Nested Plant/Controller Optimization with Application to Combined Passive/Active Automotive Suspensions

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Abstract

This paper addresses the coupling between the plant and control optimization problems for a suspension comprising both passive and active components. Suspension dynamics are modeled using a two degree-of-freedom, linear and time-invariant quarter-car model whose ability to capture the influence of the passive stiffness on rattle space depends on the ground disturbance's spectral density. Nested optimization, a strategy that guarantees system optimality, is presented and used to find the optimal suspension system design. The design exhibits superior performance compared to its passive, active and sequentially optimized passive/active counterparts because it accounts for the coupling between the passive and active suspension optimization problems.

1. Introduction

Car suspensions provide comfort by isolating passengers from road disturbances and improve handling by regulating the contact forces between the vehicle chassis, tires and the road. These requirements are mutually conflicting. Softer suspensions generally offer more ride comfort at the cost of degraded handling [1]. Suspension design often trades these requirements off by grouping them into a weighted performance function for optimization [2-8].

A suspension's optimal performance depends on whether it is *active* or *passive*. Active suspensions utilize external energy sources (e.g., hydraulic actuators), whereas passive suspensions consist solely of energy storage and dissipation components (e.g., springs and viscous dampers) [1]. Passive components can only impart forces that depend on relative chassis/tire motion, whereas active (*sky-hook*) elements can generate forces that depend on absolute chassis motion. Consequently, active suspensions can outperform their passive counterparts significantly, at the expense of using external energy [1-2, 8-9].

When a suspension comprises both passive and active components, these components "compete, rather than help each other" [10], thus failing to reach the suspension's full synergistic performance potential [11]. This competition exemplifies the well-known *coupling* between the *plant* and *controller* optimization problems [10, 12-16]. Sequentially optimizing a suspension's plant (i.e., passive components) and controller (i.e., active components) does not account for this coupling and hence fails to guarantee system optimality. To prevent suboptimality and to design synergistic suspensions, one must optimize passive and active constituents simultaneously.

Such a simultaneous optimization approach is lacking in the literature and the present paper contributes an integrated passive/active suspension optimization problem (Section 2) and its solution using a nested strategy (Section 3). The resulting system-optimal suspension outperforms its passive, active and sequentially optimized passive/active counterparts in both the time and frequency domains, as shown in Sections 4 and 5.

2. System-Level Optimization Problem

To optimize a suspension's passive and active subsystems simultaneously, one must use a system-level model capturing the influence of both subsystems on performance. The model should express design goals in terms of physical variables (e.g., passive stiffness and damping coefficient) rather than derived ones (e.g., open-loop natural frequencies and damping ratios). It must also be simple while capturing the important suspension design tradeoffs (e.g., the tradeoff between ride quality and handling).

Suspension dynamics are described by a quarter-car, two degree-of-freedom, linear and time-invariant model [2-8], Figure 1.

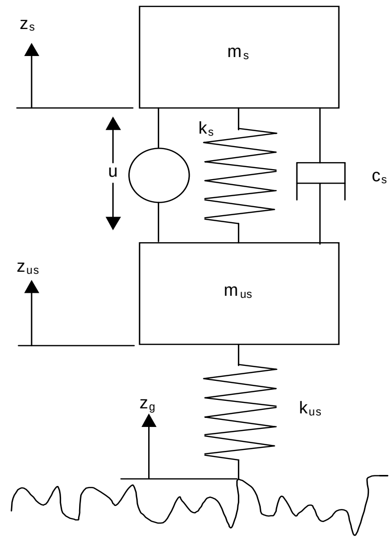

Figure 1: Dynamic Model of Combined Active/ Passive Suspension

In this model, m_s and m_{us} are the sprung and unsprung masses, k_{us} is the tire stiffness, tire damping is neglected, k_s and c_s are the passive stiffness and damping coefficient, $u(t)$ is the active control force and $z_s(t)$, $z_{us}(t)$ and $z_g(t)$ are the vertical displacements of the sprung mass, the unsprung mass and the road, respectively. This gives the suspension model:

$$\begin{bmatrix} \dot{z}_{us}-\dot{z}_g \\ \ddot{z}_{us} \\ \dot{z}_s-\dot{z}_{us} \\ \ddot{z}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{us}}{m_{us}} & -\frac{c_s}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} \end{bmatrix} \begin{bmatrix} z_{us}-z_g \\ \dot{z}_{us} \\ z_s-z_{us} \\ \dot{z}_s \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{m_{us}} \\ 0 \\ \frac{1}{m_s} \end{bmatrix} u + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dot{z}_g \quad (1)$$

In the active suspension literature the ground disturbance velocity is modeled as zero-mean, white and Gaussian with covariance W [2], i.e.,

$$E(\dot{z}_g(t)\dot{z}_g(\tau)) = W\delta(t-\tau), \quad (2)$$

where E is the expectation operator and δ is the delta function. This model is not suitable for passive or combined passive/active suspension optimization because it does not account for the physically observed influence of the passive stiffness k_s on the mean square (ms) rattlespace, defined as the expected value of $(z_s-z_{us})^2$. A "softer" suspension (i.e., one with a smaller stiffness

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k_s) can typically provide more ride comfort at the expense of larger suspension deflections, or “rattle”. With the above model, the ms rattlespace for a purely passive suspension is independent of the suspension stiffness.

Coloring the ground disturbance velocity using a first-order low-pass filter captures the influence of the passive stiffness on the ms rattlespace and produces the model below.

$$\dot{\mathbf{z}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ -\frac{k_{us}}{m_{us}} & -\frac{c_s}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 \\ 0 & 0 & 0 & 0 & -\omega_f \end{bmatrix} \mathbf{z}(t) + \begin{bmatrix} 0 \\ -\frac{1}{m_{us}} \\ 0 \\ \frac{1}{m_s} \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_f \end{bmatrix} w(t) \qquad (3)$$

Here, $\mathbf{z}(t) = [z_{us}(t) - z_g(t), \dot{z}_{us}(t), z_s(t) - z_{us}(t), \dot{z}_s(t), \dot{z}_g(t)]^T$ is the vector of suspension (and ground disturbance model) states, ω_f is the filter cutoff frequency and $w(t)$ is a hypothetical zero-mean, white and Gaussian disturbance of variance W. This modified model introduces a fifth, uncontrollable state for the ground disturbance velocity. If full state feedback is used, this state must be either measured or estimated. For simplicity, all suspension states are assumed measured, an assumption not easily implemented in practice, since directly measuring the ground disturbance velocity is equivalent to having perfect disturbance information.

Using Eq. (3), we pose the system-level combined passive/active suspension optimization problem:

$$\min_{k_s, c_s, \mathbf{K}} J = E\int_0^\infty (r_o(\ddot{z}_s)^2 + r_1(z_{us} - z_g)^2 + r_2(z_s - z_{us})^2 + r_3u^2)dt$$

subject to: $\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}u(t) + \mathbf{G}w(t)$ **,** $u(t) = -\mathbf{K}\mathbf{z}(t)$ **,**

$E(w(t)w(\tau)) = W\delta(t - \tau)$, $\mathrm{eig}(\mathbf{A} - \mathbf{B}\mathbf{K}) \le 0$ **,**

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ -\frac{k_{us}}{m_{us}} & -\frac{c_s}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 \\ 0 & 0 & 0 & 0 & -\omega_f \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{1}{m_{us}} \\ 0 \\ \frac{1}{m_s} \\ 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_f \end{bmatrix}, \qquad (4)$$

$\mathbf{z}(t) = [z_{us}(t) - z_g(t), \dot{z}_{us}(t), z_s(t) - z_{us}(t), \dot{z}_s(t), \dot{z}_g(t)]^T$,

$k_s \le 160\text{kN/m}$, $c_s \le 16\ \text{kNs/m}$.

Optimization variables for the plant are the passive stiffness k_s and damping coefficient c_s, and for the controller, the feedback gain vector $\mathbf{K}$. Linear and time-invariant full state feedback is assumed. Upper bounds of 160kN/m and 16kNs/m are imposed on the plant design variables to prevent them from deviating greatly from their typical values in [11], and closed-loop stability is imposed as a constraint on the plant and controller designs. The optimization objective is a weighted sum of the ms sprung mass acceleration, ms tire deflection (*wheel hop*), ms suspension stroke (*rattle space*) and ms active control force [11]. An optimal solution to the above problem will be referred to as the *system-optimal* suspension design.

3. Nested Suspension Optimization

Nested optimization, a strategy guaranteed to find system-optimal designs, is applied to the suspension optimization problem in Eq. (4). To understand the strategy consider the problem in Eq. (4), abstracted as:

$$\mathbf{x}^* = \underset{\mathbf{x} \in \mathbf{X}}{\arg\min}\ f(\mathbf{x}) \qquad (5)$$

where $\mathbf{x} = [k_s\ c_s\ \mathbf{K}]$ is the vector of system variables, the * superscript denotes system-level optimality, $f = J$ is the system-level optimization objective and $\mathbf{X}$ is the set of feasible systems, i.e., the set of systems satisfying all system-level constraints. Now suppose that this problem is partitioned into plant and controller optimization subproblems by partitioning the vector $\mathbf{x}$ of system variables into two vectors, $\mathbf{x}_p$ and $\mathbf{x}_c$, of plant and controller variables,

$$\mathbf{x} = (\mathbf{x}_p, \mathbf{x}_c), \quad \mathbf{x}_p = [k_s\ c_s], \quad \mathbf{x}_c = [\mathbf{K}]. \qquad (6)$$

Let $\mathbf{X}_p$ be the set of feasible plants (plants for which feasible systems exist), and $\mathbf{X}_c(\mathbf{x}_p)$ be the set of feasible controllers for a given plant (controllers which, together with the given plant, constitute a feasible system). These sets are defined formally, as

$$\mathbf{X}_p = \{\mathbf{x}_p : \exists \mathbf{x}_c : (\mathbf{x}_p, \mathbf{x}_c) \in \mathbf{X}\},\ \mathbf{X}_c(\mathbf{x}_p) = \{\mathbf{x}_c : (\mathbf{x}_p, \mathbf{x}_c) \mathbf{X}\}. \qquad (7)$$

Nested optimization solves the two “nested” problems:

$$\mathbf{x}_p^* = \underset{\mathbf{x}_p \in \mathbf{X}_p}{\arg\min}\ f(\mathbf{x}_p, \mathbf{x}_c^*(\mathbf{x}_p)),\ \mathbf{x}_c^*(\mathbf{x}_p) = \underset{\mathbf{x}_c \in \mathbf{X}_c(\mathbf{x}_p)}{\arg\min}\ f(\mathbf{x}_p, \mathbf{x}_c) \qquad (8)$$

and has been shown to yield the system optimum [12]. The outer loop optimizes the system-level objective with respect to the plant, with a controller optimal for that plant. The inner loop finds the optimal controller for every plant generated by the outer loop. To use nested optimization, one must be able to determine the set $\mathbf{X}_p$ of feasible plants *a priori*, because the outer optimization loop searches over this set. For the problem in Eq. (4), $\mathbf{X}_p$ is the set of all passive suspensions satisfying $k_s \le 160\text{kN/m}$ and $c_s \le 16\ \text{kN.s/m}$. Using this result, one can apply nested optimization (Eq. (8)) to the problem in Eq. (4) as follows.

<u>Outer Loop:</u> $(k_s^*, c_s^*) = \underset{k_s, c_s}{\arg\min}\ f^*(k_s, c_s)$ subject to:

$$k_s \le 160\text{kN/m},\ c_s \le 16\ \text{kN.s/m},\ f^*(k_s, c_s) = \text{output of inner loop} \qquad (9)$$

<u>Inner Loop:</u>

$$f^*(k_s, c_s) = \min_{\mathbf{K}} E\int_0^\infty (r_o(\ddot{z}_s)^2 + r_1(z_{us} - z_g)^2 + r_2(z_s - z_{us})^2 + r_3u^2)dt$$

subject to: $\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}u(t) + \mathbf{G}w(t)$, $u(t) = -\mathbf{K}\begin{bmatrix} \mathbf{z}(t) \\ \dot{z}_g(t) \end{bmatrix}$,

$$\mathbf{z}(t) = [z_{us}(t) - z_g(t), \dot{z}_{us}(t), z_s(t) - z_{us}(t), \dot{z}_s(t), \dot{z}_g(t)]^T, \qquad (10)$$

$E(w(t)w(\tau)) = W\delta(t - \tau)$, $\mathrm{eig}(\mathbf{A} - \mathbf{B}\mathbf{K}) \le 0$,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ -\frac{k_{us}}{m_{us}} & -\frac{c_s}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 \\ 0 & 0 & 0 & 0 & -\omega_f \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ -\frac{1}{m_{us}} \\ 0 \\ \frac{1}{m_s} \\ 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_f \end{bmatrix}.$$

Equation (10) is an LQR problem and standard LQR optimization techniques can be applied.

4. Optimization Results

This section compares four car suspension types numerically:

(i) A *(purely) passive* suspension, obtained by solving the system-level problem in Eq. (4) with the additional constraint $\mathbf{K} = 0$;

(ii) *An actively-implemented* (*"active"*, for short) suspension, obtained by solving the system-level problem in Eq. (4) with the additional constraints $k_s = 0$ and $c_s = 0$; This suspension is constrained to impart forces on the car chassis using purely active means (e.g., hydraulic actuators), but is <u>not</u> constrained to be strictly active, i.e., some of its control action could be replicated using passive means (but this will not be done here);

(iii) A *sequentially-optimized passive/active suspension* (*"sequential"*, for short), obtained by finding the optimal passive suspension design, then solving the active suspension optimization problem in Eq. (10) for that passive design;

(iv) A *system-optimal combined passive/active suspension* (*"nested"*, for short), obtained by solving the nested optimization problem in Eqs. (9-10).

Three different measures are used for comparing these designs:

(i) *Overall performance*, measured by the overall performance index J in Eq. (4);

(ii) *Suspension quality*, defined as the weighted sum of the ms sprung mass acceleration, ms rattlespace and ms wheel hop:

$$J_q = E\int_0^\infty (r_o(\ddot{z}_s)^2 + r_1(z_{us} - z_g)^2 + r_2(z_s - z_{us})^2)dt ; \tag{11}$$

(iii) *Control energy*, defined as the ms control force and, hence, a measure of how much control needs to be actively implemented by a given suspension design:

$$J_c = E\int_0^\infty (u^2)dt . \tag{12}$$

Thus, overall performance (Eq. (4)) is a weighted sum of suspension quality and control energy:

$$J = J_q + r_3 J_c . \tag{13}$$

Numerical results were obtained for the weights $r_o = 1$, $r_1 = r_2 = 3 \times 10^4$ and the range $(0, \infty)$ of values for the weight r_3, producing four *Pareto curves* of suspension quality versus control energy for the four suspension types investigated. This section presents these Pareto curves (Figure 2), studies the notion of the optimal "mix" of passive and active elements, and finally studies the Pareto points corresponding to $r_3 = 4 \times 10^{-6}$. The Pareto curve for the passive suspension is a singleton because the optimal passive design does not vary with the weight r_3.

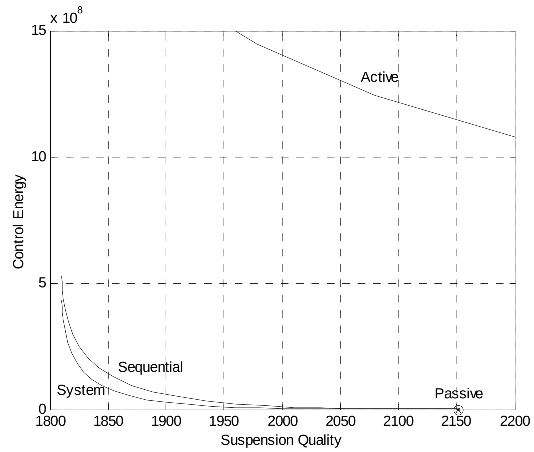

Figure 2: Susp. Quality/Contr. Energy Pareto Curves

Figure 2 reaffirms the well-known fact that active suspensions can provide significant improvements in suspension quality over their passive counterparts, at the expense of requiring significant control energy [1]. In the present case, this result is accentuated by the fact that k_s and c_s have been fixed to zero in the active suspension design problem. To barely attain the same suspension quality provided by the passive suspension, an active suspension must exert a root mean square active control force 1.73 times the static weight of the sprung mass. It is desirable to achieve the suspension qualities attainable by the active suspensions without expending such high control energy. Figure 2 indicates that this is accomplished if one utilizes an *optimal combination* of passive and active components (i.e., a system-optimal combined passive/active suspension). To see why this is so, examine the feedback gain matrix $\mathbf{K}$ for the optimal active suspension design. Some of the gains in this matrix represent *sky-hook* elements that link the sprung mass to a hypothetical "sky" or inertial frame of reference. These gains cannot be realized passively, which gives active suspensions an edge over passive suspensions and enables them to provide a better suspension quality J_q [1]. Some of these gains, however, can be realized passively. The gain $\mathbf{K}(3)$, for example, is equivalent to a linear spring linking the sprung and unsprung masses. Now suppose that, for the optimal active suspension corresponding to $r_3 = 4 \times 10^{-6}$, a fraction of this gain is replaced by a nonzero passive spring k_s. This replacement will not affect the suspension's closed-loop dynamics, since the gain $\mathbf{K}(3)$ and stiffness k_s appear as a sum in the closed-loop suspension dynamic model. Therefore, implementing a fraction of $\mathbf{K}(3)$ passively will not affect the suspension quality J_q, but it will affect the control energy J_c required for achieving this quality. The effect of replacing fractions of the gains $\mathbf{K}(3)$ and $\mathbf{K}(4)$ of the optimal active suspension corresponding to $r_3 = 4 \times 10^{-6}$ with a spring-damper assembly on the performance J and control energy J_c is plotted below versus the fractions of the gains implemented passively.

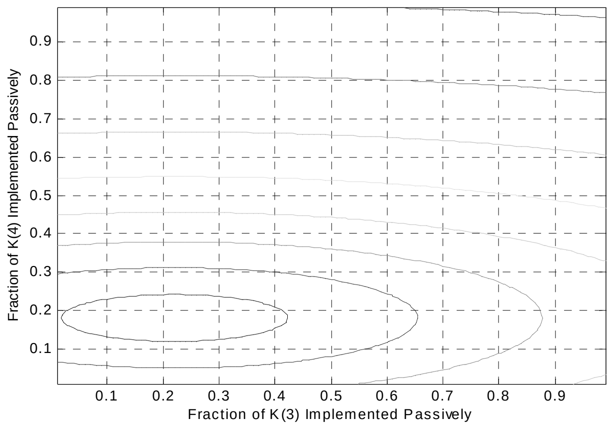

Figure 3: Contours of Control Energy vs. Fractions of Active Suspension Gains Implemented Passively

Figure 3 shows that optimally replacing fractions of the active suspension gains with a passive spring-damper assembly can improve control energy and overall performance (in the present case, by 69% and 23%, respectively). Therefore, there exists an "optimal mix" of active and passive control in suspension design, a fact which has also been observed in the space structure design field [10]. The optimal car suspension system design is hence neither purely passive nor purely active, but rather an optimal, synergistic blend of both. This is why sequentially optimized suspensions, which incorporate both passive and active components, outperform the corresponding optimal active suspensions for all values of the gain r_3, as Fig. 2 shows. Nevertheless, sequential optimization does not find the truly system-optimal mix of active and passive components because it does not explicitly account for the plant/controller optimization coupling. This optimal mix is found by nested optimization. As Fig. 2 shows, the Pareto set of system-optimal passive/active suspensions "dominates" the Pareto set of sequentially optimized passive/active suspensions in the sense that attaining any possible suspension quality using a system-optimal suspension requires less control energy.

To examine these results in more depth, Tables 1 and 2 compare the optimal passive, optimal active, sequentially optimized and system-optimal suspension designs and performance measures for the weights $r_o = 1$, $r_1 = r_2 = 3\times10^4$ and $r_3 = 4\times10^{-6}$.

Table 1: Optimal Suspension Design Comparison

Design Var.s	Pass.	Active	Seq.	Nest.
k_s [kN/m]	2.44	0	2.44	31.71
c_s [kN.s/m]	16.00	0	16.00	16.00
$\mathbf{K}(1)$ [kN/m]	0	59.78	11.72	4.90
$\mathbf{K}(2)$ [kN.s/m]	0	-1.85	0.405	0.413
$\mathbf{K}(3)$ [kN/m]	0	84.02	81.62	57.77
$\mathbf{K}(4)$ [kN.s/m]	0	17.625	77.27	5.85
$\mathbf{K}(5)$ [kN.s/m]	0	-1.824	-0.572	-0.299

Table 2: Optimal Suspension Performance Comparison

Perf. Obj.	Passive	Active	Sequent.	Nested
J_q	2152	3380	1989	1942
J_c	0	4.38x10^8	1.28x107	1.25x10^7
J	2152	5131	2040	1992

The following observations can be made.

1. The upper bound on the passive damping coefficient is active in all problems where this coefficient is varied. This is a result of the chosen optimization weights. For other choices of weights (e.g., $r_o = 1$, $r_1 = 2\times10^4$, $r_2 = 1\times10^5$ and $r_3 = 4\times10^{-6}$), this constraint need not be active for all four problems. The activity of this particular constraint does not affect the conclusions presented herein about the existence of a plant/controller optimization coupling and the ability of nested optimization to mitigate it.
2. Surprisingly, both the suspension quality and overall performance of the active suspension are significantly inferior (by 57% and 138%) to those of the passive suspension. Even though active suspensions can outperform their passive counterparts, this requires large amounts of control energy. In the present case, the value $r_3 = 4\times10^{-6}$ for the weight on control energy produces an active suspension that expends a root mean square control force equal to 1.07 times the static sprung weight, compared to the rms control force of 1.73 times that static weight needed to achieve the same suspension quality as the optimal passive design.
3. The sequential design significantly outperforms its passive and active counterparts, exhibiting a suspension quality 7.57% and 41.15% better, respectively, while expending 70.77% less control energy than the active suspension. This demonstrates the fact that the truly system-optimal suspension is neither purely passive nor purely active, but rather a synergistic mix of both.
4. The passive and active suspension optimization problems are coupled in the sense that solving them sequentially does not guarantee system-level optimality. This coupling is demonstrated by the fact that the system-optimal suspension's quality, energy and overall performance are 2.36%, 2.34% and 2.35% superior to those of the sequentially optimized suspension, respectively.

Table 3 shows that compared to its sequential counterpart, the system-optimal suspension design's resonant peaks reside at higher frequencies where the ground disturbance velocity signal dies off.

Table 3: Complex Plane Suspension Comparison

Quantity	Sequential Design	Nested Design
1st-mode ω_n	1.128 Hz	1.1315 Hz
1st-mode ζ	0.9181	0.7890
2nd-mode ω_n	9.1476 Hz	9.4104 Hz
2nd-mode ζ	0.6683	0.6565

Finally, Figures 4 and 5 depict the superiority of the system-optimal suspension design over its sequentially optimized counterpart visually using the power spectral densities of the suspension quality and control energy. From Eq. (11)

$$J_q = E\int_0^\infty r_o(\ddot{z}_s)^2 dt + E\int_0^\infty r_1(z_{us}-z_g)^2 dt + E\int_0^\infty r_1(z_s-z_{us})^2 dt\,, \qquad (14)$$

and since the mean square value of a scalar stochastic process equals the integral of its spectral density function divided by 2π,

$$J_q = \frac{1}{2\pi}\left\{\begin{array}{c} r_o\int_{-\infty}^{\infty} -\omega^2 Z_s(j\omega)Z_s(-j\omega)d\omega \\ +r_1\int_{-\infty}^{\infty}(Z_{us}(j\omega)-Z_g(j\omega))(Z_{us}(-j\omega)-Z_g(-j\omega))d\omega \\ +r_2\int_{-\infty}^{\infty}(Z_s(j\omega)-Z_{us}(j\omega))(Z_s(-j\omega)-Z_{us}(-j\omega))d\omega \end{array}\right\}. \quad (15)$$

Now let the weighted sum of the above three integrands be $S_q(\omega)$:

$$S_q = \left\{\begin{array}{c} -r_o(\omega^2 Z_s(j\omega)Z_s(-j\omega)) \\ +r_1((Z_{us}(j\omega)-Z_g(j\omega))(Z_{us}(-j\omega)-Z_g(-j\omega))) \\ +r_2((Z_s(j\omega)-Z_{us}(j\omega))(Z_s(-j\omega)-Z_{us}(-j\omega))) \end{array}\right\} \quad (16)$$

Therefore, the suspension quality is related to $S_q(\omega)$ by:

$$J_q = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_q(\omega)d\omega \quad (17)$$

The function $S_q(\omega)$ will be denoted as the *suspension quality spectral density (SQSD)*. It is an aggregate frequency-domain measure of the strengths of the signals contributing to suspension quality. Together with *control energy spectral density (CESD)* (the spectral density of the control signal $u(t)$), SQSD is used in Figures 4 and 5 to compare the four suspension designs in Tables 1 and 2 in the frequency domain, assuming unity disturbance variance.

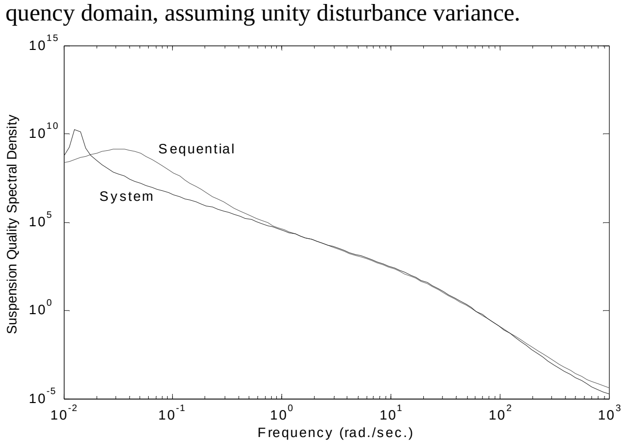

Figure 4: SQSD Comparison

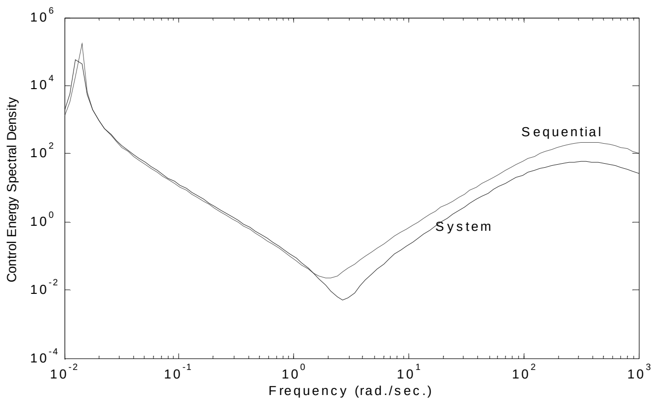

Figure 5: CESD Comparison

Figure 4 shows that the system-optimal suspension design requires approximately the same control energy at lower frequencies as its sequentially optimized counterpart to acheive better suspension qualities, and requires less control energy at higher frequencies to achieve the same suspension quality. This explains why the system-optimal suspension design supercedes its sequentially optimized counterpart in terms of overall performance. At every frequency of interest, the system-optimal suspension either achieves the same vibration attenuation (measured by SQSD) as its sequentially optimized counterpart using less control energy, or achieves better vibration attenuation using the same control energy. The superiority of the system-optimal suspension spans the entire frequency domain (except extremely small frequencies).

5. In the present case, the differences in performance between the system-optimal and sequentially optimized combined passive/active suspensions are relatively small, especially when compared to the rather large differences between the corresponding designs. For example, the system-optimal suspension's stiffness k_s is 13 times the sequentially optimized suspension's stiffness, but the difference in their overall performances is only 2.35%. The reason why such large differences in suspension design produce relatively small differences in suspension performance in the present case is that LQR control is remarkably robust [17]. Indeed, the gain margins of both the sequentially optimized and system-optimal combined active/passive suspensions are infinite (as expected [17]), and their phase margins are 84.34 and 64.44 degrees, respectively. Robust control minimizes the influence of undesirable variations and errors in the plant model on closed-loop, system-level performance, but this also results in minimizing the influence of intentional variations in the plant design on this performance, thereby making the performance difference between sequential plant/controller optimization and system-level plant/controller optimization less pronounced.

5. Discussion and Conclusions

Past research showed that active car suspensions can outperform their passive counterparts because they incorporate sky-hook elements that connect their sprung masses directly to an inertial frame of reference. As this article has shown, improved performance comes at a considerable cost in control energy.

A suspension system comprising both active and passive components can address this drawback. From a modeling viewpoint, the traditional quarter-car suspension model acted on by white ground disturbance velocities cannot be used combined passive/active suspension optimization because it is insenstitive to the influence of the passive stiffness on the mean square rattle space. Instead, the ground velocity needs to be colored using a low-pass filter to produce a model suitable for system-level, combined passive/active suspension optimization. This model was developed here and used to design a suspension that outperforms both the corresponding passive and active designs. The nested optimization solution strategy can be used for finding the system-optimal suspension for any arbitrary set of objective weights in the Pareto formulation of the combined problem. The superior design is obtained because the plant and controller optimization problems are coupled and the approach here accounts for that coupling effectively.

The superiority of the system-optimal design is evident not only using the aggregated, time-domain LQR performance measures, but also using frequency-domain measures of performance at every given frequency. Specifically, at all but the extremely small frequencies, the system-optimal suspension achieves either the same suspension quality spectral density as the sequentially optimized suspension or better, using either the same control energy spectral density or less. Also, the system-optimal suspension design is significantly different from the sequentially optimized design in terms of its physical design, but only 2.35% better in terms of its overall performance. This can be attributed to the use of LQR controllers, whose closed-loop performance is not very sensitive to the underlying plant designs owing to their robustness. This result should not discourage the designer from pursuing combined plant/controller optimization in suspension design. Even if the system-optimal suspension outperforms the sequentially optimized suspension by only 2.35%, it does outperform the optimal purely passive and optimal purely actively-implemented suspensions by 7.43% and 61.17%, respectively. Overall suspension performance is improved by optimally augmenting passive suspensions with active control rather than replacing a traditional purely passive suspension with a purely active one.

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