Proceedings of the 2003 ASME
Mechanical Engineering Congress and Exposition
Washington, DC, USA, November 15-21, 2003

IMECE 2003-42014

INTEGRATED PLANT, OBSERVER, AND CONTROLLER OPTIMIZATION WITH APPLICATION TO COMBINED PASSIVE/ACTIVE AUTOMOTIVE SUSPENSIONS

Hosam K. Fathy[1], Panos Y. Papalambros[2], and A. Galip Ulsoy[2]

1. Research fellow; 2. Professor
Department of Mechanical Engineering,
The University of Michigan,
Ann Arbor, MI 48109-2125

ABSTRACT

The plant and control optimization problems are coupled in the sense that solving them sequentially does not guarantee system optimality. This paper extends previous studies of this coupling by relaxing their assumption of full state measurement availability. An original derivation of first-order necessary conditions for plant, observer, controller, and combined optimality furnishes coupling terms quantifying the underlying trilateral coupling. Special scenarios where the problems decouple are pinpointed, and a nested optimization strategy that guarantees system optimality is adopted otherwise. Applying these results to combined passive/active car suspension optimization produces a suspension design outperforming its passive, active, and sequentially optimized passive/active counterparts.

Keywords: Nested Optimization - Plant, Observer and Controller Optimality Conditions

NOMENCLATURE

Symbol	Meaning
$\mathbf{A}$, $\mathbf{B}$	State (system) and input matrices.
c_s	Passive suspension damping coefficient.
$\mathbf{C}$	Output matrix.
$\mathbf{e}(t)$	State estimate error trajectory.
E	Expectation operator.
E_{ms}	Mean square state estimation error.
f	System optimization objective.
$\mathbf{g}_p$	Plant optimization inequality constraints.
$\mathbf{G}$	Disturbance matrix.
$\mathbf{h}_p$	Plant optimization equality constraints.
k_s	Passive suspension stiffness.
$\mathbf{K}$, $\mathbf{L}$	Time-invariant feedback and observation gains.
$\mathbf{L}(t, \tau)$	Non-clairvoyant observer mapping.
m_s	Sprung mass.
m_{us}	Unsprung mass.
$\mathbf{Q}$	State weight in LQ objective.
r_o , r_1 , r_2 , r_3	Sprung mass acceleration, wheel hop, rattle space, and passenger acceleration objective weights.
$\mathbf{R}$	Control effort weight in LQ objective.
t_o	Initial time
T	Time of mean square estimation error computation.
$u(t)$	Control input trajectory.
$\mathbf{v}(t)$	Sensor noise signal.
$\mathbf{V}$	Sensor noise autocovariance matrix.
w_p	Plant optimization weight.
w_{oc}	Observer/controller optimization weight.
$\mathbf{w}(t)$	Disturbance signal.
$\mathbf{W}$	Disturbance autocovariance matrix.
$\mathbf{x}$, $\mathbf{x}_p$, $\mathbf{x}_{oc}$	System, plant, and observer/controller optimization variables.
$\mathbf{X}$, $\mathbf{X}_p$, $\mathbf{X}_{oc}$	Set of feasible systems, plants, and observer/controller pairs.
$\mathbf{X}_p(\mathbf{x}_{oc})$	Set of feasible plants for an observer/controller pair.
$\mathbf{X}_{oc}(\mathbf{X}_p)$	Set of feasible observer/controller pairs for a plant.
$\mathbf{y}(t)$	Output trajectory.
$\mathbf{z}(t)$, $\mathbf{z}_e$	State and state estimate trajectories.
z_g	Ground disturbance displacement.
$\mathbf{z}_o$	Initial states.
z_s , z_{us}	Sprung and unsprung mass displacements.
α , β	Lagrange multipliers corresponding to plant equality and inequality constraints.
Γ	Coupling term.
λ	Costate trajectory (dual of state trajectory).
σ , τ	Time.
ω_f	Filter cutoff frequency.

1. INTRODUCTION

Controlled system design problems are traditionally simplified by partitioning them into smaller and more tractable plant and control optimization subproblems that are then solved sequentially. Research on flexible air and space structures [1-15], flexible robot manipulators [16], and other mechatronics [17-20] has unveiled a coupling between the plant and control optimization subproblems that prevents their sequential solution from guaranteeing system optimality. Generalizing from these demonstrations, the present authors:

i- quantified this coupling by comparing plant, controller, and system optimality conditions [21],

ii- classified the strategies proposed in the literature for mitigating this coupling into sequential, iterative, nested, and simultaneous ones [21],

iii- showed that unlike sequential and iterative strategies, nested ones guarantee system optimality [21], and,

iv- applied nested optimization to combined passive/active vibration attenuation for car suspensions and elevators [22, 23].

This literature primarily assumed full state measurement and feedback. A rigorous study of the coupling between plant and control optimization that incorporates observers, which are typically necessitated by state measurement inavailability and sensor noise, is lacking. This paper addresses this problem by affording the following original contributions to the literature:

i- the first quantification of the trilateral coupling between plant, observer, and controller optimization using first-order necessary conditions for optimality (Section 4),

ii- a nested optimization strategy that guarantees combined plant, observer and controller optimality (Section 3), and,

iii - the application this strategy to a combined passive/active car suspension (Section 5).

2. MOTIVATING EXAMPLE

Fathy *et al.* [23] demonstrated the coupling between plant and controller optimization using the car suspension model:.

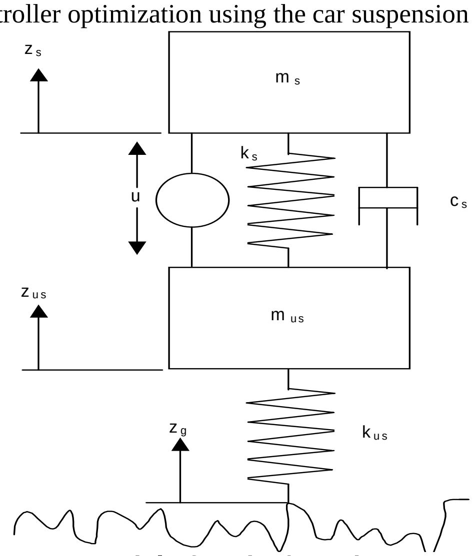

Figure 1: Model of Active/Passive Suspension

In this model, m_s and m_{us} are the sprung and unsprung masses, k_{us} is the tire stiffness, tire damping is neglected, k_s and c_s are the passive suspension stiffness and damping coefficient, $u(t)$ is the active control force and $z_s(t)$, $z_{us}(t)$ and $z_g(t)$ are the vertical displacements of the sprung mass, the unsprung mass and the road, respectively. Assuming a ground disturbance velocity signal colored by a first-order low-pass filter with cutoff frequency ω_f, Fathy *et al.* [23] solved the combined passive/active suspension optimization problem,

$$\min_{k_s, c_s, \mathbf{K}} E\int_0^\infty (r_o(\ddot{z}_s)^2 + r_1(z_{us}-z_g)^2 + r_2(z_s-z_{us})^2 + r_3u^2)dt$$

$$\text{subject to: } \dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}u(t) + \mathbf{G}w(t),\ u(t) = -\mathbf{K}\mathbf{z}(t),$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ -\frac{k_{us}}{m_{us}} & -\frac{c_s}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 \\ 0 & 0 & 0 & 0 & -\omega_f \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{1}{m_{us}} \\ 0 \\ \frac{1}{m_s} \\ 0 \end{bmatrix},\ \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_f \end{bmatrix},$$

$$\mathbf{z}(t) = [z_{us}(t)-z_g(t), \dot{z}_{us}(t), z_s(t)-z_{us}(t), \dot{z}_s(t), \dot{z}_g(t)]^T,$$

$$k_s \le 160\text{kN/m},\ c_s \le 16\ \text{kN.s/m}, \tag{1}$$

where the optimization objective was a sum of the mean square (ms) sprung mass acceleration (a passenger discomfort measure), ms tire deflection (*wheel hop*), ms suspension stroke (*rattle space*) and ms control input, weighted by constants r_o, r_1, r_2, and r_3, respectively. Optimization variables comprised the passive stiffness k_s and damping coefficient c_s for the plant and the full state feedback gain vector $\mathbf{K}$ for the controller. The resulting optimal suspension design was neither purely passive nor purely active nor a sequentially optimized mix of both, but rather a system-optimal mix of both determined by nested optimization.

This problem assumes direct full state feedback, but a typical suspension sensor package measures either the stroke, $z_s(t)-z_{us}(t)$, or sprung mass acceleration, $\ddot{z}_s(t)$, or both, with superimposed sensor noise [24]. State estimation is hence needed to estimate unmeasured states (e.g., ground disturbance velocity, $\dot{z}_g$) and filter the sensor noise. This paper formulates and solves the resulting combined plant/observer/controller optimization problem and quantifies, for the first time, its underlying trilateral coupling. This trilateral coupling is first introduced using set theory (Section 3), then quantified using an original derivation of first-order necessary conditions for combined plant, observer, and controller optimality (Section 4), and finally exemplified using a simple combined passive/active car suspension optimization example (Section 5).

3. SEQUENTIAL AND NESTED OPTIMIZATION

We introduce the aforementioned coupling using the generic system optimization problem:

$$\min_{\mathbf{x} \in \mathbf{X}} f(\mathbf{x}) , \tag{2}$$

where $\mathbf{x}$, $\mathbf{X}$, and f denote the system optimization variables, feasible set, and objective function, respectively. By partitioning the vector of system variables into subvectors of plant design variables ($\mathbf{x}_p$) and observer/controller design variables ($\mathbf{x}_{oc}$),

$$\mathbf{x} = (\mathbf{x}_p, \mathbf{x}_{oc}) , \tag{3}$$

one can define the sets $\mathbf{X}_p(\mathbf{x}_{oc})$, $\mathbf{X}_{oc}(\mathbf{x}_p)$, $\mathbf{X}_p$, and $\mathbf{X}_{oc}$ of feasible plants for a given observer/controller pair, feasible observer/controller pairs for a given plant, feasible plants, and feasible observer/controller pairs, respectively, as follows (Fig. 2).

$$\mathbf{X}_p(\mathbf{x}_{oc}) = \{\mathbf{x}_p : (\mathbf{x}_p, \mathbf{x}_{oc}) \in \mathbf{X}\} . \tag{4}$$

$$\mathbf{X}_{oc}(\mathbf{x}_p) = \{\mathbf{x}_{oc} : (\mathbf{x}_p, \mathbf{x}_{oc}) \in \mathbf{X}\} . \tag{5}$$

$$\mathbf{X}_p = \{\mathbf{x}_p : \exists \mathbf{x}_{oc} : (\mathbf{x}_p, \mathbf{x}_{oc}) \in \mathbf{X}\} . \tag{6}$$

$$\mathbf{X}_{oc} = \{\mathbf{x}_{oc} : \exists \mathbf{x}_p : (\mathbf{x}_p, \mathbf{x}_{oc}) \in \mathbf{X}\} . \tag{7}$$

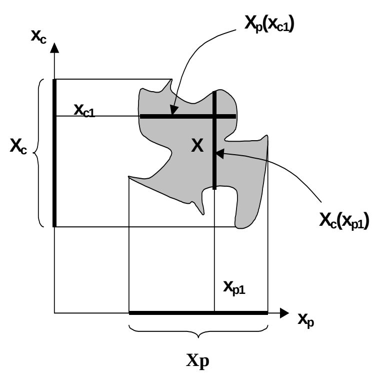

Figure 2: Basic Definitions

Given these definitions, one can partition the system optimization problem into a plant optimization problem,

$$\min_{\mathbf{x}_p \in \mathbf{X}_p(\mathbf{x}_{oc})} f(\mathbf{x}_p, \mathbf{x}_{oc}) , \tag{8}$$

and an observer/controller optimization problem,

$$\min_{\mathbf{x}_{oc} \in \mathbf{X}_{oc}(\mathbf{x}_p)} f(\mathbf{x}_p, \mathbf{x}_{oc}) . \tag{9}$$

Solving these two problems sequentially for some initial observer/controller guess $\mathbf{x}_{oc}^i$ simplifies the system optimization problem by partitioning it along interdisciplinary boundaries into smaller and hence more tractable optimization subproblems. This simplification entails implicitly weighing plant optimization infinitely more than observer/controller optimization and hence fails to guarantee system-level optimality, except when the initial controller guess fortuitously corresponds to the optimal system design [21]. The plant and observer (and controller) optimization problems are hence coupled in the sense that solving them sequentially does not guarantee system optimality. We now present an optimization strategy that mitigates such coupling and guarantees system optimality.

The *nested optimization* strategy utilizes two nested loops (hence the name): an outer loop that optimizes the plant design and an inner one that finds the optimal observer/controller pair for every plant designed by the outer loop. Defined mathematically as follows,

<u>Outer (plant) loop:</u> $\mathbf{x}_p^* = \arg\min_{\mathbf{x}_p \in \mathbf{X}_p} f(\mathbf{x}_p, \mathbf{x}_{oc}^*(\mathbf{x}_p))$, (10)

<u>Inner (observer/controller) loop:</u>

$$\mathbf{x}_{oc}^*(\mathbf{x}_p) = \arg\min_{\mathbf{x}_{oc} \in \mathbf{X}_{oc}(\mathbf{x}_p)} f(\mathbf{x}_p, \mathbf{x}_{oc}) , \tag{11}$$

nested optimization guarantees system optimality, as shown in Theorem 1.

Theorem 1 (Nested Optimization): Let $\mathbf{X}^*$, the *set of system optima*, be the set of solutions to Eq. (2). Furthermore, let $\mathbf{X}^N$, the set of *nested optima*, be the set of solutions to the nested optimization problem defined in Eqs. (10) and (11), given the definitions of the sets $\mathbf{X}$, $\mathbf{X}_p(\mathbf{x}_c)$ and $\mathbf{X}_c(\mathbf{x}_p)$ above. Then $\mathbf{X}^* = \mathbf{X}^N$. The proof follows directly from the definition of a minimum.

4. THE COUPLING BETWEEN THE PLANT, OBSERVER AND CONTROLLER OPTIMIZATION PROBLEMS

The choice between sequential and nested optimization depends on the magnitude of the given problem's coupling. When the coupling is strong, guaranteed system optimality makes nested optimization quite attractive, but when the coupling is weak, the computationally cheaper sequential strategy is more expeditious. Previous work quantified the coupling between the plant and controller optimization problems assuming full state measurement availability [21]. This paper relaxes this assumption and quantifies the trilateral coupling between the plant, observer, and controller optimization problems for the first time. The derivation is based on the generic combined plant/observer/controller optimization problem:

$$\min_{\mathbf{x}_p, \mathbf{K}, \mathbf{L}(t,\tau)} w_p f_p(\mathbf{x}_p) + w_{oc} E\int_0^{\infty} (\mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt$$

$$\text{subject to: } \mathbf{h}_p(\mathbf{x}_p) = 0 ,\ \mathbf{g}_p(\mathbf{x}_p) \le 0 , \tag{12}$$

$$\dot{\mathbf{z}} = \mathbf{A}(\mathbf{x}_p)\mathbf{z} + \mathbf{B}(\mathbf{x}_p)\mathbf{u} + \mathbf{G}(\mathbf{x}_p)\mathbf{w} ,\ \mathbf{y} = \mathbf{C}(\mathbf{x}_p)\mathbf{z} + \mathbf{v} ,\ \mathbf{u} = -\mathbf{K}\mathbf{z}_e ,$$

$$\mathbf{z}_e(t) = \int_0^t \mathbf{L}(t,\tau)\mathbf{y}(\tau)d\tau ,\ E(\mathbf{w}(t)\mathbf{w}^T(\tau)) = \mathbf{W}\delta(t-\tau) ,$$

$$E(\mathbf{v}(t)\mathbf{v}^T(\tau)) = \mathbf{V}\delta(t-\tau) ,$$

where the plant is assumed linear and time-invariant. In this problem, the plant design $\mathbf{x}_p$ dictates the state, input, output, and disturbance matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{G}$, respectively. State, input, and output trajectories (denoted by $\mathbf{z}(t)$, $\mathbf{u}(t)$, $\mathbf{y}(t)$, respectively) are afflicted by zero-mean, uncorrelated, white, and Gaussian random disturbance and noise signals, $\mathbf{w}(t)$ and $\mathbf{v}(t)$, with autocovariances $\mathbf{W}$ and $\mathbf{V}$, respectively. An observer is used to compute state estimates $\mathbf{z}_e(t)$ from output measurements, and a feedback gain matrix $\mathbf{K}$ is used to generate control inputs from these estimates. Following Kalman and Bucy's seminal work, the observer design is described by a non-clairvoyant mapping $\mathbf{L}(t, \tau)$ between the measurement and estimation Hilbert spaces [25]. This is a more general formulation that, for the present linear and time-invariant plant, is equivalent to assuming a more traditional linear and time-invariant observer gain matrix [25]. Optimization is performed over the plant, observer, and controller design variables $\mathbf{x}_p$, $\mathbf{K}$, and $\mathbf{L}(t, \tau)$, respectively. A weighted sum of a generic plant objective f_p and a standard stochastic quadratic objective is used as the system objective. Finally, the plant design is constrained by equality and inequality constraints $\mathbf{h}_p(\mathbf{x}_p)$ and $\mathbf{g}_p(\mathbf{x}_p)$ reflecting the fact that a plant cannot be arbitrarily optimized for observation and control, but must satisfy additional design constraints from other disciplines.

Applying nested optimization, Eqs. (10-11), to this system optimization problem gives:

Outer (plant) optimization loop:

$$\min_{\mathbf{x}_p} \; w_p f_p(\mathbf{x}_p) + w_{oc} f^*_{oc}(\mathbf{x}_p) \text{ subject to:} \tag{13}$$

$$\mathbf{h}_p(\mathbf{x}_p) = 0, \; \mathbf{g}_p(\mathbf{x}_p) \le 0, \; f^*_{oc}(\mathbf{x}_p) = \text{ output of inner loop.}$$

Inner (observer/controller) optimization loop:

$$f^*_{oc}(\mathbf{x}_p) = \min_{\mathbf{K}, \mathbf{L}(t,\tau)} E\int_0^{\infty}(\mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt \text{ subject to:}$$

$$\dot{\mathbf{z}} = \mathbf{A}(\mathbf{x}_p)\mathbf{z} + \mathbf{B}(\mathbf{x}_p)\mathbf{u} + \mathbf{G}(\mathbf{x}_p)\mathbf{w}, \; \mathbf{u} = -\mathbf{K}\mathbf{z}_e, \; \mathbf{y} = \mathbf{C}(\mathbf{x}_p)\mathbf{z} + \mathbf{v},$$

$$\mathbf{z}_e(t) = \int_0^t \mathbf{L}(t, \tau)\mathbf{y}(\tau)d\tau, \; E(\mathbf{w}(t)\mathbf{w}^T(\tau)) = \mathbf{W}\delta(t-\tau), \tag{14}$$

$$E(\mathbf{v}(t)\mathbf{v}^T(\tau)) = \mathbf{V}\delta(t-\tau).$$

The inner loop of this nested problem (Eq. (14)) is a standard linear quadratic Gaussian (LQG) optimal control problem [27]. Hence, the nested loops in Eqs. (13-14) collapse to:

$$\min_{\mathbf{x}_p} \; f_p(\mathbf{x}_p) + w_{oc} f^*_{oc}(\mathbf{x}_p) \text{ subject to:} \tag{15}$$

$$\mathbf{h}_p(\mathbf{x}_p) = 0, \; \mathbf{g}_p(\mathbf{x}_p) \le 0, \; f^*_{oc}(\mathbf{x}_p) = \mathrm{tr}[\mathbf{G}\mathbf{W}\mathbf{G}^T\mathbf{S} + \mathbf{K}^T\mathbf{R}\mathbf{K}\mathbf{P}],$$

$$\mathbf{L} = \mathbf{P}\mathbf{C}^T\mathbf{V}^{-1}, \; \mathbf{A}^T\mathbf{S} + \mathbf{S}\mathbf{A} - \mathbf{S}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{S} + \mathbf{Q} = 0,$$

$$\mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T - \mathbf{P}\mathbf{C}^T\mathbf{V}^{-1}\mathbf{C}\mathbf{P} + \mathbf{G}\mathbf{W}\mathbf{G}^T = 0, \; \mathbf{K} = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{S}.$$

Thus, by capitalizing on standard results from optimal estimation and control theory (namely, LQG), the nested has successfully reduced a hybrid static/variational system optimization problem, Eq. (12), into a purely static optimization problem, Eq. (15), while still guaranteeing system optimality. To simplify the system optimization problem in Eq. (12) further, one can partition it into a plant optimization subproblem,

$$\min_{\mathbf{x}_p} \; f_p(\mathbf{x}_p) \text{ subject to: } \mathbf{h}_p(\mathbf{x}_p) = 0, \; \mathbf{g}_p(\mathbf{x}_p) \le 0, \tag{16}$$

and an observer/controller optimization subproblem,

$$\min_{\mathbf{K}, \mathbf{L}(t,\tau)} E\int_0^{\infty}(\mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt \text{ subject to:}$$

$$\dot{\mathbf{z}} = \mathbf{A}(\mathbf{x}_p)\mathbf{z} + \mathbf{B}(\mathbf{x}_p)\mathbf{u} + \mathbf{G}(\mathbf{x}_p)\mathbf{w}, \; \mathbf{u} = -\mathbf{K}\mathbf{z}_e, \; \mathbf{y} = \mathbf{C}(\mathbf{x}_p)\mathbf{z} + \mathbf{v},$$

$$E(\mathbf{v}(t)\mathbf{v}^T(\tau)) = \mathbf{V}\delta(t-\tau), , \; E(\mathbf{w}(t)\mathbf{w}^T(\tau)) = \mathbf{W}\delta(t-\tau),$$

$$\mathbf{z}_e(t) = \int_0^t \mathbf{L}(t, \tau)\mathbf{y}(\tau)d\tau, \tag{17}$$

Using the certainty equivalence principle [27] the observer/controller optimization subproblem can be partitioned without further loss of optimality into an observer optimization (specifically, Kalman filtering) subproblem [25],

$$\min_{\mathbf{L}(t,\tau)} E\left\{\left(\mathbf{z}(T) - \int_0^T \mathbf{L}(\tau, T)\mathbf{y}(\tau)d\tau\right)^T\left(\mathbf{z}(T) - \int_0^T \mathbf{L}(\tau, T)\mathbf{y}(\tau)d\tau\right)\right\}$$

$$\text{subject to: } \dot{\mathbf{z}} = \mathbf{A}(\mathbf{x}_p)\mathbf{z} + \mathbf{G}(\mathbf{x}_p)\mathbf{w}, \; \mathbf{y} = \mathbf{C}(\mathbf{x}_p)\mathbf{z} + \mathbf{v}, \tag{18}$$

$$E(\mathbf{w}(t)\mathbf{w}^T(\tau)) = \mathbf{W}\delta(t-\tau), E(\mathbf{v}(t)\mathbf{v}^T(\tau)) = \mathbf{V}\delta(t-\tau),$$

and a controller optimization (specifically, linear quadratic regulation, LQR) subproblem [27],

$$\min_{\mathbf{K}} \int_0^{\infty}(\mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt \text{ subject to:}$$

$$\dot{\mathbf{z}} = \mathbf{A}(\mathbf{x}_p)\mathbf{z} + \mathbf{B}(\mathbf{x}_p)\mathbf{u} + \mathbf{G}(\mathbf{x}_p)\mathbf{w}, \; u = -\mathbf{K}\mathbf{z}(t). \tag{19}$$

Solving the plant, observer, and controller optimization problems in Eqs. (16, 18, 19) sequentially simplifies system optimization by partitioning it along interdisciplinary boundaries into tractable and well-understood design, estimation, and control optimization problems. Since it solves each of these three optimization subproblems only once, sequential optimization is computationally cheaper than nested optimization. When the coupling between the plant and observer/controller optimization problems is weak, this reduced computational cost is desirable. To quantify the strength or weakness of the coupling, recall that, assuming regularity, an optimal plant must satisfy the Karush-Kuhn-Tucker (KKT) conditions [26],

$$\mathbf{h}_p(\mathbf{x}_p) = 0, \; \mathbf{g}_p(\mathbf{x}_p) \le 0, \; \exists(\alpha \ne 0, \beta \ge 0):$$

$$\beta^T \mathbf{g}_p(\mathbf{x}_p) = 0,$$

$$\frac{dF_p}{d\mathbf{x}_p} + \left(\frac{d\mathbf{h}_p}{d\mathbf{x}_p}\right)^T\alpha + \left(\frac{d\mathbf{g}_p}{d\mathbf{x}_p}\right)^T\beta = 0 \quad , \tag{20}$$

an optimal observer must satisfy the Wiener-Hopf equation [25]:

$$\mathrm{cov}[\mathbf{z}(T), \mathbf{y}(\sigma)] + \int_0^T \mathbf{L}(T, \tau)\mathrm{cov}[\mathbf{y}(\tau), \mathbf{y}(\sigma)]d\tau = 0 \;\; \forall\sigma \in [0, T], \tag{21}$$

and optimal control must satisfy the Pontryagin principle [27],

$$\dot{\mathbf{z}} = \mathbf{A}(\mathbf{x}_p)\mathbf{z} + \mathbf{B}(\mathbf{x}_p)\mathbf{u} + \mathbf{G}(\mathbf{x}_p)\mathbf{w}\,,\ \mathbf{u}(t) = -\mathbf{K}\mathbf{z}(t)\,,$$
$$\exists(\lambda(t) \neq 0, L := \mathbf{z}^T\mathbf{Q}\mathbf{z} + \mathbf{u}^T\mathbf{R}\mathbf{u}, \mathbf{f} := \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u}, H := L + \lambda^T\mathbf{f}): \qquad (22)$$
$$\frac{\partial H}{\partial \mathbf{u}} = 0\,,\ \frac{d\lambda}{dt} = \frac{\partial H}{\partial \mathbf{z}}\,,\ \lim_{t \to \infty} \lambda(t) = 0\,.$$

Optimizing a plant, its observer, and its controller sequentially produces a system satisfying these three sets of first-order necessary conditions for optimality. For the resulting system to be truly optimal, it must satisfy first-order necessary conditions for system optimality, which can be shown to consist of the Weiner-Hopf equation, Pontryagin conditions, plus system optimality KKT conditions differing from the plant optimality KKT conditions by a *coupling term* Γ, as shown below.

$$\mathbf{h}_p(\mathbf{x}_p) = 0\,,\ \mathbf{g}_p(\mathbf{x}_p) \leq 0\,,\ \exists(\alpha \neq 0, \beta \geq 0):$$
$$\beta^T \mathbf{g}_p(\mathbf{x}_p) = 0\,,\ \frac{dF_p}{d\mathbf{x}_p} + \left(\frac{d\mathbf{h}_p}{d\mathbf{x}_p}\right)^T \alpha + \left(\frac{d\mathbf{g}_p}{d\mathbf{x}_p}\right)^T \beta + \Gamma = 0\,. \qquad (23)$$

This coupling term has several important interpretations:

1. It is a measures the sensitivity of optimal closed-loop performance with respect to the underlying plant.
2. It quantifies the coupling between the plant, observer, and controller optimization problems (the larger the coupling term, the more coupled the problems).
3. It quantifies *ease of observation and control*. A large coupling term implies that the given plant can be modified to signficantly improve closed-loop performance and is, hence, <u>not</u> easy to observe and control.
4. It can be used to classify the scenarios where the plant and observer/controller optimization problems decouple [21].

For combined plant/controller optimization assuming full state measurement, the coupling term is given by [21]:

$$\Gamma = -\frac{w_{oc}}{w_p}\int_{t_o}^{T}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}_p}\right)^T \lambda dt\,. \qquad (24)$$

Furthermore, for combined plant/observer optimizationassuming open-loop control, the coupling term is given by:

$$\Gamma = \frac{w_{oc}}{w_p}\frac{dE_{ms}^*}{d\mathbf{x}_p}\,. \qquad (25)$$

Finally, for the combined plant/observer/controller optimization problem in Eq. (12), the coupling term is given by:

$$\Gamma = \frac{w_o}{w_p}\frac{d}{d\mathbf{x}_p}f_{oc}^*(\mathbf{x}_p)\,. \qquad (26)$$

These three terms quantify a plant's *ease of observation*, *ease of control*, and *ease of observation and control*, respectively.

5. SUSPENSION OPTIMIZATION EXAMPLE

To demonstrate the coupling, this section optimizes a combined passive/active car suspension for ease of control, ease of observation, and ease of observation and control using the nested strategy, quantifies the coupling for these three problems, and draws some conclusions from the optimization results.

The combined suspension plant/controller optimization problem in Eq. (1) was solved by Fathy *et al.* [23] for the weights $r_o = 1$, $r_1 = r_2 = 3 \times 10^4$ and $r_3 = 4\text{x}10^{-6}$ and the filter cutoff frequency $\omega_f = 20.91\,\text{rad/sec}$. The system-optimal passive suspension plant design was found to be

$$k_s^* = 31.71\,\text{kN/m}\,,\ c_s^* = 16\,\text{kNs/m}\,, \qquad (27)$$

and, together with the corresponding system-optimal controller, this plant outperformed its purely passive, purely active and sequentially optimized passive/active counterparts by 7.43%, 61.17% and 2.35%, respectively.

Using the same parameter values and assuming only the availability of only one state measurement (namely, suspension stroke) and unity disturbance and noise variances, this section solves the combined plant/observer optimization problem,

$$\min_{k_s, c_s, \mathbf{L}(t,\tau)} E_{ms} = E\{\mathbf{e}^T(T)\mathbf{e}(T)\} \text{ subject to:}$$
$$\mathbf{z}(t) = [z_{us}(t) - z_g(t), \dot{z}_{us}(t), z_s(t) - z_{us}(t), \dot{z}_s(t), \dot{z}_g(t)]^T\,,$$
$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{G}w\,,\ \mathbf{y} = \mathbf{C}\mathbf{z} + \mathbf{v}\,, \mathbf{e} = \mathbf{z} - \mathbf{z}_e\,,$$
$$E(w(t)w(\tau)) = W\delta(t-\tau)\,,\ E(v(t)v(\tau)) = V\delta(t-\tau)\,,$$
$$\mathbf{z}_e(t) = \int_0^t \mathbf{L}(\tau, t)\mathbf{y}(\tau)d\tau\,,\ k_s \leq 160\text{kN/m}\,,\ c_s \leq 16\ \text{kN.s/m}\,,$$
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ -\frac{k_{us}}{m_{us}} & -\frac{c_s}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 \\ 0 & 0 & 0 & 0 & -\omega_f \end{bmatrix},\ \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_f \end{bmatrix}, \qquad (28)$$

and the resulting optimal combined plant/observer design is found to comprise the plant,

$$k_s^* = 0\,,\ c_s^* = 13.31\,\text{kN.s/m}\,, \qquad (29)$$

and the corresponding optimal observer. The fact that the system-optimal solutions to the combined suspension plant/controller optimization problem in Eq. (1) and the combined suspension plant/observer optimization problem in Eq. (28) do not match implies that these two problems are not separable. In other words, even though the well-known separation theorem [27] allows a fixed plant's observer and controller to be optimized separately without loss of system optimality, it is not possible to optimize a plant for ease of observation and ease of control separately and obtain consistent, system-optimal results.

To find the truly optimal combined suspension plant, observer and controller design, we pose and solve the combined plant/observer/controller optimization problem below using nested optimization for the weights $r_o = 1$, $r_1 = r_2 = 3 \times 10^4$ and $r_3 = 4\text{x}10^{-6}$, the filter cutoff frequency $\omega_f = 20.91\,\text{rad/sec}$, and three different sensor configurations (namely, measurement of suspension stroke only, measurement of sprung mass acceleration only, and measurement of both). Unity disturbance signal variance was assumed, and optimization was carried out for different values of the sensor noise variance V (in the case where both the stroke and sprung mass acceleration are measured, the sensor noise covariance matrix was assumed to equal $V\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, and optimization was carried out for different values of V).

$$\min_{k_s,\, c_s,\, \mathbf{K},\, \mathbf{L}(t,\,\tau)} E\int_0^\infty (r_o(\ddot{z}_s)^2 + r_1(z_{us} - z_g)^2 + r_2(z_s - z_{us})^2 + r_3u^2)dt$$

subject to: $\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}u + \mathbf{G}w$, $u = -\mathbf{K}\mathbf{z}_e$, $y = \mathbf{C}\mathbf{z} + v$,

$$\mathbf{z}(t) = [z_{us}(t) - z_g(t), \dot{z}_{us}(t), z_s(t) - z_{us}(t), \dot{z}_s(t), \dot{z}_g(t)]^T,$$

$$E(w(t)w(\tau)) = W\delta(t-\tau),\; E(v(t)v(\tau)) = V\delta(t-\tau),$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ -\frac{k_{us}}{m_{us}} & -\frac{c_s}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 \\ 0 & 0 & 0 & 0 & -\omega_f \end{bmatrix},\; \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{1}{m_{us}} \\ 0 \\ \frac{1}{m_s} \\ 0 \end{bmatrix},\; \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_f \end{bmatrix}, \quad (30)$$

$$k_s \le 160\text{kN/m},\; c_s \le 16\text{ kN.s/m},\; \mathbf{z}_e(t) = \int_0^t \mathbf{L}(\tau, t)\mathbf{y}(\tau)d\tau.$$

The results are depicted in Figure 3, which shows the percent improvement in performance that nested plant/observer/controller optimization gives over sequential optimization for the three different sensor configurations (two of the three curves are identical) and the various values of the sensor noise variance V.

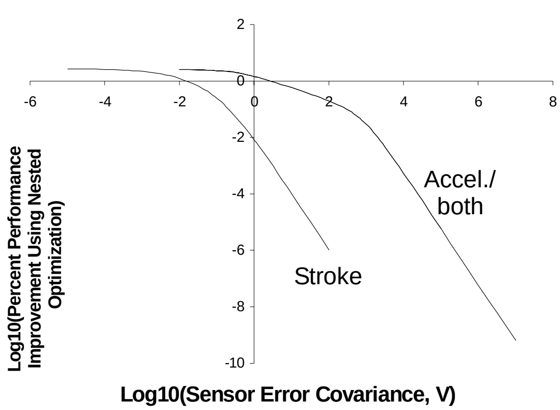

Figure 3: Percent Gain in Performance vs. Sensor Error

Two interesting results can be seen from Figure 3. First, for any of the three selected sensor configurations, increasing the sensor error decreases the percent difference in performance between the sequentially optimized and system-optimal suspension designs. This result can be explained as follows. As the sensor measurements become noisier, feedback control becomes less effective for disturbance rejection. Hence, the system performance becomes more dependent on the passive suspension design, and passive suspension optimization becomes more solely instrumental for optimizing the overall system performance, which decouples the plant and observer/controller optimization problems.

The second interesting result is that the percent improvement attainable using nested optimization, for extremely small sensor errors, is larger than the percent improvements obtained for the suspension plant/controller optimization problem in Eq. (1). For instance, using only a stroke sensor afflicted by white measurement noise whose standard deviation is only $1\text{x}10^{-5}\ \text{m}^2$, the system-optimal suspension design's performance is superior to that of the sequentially optimized suspension design by 2.71%, as opposed to the 2.35% improvement attained when system optimization is used instead of sequential optimization for the combined suspension plant/controller optimization problem in Eq. (1). Furthermore, the plant/observer/controller coupling term obtained when solving Eq. (3) using nested optimization is 12.6% larger in magnitude than the coupling term obtained when solving the combined plant/controller optimization problem in Eq. (1) using nested optimization for the same parameter values. In other words, the combined passive/active suspension optimization problem studied in this paper can exhibit a stronger coupling between its passive and active subsystem optimization problems when LQG control is used for the active subsystem instead of LQR control. This result can be explained as follows. Unlike LQG control, LQR control is guaranteed to be remarkably robust [27], in the sense that the closed-loop sensitivity of its performance to the underlying plant design is not very large. Therefore, the coupling between the plant and controller optimization problems for LQR-controlled systems, measured by a coupling term that reflects (among other factors) the systems' closed-loop performance sensitivity to the underlying plant, tends to be rather small. On the other hand, for LQG-controlled systems, these guaranteed robustness properties and their influence on the coupling term cease to exist. In sumamry, there is an interesting connection between a system's robustness and the coupling between the problems of optimizing its passive and active constituents, and this link will be pursued in more depth in future research.

6. DISCUSSION AND CONCLUSIONS

Research in several application areas has unveiled a coupling between the plant and control optimization problems. Due to this coupling, optimizing a system's plant and control sequentially does not guarantee system optimality. Recent work has quantified this coupling and constructed nested plant/control optimization strategies guaranteed to mitigate it and find system optima. This research mostly assumes full state measurement.

This paper presents, for the first time, a study of the coupling between plant and control optimization that incorporates observers, which are necessitated in practice by limited and noisy sensor measurements. The trilateral coupling between the plant, observer, and controller optimization problems is quantified in terms of coupling terms derived from a comparison of first-order necessary conditions for optimality. These coupling terms measure a plant's ease of observation and control and provide numerical means for deciding whether to utilize nested or sequential plant/observer/controller optimization. The results are demonstrated using a simple combined passive/active car suspension case study. From the case study's results, we conjecture the existence of a connection between a plant's robustness and the coupling between the optimization of its plant, observation, and control. Future work will address this conjecture in depth.

7. ACKNOWLEDGMENTS

The authors would like to thank the Ford Research Laboratories and the Automotive Research Center (US Army Center of Excellence) and NSF Engineering Research Center for Reconfigurable Manufacturing Systems (NSF grant # EEC9529125) at The University of Michigan for their generous financial support to this research.

8. REFERENCES

1. Belvin, W. K., and Park, K. C., 1990, "Structural Tailoring and Feedback Control Synthesis: An Interdisciplinary Approach," *Journal of Guidance, Control and Dynamics*, vol. 13, pp. 424-429, 1990.
2. Eastep, F. N., Khot, N., and Grandhi, R., 1987, "Improving the Active Vibrational Control of Large Space Structures Through Structural Modifications," *Acta Astronautica*, vol. 15, pp. 383-389, 1987.
3. Grigoriadis, K. M., Carpenter, M. J., Zhu, G., and Skelton, R. E., 1996, "Optimal Redesign of Linear Systems," *Transactions of the ASME*, vol. 118, pp. 598-605, Sep, 1996.
4. Haftka, R. T., Martinovic, Z. N., Hallauer, W. L., and Schamel, G., 1987, "An Analytical and Experimental Study of a Control System's Sensitivity to Structural Vibrations," *AIAA Journal*, vol. 25, pp. 310-315, Feb, 1987.
5. Hale, A. L., Lisowski, R. J., and Dahl, W. E., 1985, "Optimal Simultaneous Structural and Control Design of Maneuvering Flexible Aircraft," *Journal of Guidance, Control and Dynamics*, vol. 8, pp. 86-93, Feb, 1985.
6. Khot, N. S., Oz, H., Grandhi, R. V., Eastep, F. E., and Venkayya, V. B., 1988, "Optimal Structural Design with Control Gain Norm Constraint," *AIAA Journal*, vol. 26, pp. 604-611, 1988.
7. Maghami, P. M., Joshi, S. M., and Price, D. B., 1995, "Integrated Controls-Structures Design Methodology for Flexible Aircraft," *Journal of Spacecraft and Rockets*, vol. 32, pp. 839-844, 1995.
8. Maghami, P. G., Gupta, S., Elliot, K. B., and Waltz, J. E., 1996, "Experimental Validation of an Integrated Controls-Structure Design Methodology," *Journal of Guidance, Control and Dynamics*, vol. 19, pp. 324-333, 1996.
9. McLaren, M. D., and Slater, G. L., 1990, "A Covariance Approach to Integrated Control/Structure Optimization," *Proceedings of the AIAA Dynamic Spec. Conference*, pp. 189-205, 1990.
10. Onoda, J., and Haftka, R. T., 1987, "An Approach to Structure/ControlSimultaneous Optimization for Large Flexible Spacecraft", *AIAA Journal*, vol. 25, pp. 1133-1138, Aug, 1987.
11. Rao, S. S., 1988, "Combined Structural and Control Optimization of Flexible Structures," *Engineering Optimization*, vol. 13, pp. 1-16, 1988.
12. Smith, M. J., Grigoriadis, K. M., and Skelton, R. E., 1992, "Optimal Mix of Active and Passive Control in Structures," *Journal of Guidance, Control and Dynamics*, vol. 15, pp. 912-919, 1992.
13. Sunar, M. and Rao, S. S., 1993, "Simultaneous Passive and Active Control Design of Structures Using Multiobjective Optimization Strategies", *Computers and Structures*, vol. 48, pp. 913-924, 1993.
14. Karpel, M., 1992, "Multidisciplinary Optimization of Aeroservoelastic Systems Using Reduced-Size Models," *Journal of Aircraft*, vol. 29, pp. 939-946, 1992.
15. Sahasrabudhe, V., Celi, R., and Tits, A. L., 1997, "Integrated Rotor-Flight Control System Optimization with Aeroelastic and Handling Qualities Constraints," *Journal of Guidance, Control and Dynamics*, vol. 20, Mar, 1997-Apr 30, 1997.
16. Asada, H., Park, J.-H., and Rai, S., 1991, "A Control-Configured Flexible Arm: Integrated Structure/Control Design", *Proceedings of the 1991 IEEE International Conference on Robotics and Automation*, Sacramento, California, pp. 2356-2362, Apr. 1991.
17. Youcef-Toumi, K., 1996, "Modeling, Design and Control Integration: A Necessary Step in Mechatronics," *IEEE/ASME Transactions on Mechatronics*, vol. 1, pp. 29-38, 1996.
18. Reyer, J. A., and Papalambros, P. Y., 1999, "Optimal Design and Control of an Electric DC Motor," *Proceedings of the 1999 ASME Design Engineering Technical Conferences*, 1999.
19. Reyer, J. A., and Papalambros, P. Y., 2000, "An Investigation into Modeling and Solution Strategies for Optimal Design and Control," *Proceedings of the 2000 ASME Design Engineering Technical Conferences*, 2000.
20. Reyer, J. A., 2000, *Combined Embodiment Design and Control Optimization: Effect of Cross-Disciplinary Coupling*. (Dissertation, The University of Michigan, Ann Arbor, Michigan, 2000).

21. Fathy, H. K., Reyer, J. A., Papalambros, P. Y. and Ulsoy, A. G., 2001, “On the Coupling between the Plant and Controller Optimization Problems”, *Proceedings of the 2001 American Control Conference*, pp. 1864-1869, 2001.
22. Fathy, H. K., Bortoff, S. A., Copeland, G. S., Papalambros, P. Y. and Ulsoy, A. G., 2002, “Nested Optimization of an Elevator and its Gain-Scheduled LQG Controller”, *Proceedings of the 2002 ASME International Mechanical Engineering Congress and Exposition. Also submitted to the ASME/IEEE Transactions on Mechatronics for review.*
23. Fathy, H. K., Papalambros, P. Y., Ulsoy, A. G., and Hrovat, D., 2003, “Nested Plant/Control Optimization with Application to Combined Passive/Active Car Suspensions”, *Proceedings of the 2003 American Control Conference.*
24. Ulsoy, A. G., Hrovat, D., and Tseng, T., 1994, “Stability Robustness of LQ and LQG Active Suspensions,” *Journal of Dynamic Systems, Measurement and Control*, vol. 116, pp. 123-131.
25. Kalman, R. E., and Bucy, R. S., 1961, “New Results in Linear Filtering and Prediction Theory,” *Journal of Basic Engineering (Transactions of ASME)*, pp. 95-108.
26. Papalambros, P. Y., and Wilde, D. J., *Principles of Optimal Design : Modeling and Computation*. (Cambridge University Press, Cambridge, New York: 2000).
27. Lewis, F. L., and Syrmos, V. L., *Optimal Control*. (John Wiley & Sons, New York: 1994).