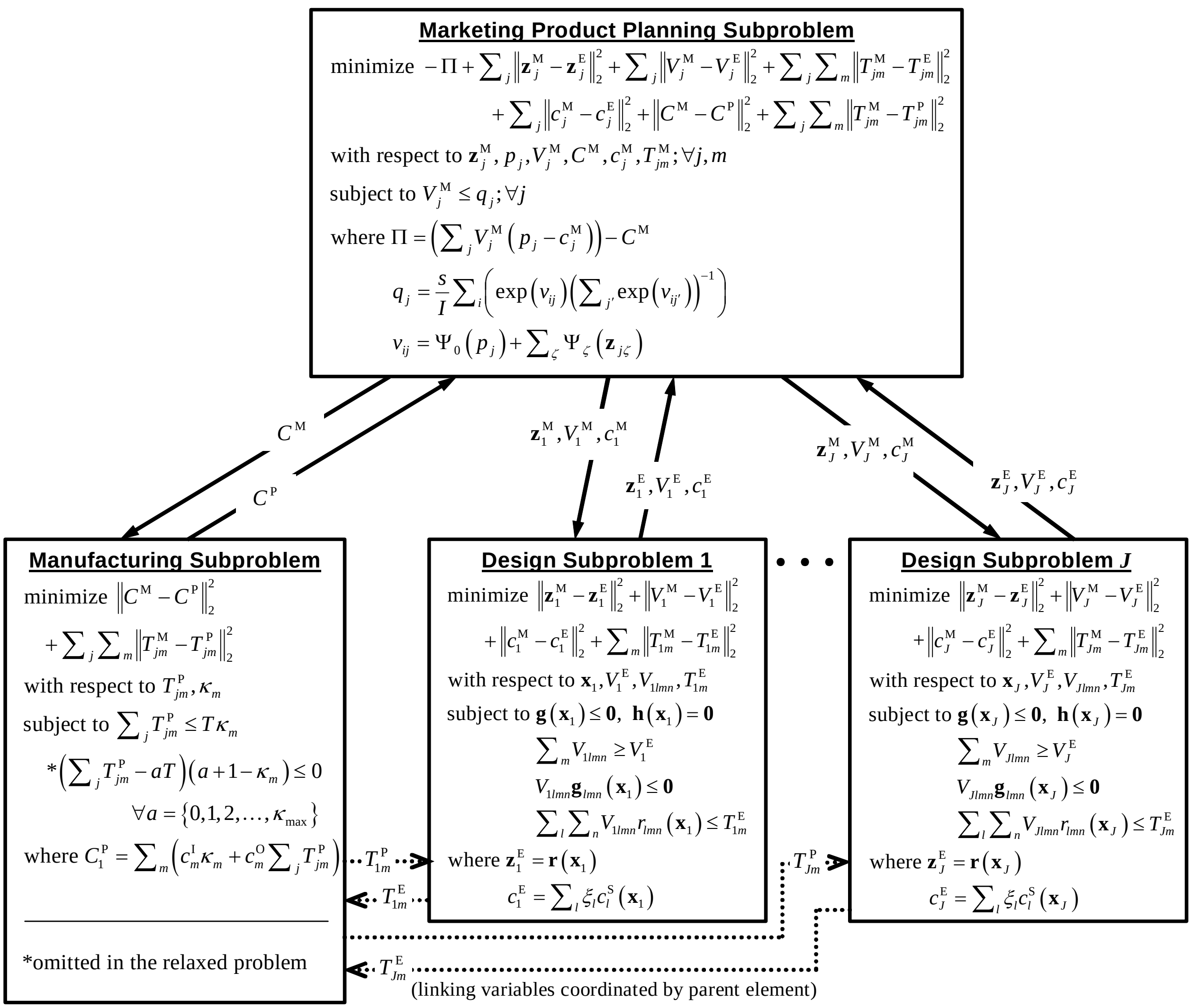

Figure 3: ATC coordination of marketing, engineering design, and manufacturing decisions

the development of the model is available in Michalek *et al*. (2005b).

If the target production volume V_j^M of each product j is less than or equal to demand, the resulting profit Π can calculated as

$$\Pi = \left(\sum_j V_j^M \left(p_j - c_j^M \right) \right) - C^M . \quad (2)$$

If production volume were to be greater than demand, profit would be calculated in terms of demand, but here a constraint is included to ensure that $V_j^M \leq q_j$. It is true that $V_j^M = q_j$ at the solution, since it is not profitable to produce more or less than demanded, so it is not necessary to allow V_j^M to deviate from q_j in the formulation. However, if $V_j^M = q_j$ is enforced during optimization, this can result in the marketing subproblem working to make product characteristics less desirable in order to match predicted demand to the lower production volumes achieved by engineering at intermediate iterations. This can increase computational time and also result in driving the marketing subproblem into an undesirable area of the design space where it may settle to a local minimum of lower global quality. Allowing $V_j^M \leq q_j$ is simply a convenience to speed up computation time by allowing each product to attract more than V_j^M individuals at intermediate iterations. At the solution, the profit objective will ensure that the constraint is active so that $V_j^M = q_j$.

Finally, the marketing subproblem includes a coordinating linking variable T_{jm}^M to coordinate machine time allocation for each product j and each machine type m between the manufacturing subproblem T_{jm}^P and each engineering design subproblem T_{jm}^E. More detail about this variable will be provided in later sections as the marketing subproblem serves only as a parent coordination element with respect to T_{jm}^P and T_{jm}^E, as described in Michalek and Papalambros (2005).

The objective function of the marketing subproblem is then to maximize profit and minimize deviation between targets and responses of the unit material cost c_j^M, investment and operating cost C^M, product characteristics $\mathbf{z}_j^M$, production volume V_j^M, and machine time T_{jm}^M variables for all products j and machines m. The full formulation of the marketing planning subproblem and its relationship to the other subproblems is shown in Figure 3.

2.2 Manufacturing Investment Subproblem

It is assumed that a fixed number of machine types $m = \{1, 2, \ldots, M\}$ is available from which to choose, and the firm must decide how many machines κ_m of each machine type to purchase. The possibility of leasing equipment is not considered, and the cost of product-specific tooling is ignored. The manufacturing subproblem is tasked with dividing up the purchased machine time among products in the line by setting decision variables T_{jm}^P, indicating the amount of time on machine m allocated to product j. Only allocation of machine time is considered here. Production issues such as machine configuration and reliability (Hu and Koren, 2005) and sequencing (Kurnaz *et al*., 2005) are left for future work. If the parameter T represents the amount of machine time available per machine in a fixed period (i.e., the number of working hours over the period), then $\kappa_m T$ is the total time available from κ_m machines. Therefore, T_{jm}^P is constrained such that

$$\sum_j T_{jm}^P \leq \kappa_m T . \quad (3)$$

In practice, each κ_m must be a nonnegative integer (0, 1, 2, …) because it is not possible to pay for a fraction of a machine at a fraction of the cost to receive a fraction of the capacity. However, the formulation is designed so that this requirement can be relaxed, permitting purchase of fractional numbers of machines. The solution to this relaxed problem will provide an upper bound on the amount of profit achievable by the more realistic situation where κ_m is restricted to integers. One way to restrict κ_m to integer values is to do so explicitly in the formulation, resulting in a mixed integer nonlinear programming problem (MINLP). However, ATC currently has not been proven to converge for discrete formulations, and further research is necessary to extend the applicability of ATC to these problems. To avoid the use of integer variables, it is possible to restrict the κ_m terms to integer values while working entirely in a continuous space: For a particular value of a, the following constraint

$$\left(\sum_j T_{jm}^P - aT \right) \left(a + 1 - \kappa_m \right) \leq 0 \quad (4)$$

coupled with simple boundary constraints restricting $T_{jm}^P \geq 0$, ensures that when fewer than (a+1) machines are purchased (i.e., when $\kappa_m < a+1$), the total machine time allocated must not be greater than aT, the time provided by a machines. A set of these constraints for all $a = \{0, 1, 2, \ldots\}$ enforced together ensures that at least a machines must be purchased in order to use a machines worth of time, for all values of a. In implementation, values of a need only be considered up to the maximum number of machines that may be purchased. While this set of constraints enables operation in a continuous domain and results in integer solutions for κ_m, it does not resolve all difficulties. This set of constraints creates a “stair step” shaped feasible region, and given the shape of the objective function, there are many cases where the shape of the feasible region creates several local minima: each at an integer value. Therefore, while the formulation allows operation in a continuous domain, solving for the optimum integer value of κ_m requires global search.

The strategy used here is to solve the relaxed problem (without the constraints in Eq.(4)) to obtain an upper bound on the profit achievable by the more restrictive problem. Next, starting from the optimum of the relaxed problem, penalty functions representing Eq.(4) are added to the objective function with a penalty coefficient parameter that increases over time until the solution is forced out of the infeasible region. This procedure results in a local minimum that is nearby the solution to the relaxed problem. The solution is not guaranteed to be the global solution; however, if it is within an acceptable deviation from the solution of the relaxed problem, it may be considered an acceptable and useful local solution.

Additionally, the cost of purchasing κ_m machines of type m is given by $\kappa_m c_m^I$, where c_m^I is the investment cost per machine of type m. The cost to operate the machines of type m is $\Sigma_j(c_m^O T_{jm}^P)$, where c_m^O is the cost per unit time to operate machine type m (labor cost plus machine use cost). The total production cost C^P is composed of investment and operating cost, so that

$$C^{\mathrm{P}} = \sum_m \left(c_m^{\mathrm{I}} \kappa_m + \sum_j c_m^{\mathrm{O}} T_{jm}^{\mathrm{P}} \right). \quad (5)$$

Finally, the objective of the manufacturing subproblem is to minimize deviation from the cost targets C^{M} passed from the marketing subproblem and minimize deviation from the machine time allocation linking variables T_{jm}^{M}, with values requested by each engineering design subproblem and coordinated by the parent marketing subproblem. The full formulation of the manufacturing subproblem is provided in Figure 3.

2.3 Engineering Design Subproblems

In each engineering design subproblem, the product characteristics $\mathbf{z}_j^{\mathrm{E}}$ (the aspects observable by the customer) are predicted as a function of the design variables $\mathbf{x}_j$ (the aspects manipulated by the engineer) so that $\mathbf{z}_j^{\mathrm{E}} = \mathbf{r}(\mathbf{x}_j)$, where $\mathbf{r}$ is a typical parametric engineering model or simulation. The engineering variables defining the design $\mathbf{x}_j$ are optimized to achieve resulting product characteristics $\mathbf{z}_j^{\mathrm{E}}$ as close as possible to the targets $\mathbf{z}_j^{\mathrm{M}}$ set by marketing, as shown in Figure 3. Secondly, each engineering design subproblem must attempt to meet production volume targets set by marketing V_j^{M} by manipulating the production volume of the product V_j^{E}. However, production volume of each product j is achieved by producing sufficient quantities of the components that comprise the product, so the individual components $l = \{1, 2, \ldots, L\}$ composing each product j must be considered. The parameter ξ_l defines the number of units of component l contained in each product. Each component l may require several manufacturing operations $n = \{1, 2, \ldots, N_l\}$; for example, production of a single component may require shearing, drawing, and bending operations. Production of the components $l = \{1, 2, \ldots, L\}$ that make up the product j must be allocated to machines $m = \{1, 2, \ldots, M\}$ in such a way that each component design meets the capability requirements of each machine on which it is made and the total time requests made for each machine do not exceed the amount of time allocated. It is assumed that none of the designs in the product line share components. This is a limitation since it is common to design product families that share specific components among different product designs in a line to save costs (Kota *et al*., 2000; Thonemann and Margaret, 2000; Fellini, 2003; Simpson, 2004); however, questions of commonality add significant complexity, and it is a reasonable first step to rule out this possibility.

The component production volume variable V_{jlmn}, represents the number of units of component l in design j on which operation n is performed by machine m. The production volume target V_j^{M} passed from marketing is achieved by producing enough of each component to assemble V_j^{M} complete products. To do this, a decision variable V_j^{E} is included to represent the total number of products of type j produced, and this value is constrained so that the manufacturing operations performed for each component V_{jlmn} are sufficient to generate the parts for V_j^{E} products.

$$\sum_m V_{jlmn} \geq \xi_l V_j^{\mathrm{E}}; \forall j, l, n. \quad (6)$$

Secondly, the total amount of time needed to execute manufacturing operations specified by V_{jlmn} must not exceed the amount of time T_{jm} allocated to product j on machine type m. If $r_{lmn}(\mathbf{x}_j)$ is a function specifying the time per unit to execute operation n on component l with machine m for a design with variables $\mathbf{x}_j$, this constraint can be represented as

$$\sum_l \sum_n V_{jlmn} r_{lmn}\left(\mathbf{x}_j\right) \leq T_{jm}^{\mathrm{E}}; \forall j, m. \quad (7)$$

Finally, V_{jlmn} may be greater than zero only if machine m has the capability to execute operation n on component l of product j. If $\mathbf{g}_{lmn}(\mathbf{x}_j)$ is a vector of constraint functions that define the feasibility of executing operation n on component l with machine m as a function of the design $\mathbf{x}_j$ of product j, then V_{jlmn} can be greater than zero only if $\mathbf{g}_{lmn}(\mathbf{x}_j) \leq \mathbf{0}$. If any constraint in $\mathbf{g}_{lmn}(\mathbf{x}_j)$ is positive, then the machine constraints are not satisfied by the product component, so operation n of component l cannot be performed on machine m, and V_{jlmn} must be exactly zero. This restriction can be represented by the following constraint

$$V_{jlmn} \mathbf{g}_{lmn}\left(\mathbf{x}_j\right) \leq \mathbf{0}; \forall j, l, m, n. \quad (8)$$

Taken in conjunction with the condition that $V_{jlmn} \geq 0$, this constraint ensures the specified relationship, allowing designs $\mathbf{x}_j$ the freedom to be altered to meet machine constraints and ensuring that components are not produced on machines if the design does not meet machine constraints. While this constraint can be implemented directly, it is advisable to implement it as a penalty function to avoid numerical problems with the near-colinearity of the gradients of Eq.(8) and the $V_{jlmn} \geq 0$ constraint for large values of $\mathbf{g}_{lmn}$. The entire formulation for each engineering design subproblem is shown in Figure 3.

Table 1: Engineering design model parameters

Description	Value
y_1: Gap between base and cover	0.30 in.
y_2: Minimum distance between spring and base	0.50 in.
y_3: Internal thickness of scale	1.90 in.
y_4: Minimum pinion pitch diameter	0.25 in.
y_5: Length of window	3.0 in.
y_6: Width of window	2.0 in.
y_7: Dist. top of cover to window	1.13 in.
y_8: Number of lbs measured per tick mark	1.0 lbs.
y_9: Horizontal dist. spring to pivot	1.10 in.
y_{10}: Length of tick mark + gap to number	0.31 in.
y_{11}: Number of lbs that number spans	16 lbs.
y_{12}: Aspect ratio of number (length/width)	1.29
y_{13}: Min. allow lever dist. base to centerline	4.0 in.

3. EXAMPLE

To demonstrate the methodology described in the previous section, the example from Michalek *et al*. (2005b) using a model of dial-readout scale design is extended to develop manufacturing models. Details of the model are consistent with the reference: Figure 4 shows the design variables $\mathbf{x}_j$ used to define the design, Table 1 lists fixed parameters, and the case of four products in the product line ($J = 4$) is examined. The product characteristics observed by the customer $\mathbf{z}_j$ include $z_1 =$

Table 2: Constraint and response functions

Formula	Description
$z_1 = \dfrac{4\pi x_6 x_9 x_{10}(x_1+x_2)(x_3+x_4)}{x_{11}(x_1(x_3+x_4)+x_3(x_1+x_5))}$	Weight Capacity (lbs)
$z_2 = x_{14}/x_{15}$	Platform aspect ratio
$z_3 = x_{14}x_{15}$	Platform Area (in^2)
$z_4 = \pi x_{12}/z_1$	Size of gap between 1-lb interval marks (in)
$z_5 = \dfrac{(2\tan(\pi y_{11}/z_1))(\frac{1}{2}x_{12} - y_{10})}{(1+(2/y_{12})\tan(\pi y_{11}/z_1))}$	Size of number (length, in)
$g_1 : x_8 \geq (x_{14} - 2y_1) - (\frac{1}{2}x_{12} + y_7) - x_7 - y_9 - x_{10}$	Sufficient rack length to span pivot and pinion
$g_2 : (x_1+x_2)^2 \leq (x_{14} - 2y_1 - x_7)^2 + (\frac{1}{2}x_{15} - y)^2$	Long lever attaches to top edge of scale
$g_3 : x_7 + y_9 + x_{11} + x_8 \leq x_{14} - 2y_1$	Rack shorter than base when pivot is rotated 90 degrees
$g_4 : (x_3 + x_4) \leq x_{14} - 2y_1$	Short lever length less than base length
$g_5 : x_5 \leq x_1 + x_2$	Lever joint location less than lever length
$g_6 : x_{12} \leq x_{15} - 2y_1$	Dial diameter less than base width
$g_7 : x_{12} \leq x_{14} - 2y_1 - x_7 - y_9$	Dial diameter less than base length minus spring plate
$g_8 : (x_{14} - 2y_1 - x_7)^2 + y_{13}^2 \leq (x_1 + x_2)^2$	Long lever at least y_{13} away from centerline for balance

weight capacity, z_2 = aspect ratio, z_3 = platform area, z_4 = gap size between dial tick marks, and z_5 = size of dial numbers. The functions $\mathbf{r}(\mathbf{x})$ mapping $\mathbf{x}$ to $\mathbf{z}$, and the constraints $\mathbf{g}(\mathbf{x})$ maintaining design feasibility are listed in Table 2. Additionally, the heterogeneous demand model described in the previous section was implemented by Michalek *et al.* (2005b) using real choice-based conjoint survey data, and the same model is used here in all calculations.

The software package DFMA: Design For Manufacture and Assembly, by Boothroyd Dewhurst (DFMA, 2004) was used to provide estimates of the manufacturing steps involved in producing the components of dial readout scales. For the example, the scope was limited to the manufacture of five components: l = 1, the cover; l = 2, the base; l = 3, the (two identical) long levers; l = 4, the (two identical) short levers; and

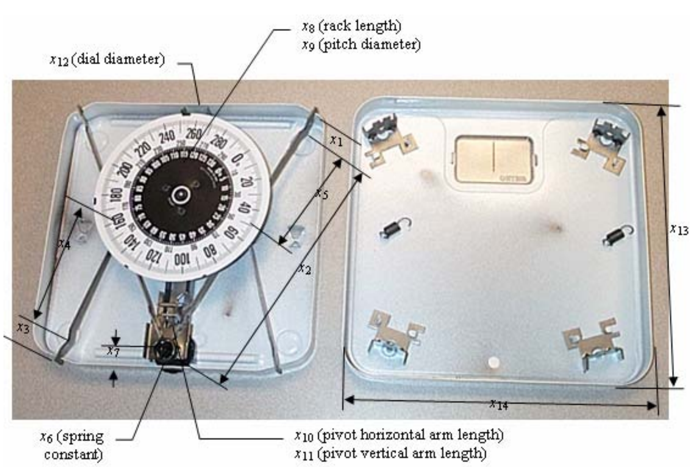

Figure 4: Design variables of the dial-readout scale

Table 4: Machine characteristics

m	Machine	Bed Width (in)	Bed Length (in)	FORCE (tons)	Press Speed (strokes/min)	Machine Rate ($/hr)	Operator Rate ($/hr)	Machine Cost ($Thousands)
1	Minster P2H-160	33.5	63	180	40	$22.10	$25.00	$335
2	Minster P2H-100	26	48	112	60	$19.40	$25.00	$250
3	Minster OBI #4F	9	12	32	90	$16.30	$25.00	$75
4	Minster OBI #5F	12	16	45	85	$16.70	$25.00	$60
5	Minster OBI #6F	14	18	60	75	$17.40	$25.00	$90
6	Minster OBI #7F	14	19	75	70	$18.00	$25.00	$100
7	Minster E2-200	36	60	200	36	$22.80	$25.00	$200
8	Minster E2-300	42	96	300	36	$26.70	$25.00	$300
9	Minster E2-400	48	108	400	36	$30.60	$25.00	$400

l = 5, the rack. There are two of each lever and one of each other component in each complete scale, so the number of components per unit produced ξ_l = {1, 1, 2, 2, 1} for l = {1, 2, 3, 4, 5} respectively. Each of these components is produced with stamping machines. The cover and base require two operations ($N_1 = N_2 = 2$): a shearing operation (n = 1) followed by a bending operation (n = 2), each performed with a compound die. The levers and rack are each produced with a single shearing step in a progressive die ($N_3 = N_4 = N_5 = 1$).

Material cost c_l^S was also estimated per part. For simplicity, the unit material cost was treated here as constant, rather than as a function of the component dimensions; however, inclusion of unit material cost as a function of design dimensions is straightforward if data are available. Since the unit material cost is treated as constant in this case, it need not be passed back and forth as a target, so the material cost calculation is included directly in the marketing subproblem to reduce computational load. Finally, the force required to perform each operation was estimated based on the machine suggestions made by the software, along with the time to load and unload each part. These data are summarized in Table 3.

A set of nine available machine alternatives (M = 9) was compiled using the software, which provided information on machine dimensions, force capacity, speed, and operating costs. Machine purchase cost estimates were obtained through informal discussions with Minster Machine Company. These machine data are summarized in Table 4.

Given these data, the rate function r_{lmn} can be calculated for each operation n on each machine m for each component l by dividing the number of strokes required per part by the machine press speed and adding the load / unload time. In general, r_{lmn} may be a function of the design variables $\mathbf{x}_j$; however, for simplicity in this example it is taken to be

Table 3: Component and operation data

l	Part	Parts per product	Material Cost ($/part)	n	Machine Operation	Force Required (tons)	Process	Strokes Per Part	Load / Unload Time Per Part (s)
1	Cover	1	$2.35	1	Shearing + Hole	100	Compound Die	3	8.35
				2	Bending	100	Compound Die	3	8.80
2	Base	1	$1.93	1	Shearing + Hole	100	Compound Die	3	8.32
				2	Bending	100	Compound Die	3	8.71
3	Long Lever	2	$0.28	1	Shearing	60	Progressive Die	1	NA
4	Short Lever	2	$0.16	1	Shearing	32	Progressive Die	1	NA
5	Rack	1	$0.07	1	Shearing	45	Progressive Die	1	NA

Table 5: Comparison of the relaxed solutions with the final solution

	Relaxed Soln	Final Soln
Revenue ($Mil)	$95.5	$94.8
Cost ($Mil)	$28.1	$27.6
Profit ($Mil)	$67.4	$67.1
κ_1	0.0	0.0
κ_2	23.3	23.0
κ_3	0.2	0.0
κ_4	0.9	1.0
κ_5	0.8	1.0
κ_6	0.0	0.0
κ_7	0.1	0.0
κ_8	0.0	0.0
κ_9	0.0	0.0

constant with respect to $\mathbf{x}_j$. The time period of interest T is set to one year, encompassing 52 weeks, five days per week without holidays, and eight hours per day, for a total of 7,488,000 seconds of machine time per machine purchased. It is assumed that all machines are purchased in full at the beginning of the year for production during that year only. This is quite conservative, since most machines in industry are purchased with multiple years of production in mind; however, changing time periods or including machine leasing or resale options could be accommodated using financial discounting. The machine constraints $\mathbf{g}_{lmn}$ are of two types: (1) ensure that the component is small enough to fit in the machine bed, and (2) ensure that the machine has sufficient force capacity to meet the component force requirements. Both of these conditions are enforced only for cases where $V_{jlmn} > 0$, as described previously. Specifically, the machine bed constraints applied to the cover, base, long lever, short lever, and rack respectively specify that

$$\begin{aligned} l=1: &\quad x_{13}, x_{14} \le \text{bed width} \\ l=2: &\quad x_{13}-2y_1,\ x_{14}-2y_1 \le \text{bed width} \\ l=3: &\quad x_1 + x_2 \le \text{bed width} \\ l=4: &\quad x_3 + x_4 \le \text{bed width} \\ l=5: &\quad x_8 \le \text{bed width} \end{aligned} \tag{9}$$

Additionally, force capacity constraints specify that the machine force is greater than or equal to the component required force for each component, operation, and machine, using the relevant data from Table 3 and Table 4.

4. RESULTS

The ATC hierarchy was solved using as a starting point the solution from Michalek *et al.* (2005b) with a value of zero for all machine purchases κ_m and time allocations (T^{M}_{jm}, T^{D}_{jm}, and T^{P}_{jm}). The solution was obtained in three stages: First the relaxed problem (omitting Eq.(4) and Eq.(8)) was solved. Next, the penalty functions representing the machine feasibility constraints in Eq.(8) were added to the objective function with a penalty coefficient iteratively increasing, gradually forcing the solution out of infeasible regions to achieve the machine-feasible solution. Finally, the penalty function forcing κ to integer values (Eq.(4)) was added to the objective function with a penalty coefficient iteratively increasing, gradually forcing κ to integer values and achieving the final feasible integer solution. The final resulting solution is not necessarily the global optimum, but it is a local optimum near the solution to the relaxed problem. Table 5 shows a comparison of the revenue, cost, profit, and machine purchase variables of the relaxed and final optimal solutions. The profit of the final integer solution is lower than the relaxed solution, as expected, since the relaxed solution is an upper bound on the final integer solution. However, the resulting profit of the final solution is within 0.4% of the relaxed solution; therefore, the result is at least a good engineering solution of high quality, and likely a global solution. In the relaxed solution the κ variables are real-valued, while in the final solution they are integers. Although the final solution in this case appears to be simply a rounding of the relaxed solution for each value of κ, this simple relationship does not hold in general.

Table 6: Product line design solution

	PRODUCT (j) 1	2	3	4
V^{M}_j (mil)	1.23	1.01	0.91	0.57
Share	25%	20%	18%	12%
z1	292	258	200	258
z2	0.980	1.155	0.924	0.975
z3	140	123	106	140
z4	0.103	0.119	0.121	0.115
z5	1.22	1.37	1.30	1.33
p	$24.13	$25.40	$24.57	$30.00
x1	11.78	0.125	9.351	9.846
x2	0.192	11.5	0.809	2.149
x3	3.364	5.981	5.745	3.778
x4	4.754	2.264	2.951	5.068
x5	0.125	0.186	0.125	0.135
x6	147.88	1.00	117.22	97.36
x7	0.50	0.50	0.50	0.50
x8	5.65	5.25	3.66	4.48
x9	0.353	0.766	0.478	0.345
x10	1.052	1.432	0.741	1.349
x11	1.696	1.776	1.696	1.878
x12	9.515	9.721	7.714	9.422
x13	11.71	11.92	9.914	11.68
x14	11.95	10.32	10.73	11.98

The products that result from this optimization are shown in Table 6, along with their predicted market shares, production volumes, and selling prices. Differences in design variables between these results and those in the Michalek *et al*., (2005b) occur because the design space in this problem does not map one to one with the product characteristics space, and multiple product designs exist that yield identical product characteristics. The specific design found by the algorithm on any given iteration is a matter of chance, but the coordination ensures that at least one feasible design exists that can attain the target product characteristics. Additionally, in this example, the machine constraints related to part dimensions are not binding, so the product characteristics shown in Table 6 are not compromised. The design itself is robust to dimensional constraints on many of its components because, for example, constraints on the length of the levers can be compensated for by changing the dimensions of other components in order to achieve the same product characteristics. In other case studies

Table 7: Product line manufacturing investment and allocation solution

	j	*l*	*n*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
				MACHINE (m)								
κ					23		1	1				
T_{jm} (million sec.)	**1**				56.77		2.50	1.97				
	2				46.60		2.05	1.60				
	3				42.19		1.83	1.46				
	4				26.66		1.12	0.92				
V_{jlmn} (million units)	**1**	**1**	**1**		1.23							
	1	**1**	**2**		1.23							
	1	**2**	**1**		1.23							
	1	**2**	**2**		1.23							
	1	**3**	**1**					2.41				
	1	**4**	**1**				2.32					
	1	**5**	**1**				1.21					
	2	**1**	**1**		1.01							
	2	**1**	**2**		1.01							
	2	**2**	**1**		1.01							
	2	**2**	**2**		1.01							
	2	**3**	**1**					1.97				
	2	**4**	**1**				1.92					
	2	**5**	**1**				0.98					
	3	**1**	**1**		0.91							
	3	**1**	**2**		0.91							
	3	**2**	**1**		0.91							
	3	**2**	**2**		0.91							
	3	**3**	**1**					1.78				
	3	**4**	**1**				1.69					
	3	**5**	**1**				0.90					
	4	**1**	**1**		0.57							
	4	**1**	**2**		0.57							
	4	**2**	**1**		0.57							
	4	**2**	**2**		0.57							
	4	**3**	**1**					1.11				
	4	**4**	**1**				1.03					
	4	**5**	**1**				0.56					

where the design is less robust, machine constraints could influence the product characteristics significantly.

The variables associated with machine purchase and manufacturing allocation are provided in Table 7. In the table, the V_{jlmn} terms are shown in millions of units and the T_{jm} terms are shown in millions of seconds. In the final solution, the time available from purchased machines is allocated to the four products via the T_{jm} terms, and each product allocates its components to the most cost-effective available machines via the V_{jlmn} terms.

5. CONCLUSIONS

This paper has presented a method to coordinate manufacturing investment decisions with marketing and product design decisions to achieve jointly optimal product line solutions. The approach aims to facilitate communication in concurrent engineering at design stages where parametric models can be called upon in order to resolve tradeoffs toward the pursuit of firm-level objectives. The modularity of the ATC-based methodology allows additional considerations, such as the manufacturing subproblem introduced in this paper, to be added to an existing hierarchy without starting from scratch. This modularity provides an opportunity for models in various disciplines to be built and used when available and appropriate to the scope of interest with minimal restructuring.

Manufacturing decisions typically involve a number of inherently discrete decisions, such as how many machines to purchase. In this formulation, these discrete decisions were represented by relaxing the problem to a continuous space and imposing constraints to enforce solutions with discrete values; however, the formulation creates multiple local minima, and gradient-based search algorithms guarantee only local optimality. The strategy employed is to solve the relaxed problem and then impose interior penalty functions to achieve a feasible solution close to the relaxed solution. In the example this strategy was successful, resulting in a final solution with a profitability within 0.4% of the relaxed solution; however, the application highlights the need for further research to extend the ATC methodology to problems with discrete variables so that mixed-integer programming can be utilized and more complex problems involving manufacturing can be solved.

The example presented was examined only for the case of four products in the product line. Determination of the optimal number of products in the line requires a comparison of separate optimization runs for each case, as in Michalek *et al*. (2005b); however, the manufacturing formulation presented here does not allow determination of the optimal number of products because tooling costs, such as the purchase of dies, and setup costs are not included in the formulation. Without these costs represented, the model predicts that more product variety is always better. It is left for future research to incorporate setup costs and tooling costs into the model. Also, the example examines only the case of a single producer; however, competitor products can be included in the existing demand model to test other scenarios, and competitive effects can be studied using game theory as in Michalek *et al*. (2004).

Finally, there exist alternative ways to decompose the marketing, engineering design, and manufacturing subproblems in this example. The formulation presented was designed to allocate as much complexity as possible to the engineering design subproblems in order to improve scalability to lines of many products: With the inclusion of many products, the marketing and manufacturing subproblems grow in dimensionality; however, each engineering design subproblem remains constant in size. Scalability is also supported because the relaxed manufacturing subproblem is linear in constraints and quadratic in the objective function. Additionally, choice of decomposition was made to minimize the number of variables shared among subproblems, improving computational properties. However, a model-based methodology similar to Michelena and Papalambros (1997) for determining how to decompose systems with multiple products for best scalability would be helpful.

This application of ATC has the potential to bridge gaps between design, manufacturing, and business perspectives of product development and production. The current model is static in the sense that market share is a deterministic function of the product characteristics and price, and demand does not vary over the time period in question. A number of potential extensions are possible such as modeling market dynamics by considering investment time (Georgiopoulos *et al*., 2002) and demand fluctuation (Asl and Ulsoy, 2002), or by including considerations of product life cycle economic modeling (Birge *et al*., 2000) and machine reconfiguration (Koren *et al*., 1999).

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