Panayotis Georgiopoulos
Program In Manufacturing,
University of Michigan,
Ann Arbor, MI 48109
e-mail: panayio@umich.edu

Mattias Jonsson
Department of Mathematics,
University of Michigan,
Ann Arbor, MI 48109
e-mail: mattiasj@umich.edu

Panos Y. Papalambros
Department of Mechanical Engineering,
University of Michigan,
Ann Arbor, MI 48109
e-mail: pyp@umich.edu

Linking Optimal Design Decisions to the Theory of the Firm: The Case of Resource Allocation

Resource allocation is a core business milestone in a firm's product development process: Maximize the final value derived from allocating resources into an appropriate product mix. Optimal engineering design typically deals with determining the best product based on technological (and, occasionally, cost) requirements. Linking technological with business decisions allows the firm to follow a resource allocation process that directly considers not only the resources to invest in different products but also the appropriate physical properties of these products. Thus, optimal designs are determined within an enterprise context that maximizes the firm's value. The article demonstrates how this integration can be accomplished analytically using a simple example in automotive product development. [DOI: 10.1115/1.1862679]

1 Introduction

The resource allocation process is a core decision-making process within a firm. Yet, engineering information produced from time-consuming and capital-intensive technical studies tends to have a largely passive role in the planning department of the firm. Occasionally, depending on the business cycle and the mood of management, we may see the other extreme, where product performance considerations drive resource allocations decisions too strongly. In either case the quality of the enterprise decisions suffers. Although it is well understood that business and engineering decisions are interdependent in product development, quantification of this relationship is a major challenge, in part due to the different cultures and modeling approaches prevailing within the financial and engineering communities. A balanced, concurrent decision process is desirable. To approach this challenge quantitatively one must maintain credibility within both communities, a quest that can be presumably achieved by using modeling practices acceptable and accessible to both the financial and the engineering communities. In this spirit the design methodology proposed here is based on the standard capital budgeting and microeconomics practice. The goal is to aid the decision-maker in understanding (at least some of) the trade-offs between the engineering and the business aspects of a design decision.

Using a simple financial model and an engineering performance simulation model we address modeling and solution of resource allocation decisions in product development. In operations research and investment science the resource allocation decision-making process is defined as the process of determining the percentage of resources to invest in different products. We broaden this definition by adding the simultaneous determination of the physical properties of the products corresponding to these assets, decisions typically considered within the domain of engineering design.

Engineering considerations in investment science are mainly associated with cost. The term technical uncertainty is used to describe the physical difficulty of completing a project [1]. Depending on the assessment of technical uncertainty, cost estimates are assumed to be either fixed capital investments [2,3] or stochastic elements in the investment problem [4].

Hazelrigg [5] proposed conceptually the idea of evaluating design decisions based on expected firm's owners utility maximization. Researchers from the engineering design community have demonstrated [6,7], enhanced [8,9], and modified [10] Hazelrigg's initial ideas.

Marston et al. [11] acknowledging the lack of resources in the early design stage, positioned utility theory in a larger framework that encompasses both descriptive and normative approaches in making design decisions. Regardless of the design stage, evaluating design decisions based on expected utility maximization stumbles on the limited acceptance of decision analysis in business decision-making practice [12]. In publicly held companies the owners of the firm are its shareholders. Their individual preferences are impossible to estimate. But one could observe them from the financial markets. The market mechanism allows shareholders to unanimously agree on the appropriate amount of investment needed for a future project. The decision-makers do not need to know anything about the tastes of the shareholders. Instead, according to standard capital budgeting theory [13] and practice [14] the decision-makers need to compare any investment opportunity with the return of financial instruments in the financial markets. They would select those that maximize net present value and therefore the market value of the firm's stock. This process allows delegation of the decision-making process from the owners to professional managers and enables the firm's stock owners to maximize their wealth and their utility of consumption over time [15].

In the present work we formulate an "enterprise-wide" decision problem for a publicly held firm, with the net present value resulting from the decisions as the objective function. This objective depends on both resource allocation and product design decisions, thus linking product design properties with the firm's goals. Constraints are imposed based on regulations, available resources, and product performance. The analysis models used to compute the outcomes of those decisions are computationally intensive simulations based on both engineering and investment science. The specific decision is modeled as a simple resource allocation problem: Given a production capacity and cost structure, historical data on sales, and two possible products that can be built in the existing production facility, what percent of production capacity should be allocated to each product and what should be the design specifications for these products? The interesting new element here is that asset allocation and asset properties determination are treated in a concurrent fashion. The end result is an estimate of what worth do the design decisions have, as an investment, to the firm's stock owners.

In the following sections we present the general formulation of resource allocation as an optimal design problem, and then pro-

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ceed to show how this model can be built in a particular resource allocation problem involving two automotive products that must operate under government regulations and associated penalties for violation. Computational results show the quantitative interdependence of engineering and investment decisions. A number of simplifying assumptions is made to afford a relatively simple study. These assumptions are noted as the models are developed.

2 Optimal Design in Resource Allocation

There is an underlying argument in the approach presented here: The sense that the firm's profitability π depends on product decisions $\mathbf{x}$ that can be partitioned into engineering design, $\mathbf{x}_d$, and business, $\mathbf{x}_b$, ones whose optimal combination leads increased profitability.

A typical engineering design problem is posed as

$$\begin{array}{ll} \text{maximize} & \text{engineering performance} \\ \text{with respect to} & \text{design decisions} \\ \text{subject to} & \text{engineering constraints} \end{array} \quad (1)$$

where all the relevant decisions are engineering ones. The typical investment decision model is

$$\begin{array}{ll} \text{maximize} & \text{firm's total value} \\ \text{with respect to} & \text{what to produce,} \\ & \text{how much to produce} \\ \text{subject to} & \text{available resources} \end{array} \quad (2)$$

where the relevant investment decisions relate to allocation of scarce resources. The engineering decision model is assumed usually to be deterministic; even though this can be a questionable assumption, in many situations the costs of dealing with uncertainty are so high that they overshadow the need for a more robust solution. On the contrary, the investment model is really meaningful only under uncertainty, and so in order to pose Eq. (2) the sources of uncertainty must be identified and described analytically to the extent possible. The time horizon for the investment opportunity must be also defined.

To link revenues and expenditures associated with asset allocation with the engineering performance of those assets, the two problems above are combined in an enterprise-wide decision model

$$\begin{array}{ll} \text{maximize} & \text{firm's total value} \\ \text{with respect to} & \text{investment decisions} \\ & \text{engineering decisions} \\ \text{subject to} & \text{investment constraints} \\ & \text{engineering constraints} \end{array} \quad (3)$$

This problem is solved with an appropriate optimization method. The term "appropriate" is used above because the nature of the optimization problem may require some judicious choices; for example, use of surrogate models instead of full simulations or use of global methods instead of gradient-based ones. This is not a central point here, but one needs to keep in mind that the resulting problem postulated in the general terms of Eq. (3) may present its own solution challenges, in addition to its formulation challenges.

We now consider this general approach for the specific case of an automotive manufacturing firm that markets premium-compact (PC) and full-size sport utility (SUV) vehicles. This market segmentation follows the J.D. Power classification for vehicles in the United States. We assume that the market segmentation during the product life-cycle will remain the same. The firm is assumed to operate in a mature industry where complementary assets [16], such as access to distribution channels, service networks, etc., are given. The decision-maker is assumed to be playing a game against nature, namely, strategy is affected by an exogenously generated random state not by competitive interaction.

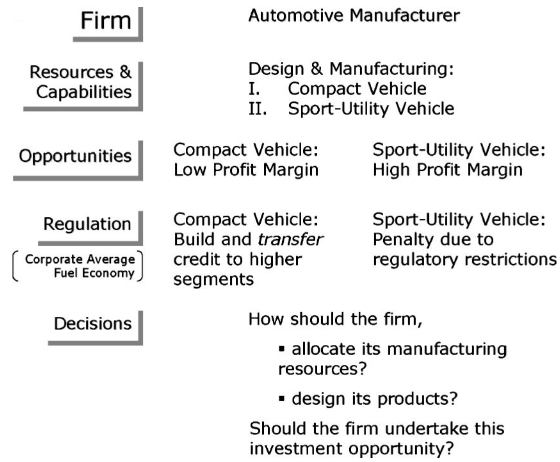

Fig. 1 Decisions modeled

The firm wishes to design new engines and transmissions for both PC and SUV segments. The PC and SUV segments are low and high profit margin segments, respectively. The Energy Policy and Conservation Act of 1975 required passenger car and light truck manufacturers to meet corporate average fuel economy (CAFE) standards applied on a fleet-wide basis. There are K units of monthly capacity currently in place for both segments, and so K is fixed, representing a capacity constraint. It is assumed that this capacity is not expandable. The decision-maker faces the following decisions: How should the units of capacity be allocated between the two segments in order to maximize the firm's value? What should the performance specifications for engines and transmissions be and how do these specifications affect the resource allocation decision? How much is this investment worth to the firm's stock owners? Figure 1 presents an overview of the decisions that the firm is facing.

Consistent with capital budgeting practice we assume that the firm under consideration evaluates investment opportunities with a single dominant objective: Maximize the market value of the firm's stock. This implies investments with positive net present value. The enterprise-wide decision model is then formulated as follows.

$$\begin{array}{ll} \text{maximize} & NPV \\ \text{with respect to} & \{\mathbf{x}^{\tau}, \mathbf{q}^{\tau}\}, \ \tau = 1,2 \\ \text{subject to} & \sum_{\tau=1}^{2} q^{\tau} = K \\ & \sum_{\tau=1}^{2} \mathrm{Cost}_{\mathrm{CAFE}}^{\tau} \leqslant 0 \\ & \mathbf{g}^{\tau}(\mathbf{x}^{\tau}) \leqslant 0, \ \tau = 1,2, \end{array} \quad (4)$$

This statement is a special case of the model in Eq. (3) where NPV is the net present value, $\mathbf{q}$ is the vector of production variables (monthly production quantities q_1, q_2), and $\mathbf{x}_{\tau}$ is the vector of engineering design decisions (engine sizes and final drive ratios) for each vehicle. The equality is a production constraint that fixes the total available production, the first inequality is an enterprise constraint that will not allow Corporate Average Fuel Economy (CAFE) penalties to be paid for the selected product mix, and the two vector inequalities are engineering constraints that will be discussed in more detail in Sec. 5.2. Definitions of the symbols used are given in the nomenclature. We will use the index $\tau = 1,2$ for the two products, but we will also use the superscripts PC and SUV instead of $\tau = 1$ and $\tau = 2$, respectively, when convenient.

Table 1 Price and quantities for the PC and SUV segments

	$P_{1/99}$	$q_{1/99}$	$P_{1/00}$	$q_{1/00}$
PC	$14,512	43,507	$ 15,015 ('99 adj.)	36,755
	$P_{1/98}$	$q_{1/98}$	$P_{1/99}$	$q_{1/99}$
SUV	$28,628	24,658	$29,596 ('98 adj.)	22,813

In Sec. 3 we will develop a solution for optimal production in a "static" problem, i.e., at a specific point in time. In Sec. 4 we will address the market uncertainties associated with time and the future, and link them to the design decisions. In Sec. 5 we will assemble the complete model for evaluating the functions in Eq. (4). In Sec. 6 we will present computed results and discuss them.

3 Partial Optimization: Optimal Production

Net present value is the aggregation of future monthly cash flows or profits π^τ minus the investment cost I over the life H of product i with a weighted average cost of capital:

$$NPV = -I + \int_0^H \pi^\tau e^{-(WACC)t} dt \tag{5}$$

The monthly profit is defined as

$$\pi^\tau = P^\tau q^\tau - C^\tau \tag{6}$$

where P^τ is the price, q^τ the quantity, and C^τ the total cost of producing q^τ vehicles.

Before addressing the problem in Eq. (4) we will first derive the model for the problem where instead of the NPV we maximize simply the monthly profits $\Sigma\pi^\tau$.

3.1 Estimation of the Demand Curve. We draw the relationship between the price P of each product and the quantity q demanded from the observed demand for final goods. Knowing two different points on the demand curve we can use the arc elasticity of demand, which is defined as:

$$E_P = \frac{\Delta q}{\Delta P} \frac{\overline{P}}{\overline{q}}, \tag{7}$$

where $\overline{P}$, $\overline{q}$ are the averages of the known prices and quantities, at the two points.

In years 2000 and 1999, General Motors PC vehicles (Chevrolet Cavalier and Prizm, Pontiac Sunfire, Saturn S-series) did not undergo a major design change. Using two pairs of data points $(P_{1/99}, q_{1/99})$, $(P_{1/00}, q_{1/00})$ (see Table 1) [17] and Eq. (7) we computed the price elasticity of the GM PC segment as equal to −4.9. The subscripts for P and q indicate month and year of data point collection. The price elasticity of all US automobiles has been found to be between −1 and −1.5 [18]. In [19] the price elasticity of the Chevrolet Cavalier was found to be −6.4. It is reasonable to assume that the price elasticity of demand for all GM PC vehicles is less than the elasticity of an individual model in that segment. For the GM SUV segment (Chevrolet Tahoe and Suburban, GMC Suburban/Yokon and Yukon XL) we used data points from the years 1999 and 1998 $(P_{1/98}, q_{1/98})$, $(P_{1/99}, q_{1/99})$ [17] where no major design change took place as well, finding the elasticity to be equal to −2.3.

Although there was no observed change in quality of vehicles in GM PC and SUV segments from 1999 to 2000, and 1998 to 1999, respectively,—which could be a reason for a shift in the demand curve—other factors that affect demand may have taken place. To use the estimated elasticities we assumed that for each segment between the two years there was no major change in consumer's income, product advertising, product information available to consumers, price and quality of substitutes and complementary goods, and population [20]. We further assumed that the two goods are independent, namely, a change in the price of the compact car has no effect on the quantity demanded for the sport utility vehicle.

The design of new engines and transmissions allows the firm to market a product with improved performance. From [19] we know the miles per dollar and horsepower to vehicle weight elasticities of demand for Chevrolet Cavalier are 0.52 and 0.42, respectively. That is, a 10% increase in miles per dollar and horsepower to vehicle weight ratio will boost demand by 5.2% and 4.2%, respectively. We will use these elasticities as representative for the PC segment. For demonstration purposes we assume that the horsepower to weight elasticity of demand of the traditional luxury segment is close to the SUV one. In [19] the Cadillac Seville horsepower to weight elasticity of demand is found to be 0.09. In this segment the miles per dollar elasticity of demand is found to be close to 0. This essentially means that the customer of that segment is satisfied with the current level of fuel economy performance. It is assumed that consumer behavior and therefore elasticities will not change during the life-cycle of the product. This is a reasonable assumption in the context of evaluating an investment decision. If a short-term pricing decision was under review then a frequent update of elasticities in Eq. (16) would be necessary.

We assume a linear multidimensional demand curve of the firm for a given market segment [21,22], namely:

$$q = \theta - \lambda_P P + \boldsymbol{\lambda}_{\boldsymbol{\alpha}}^T \boldsymbol{\alpha} \tag{8}$$

where θ is the intercept, $\boldsymbol{\alpha}$ are product characteristics observed by the consumer, and λ is the slope of the demand curve with respect to product attribute or price, i.e., $\lambda_P = \Delta q / \Delta P$ is the change in quantity associated with a change in price. The inverse demand curves for both segments are

$$\begin{aligned} P^{\mathrm{PC}} &= 14943 - 0.075 q^{\mathrm{PC}} + 2401 \frac{HP}{w} + 805 \frac{M}{\$} \\ P^{\mathrm{SUV}} &= 40440 - 0.525 q^{\mathrm{SUV}} + 2071 \frac{HP}{w} \end{aligned} \tag{9}$$

where HP is horsepower, w weight in tens of pounds, and $M/\$$ the number of ten miles increment one could travel for one dollar.

Assuming a linear relationship between cost and output,

$$C^\tau = c_0^\tau q^\tau \tag{10}$$

monthly profit is calculated as

$$\pi^\tau = \left(\frac{\theta^\tau}{\lambda_{P^\tau}} - \frac{1}{\lambda_{P^\tau}} q^\tau + \frac{1}{\lambda_{P^\tau}} (\boldsymbol{\lambda}_\alpha^\tau)^T \boldsymbol{\alpha}^\tau \right) q^\tau - c_0 q^\tau \tag{11}$$

Eq. (10) assumes that the marginal cost is constant, that is, for every unit increase in output, the average total cost increases by c_0, which is set at $13,500 and $18,500[1] for the PC and SUV segments, respectively [23]. We have assumed that the firm is operating at its minimum efficient scale [24].

3.2 CAFE Regulation Model. The fuel economy ratings for a manufacturer's entire line of passenger cars must average at least 27.5 miles per gallon (mpg). The 1993 CAFE standard was 20.3 mpg for light trucks (including vans and sport utility vehicles). Failure to comply with the limit, L^τ, results in a civil penalty of $5 for each 0.1 mpg the manufacturer's fleet falls below the standard, multiplied by the number of vehicles it produces. For example, if a manufacturer produces 2 million cars in a particular model year, and its CAFE falls 0.5 mpg below the standard, it would be liable for a civil penalty of $50 million. Specifically, for each vehicle τ, the penalty (or credit) due to CAFE is

$$(\mathrm{Cost}_{\mathrm{CAFE}})^\tau = (c_{\mathrm{CAFE}})^\tau q^\tau, \tag{12}$$

[1]These are estimates from private discussions with automotive industry professionals.

$$c_{\text{CAFE}}^{\tau} = \left[5 \times \frac{L^{\tau} - fe^{\tau}}{0.1}\right]$$

Fuel economy fe^{τ} is an engineering attribute computed in terms of the design variables by the ADVISOR model [25].

For the period 1992 to 2001 the CAFE penalty was nonpositive and approximately zero for DaimlerChrysler, Ford, and General Motors [24]. This is represented as follows:

$$\sum_{\tau=1}^{2} \text{Cost}_{\text{CAFE}}^{\tau} \leqslant 0 \tag{13}$$

Note that the CAFE regulations can only hurt the firm's profits, not contribute to them. They function as a set of internal taxes (on fuel inefficient vehicles) and subsidies (on fuel efficient vehicles) within each firm [27].

Hence, the cost of each product, Eq. (10), is modified to be

$$C^{\tau} = c_0^{\tau} q^{\tau} + \text{Cost}_{\text{CAFE}}^{\tau} = (c_0^{\tau} + c_{\text{CAFE}}^{\tau}) q^{\tau} = \xi^{\tau} q^{\tau} \tag{14}$$

where $\xi^{\tau} \triangleq c_0^{\tau} + c_{\text{CAFE}}^{\tau}$, and c_{CAFE}^{τ} is equal either to CAFE penalty or to contribution of credit to the portfolio. For example, if the PC vehicle generates $3 M of credit but the penalty incurred to the SUV is $2.5 M then $\text{Cost}_{\text{CAFE}}^{\text{PC}}$ would be equal to $2.5 M. For a portfolio of τ products the value of $\text{Cost}_{\text{CAFE}}^{\tau}$ is then redefined as:

$$c_{\text{CAFE}}^{\tau} q^{\tau} = \begin{cases} c_{\text{CAFE}}^{\tau} q^{\tau} & \sum_{1}^{\tau} c_{\text{CAFE}}^{\tau} \geqslant 0 \\ c_{\text{CAFE}}^{\tau} q^{\tau} + \dfrac{\mathcal{U}(c_{\text{CAFE}}^{\tau}) c_{\text{CAFE}}^{\tau} q^{\tau}}{\sum_{1}^{\tau} \mathcal{U}(c_{\text{CAFE}}^{\tau}) |c_{\text{CAFE}}^{\tau}| q^{\tau}} \sum_{1}^{\tau} c_{\text{CAFE}}^{\tau} q^{\tau} & \sum_{1}^{\tau} c_{\text{CAFE}}^{\tau} q^{\tau} < 0 \end{cases} \tag{15}$$

where $\mathcal{U}(c_{\text{CAFE}}^{\tau})$ is equal to 1 when c_{CAFE}^{τ} is negative and 0 when non-negative.

3.3 Economic Decision Model. As mentioned early in this section, the microeconomic model suggests that in each monthly period the firm should produce the quantity that maximizes total profit during that period:

$$\begin{aligned} &\text{maximize} && \sum_{\tau=1}^{2} \pi^{\tau} = \sum_{\tau=1}^{2} \left(\frac{\theta^{\tau}}{\lambda_P^{\tau}} - \frac{1}{\lambda_P^{\tau}} q^{\tau} + \frac{1}{\lambda_P^{\tau}} (\boldsymbol{\lambda}_{\alpha^{\tau}}^{\tau})^T \boldsymbol{\alpha}^{\tau} \right) q^{\tau} - \xi^{\tau} q^{\tau} \\ &\text{with respect to} && \{\mathbf{q}^{\tau}\}, \tau = 1,2 \\ &\text{subject to} && \sum_{\tau=1}^{2} q^{\tau} = K \\ &&& \sum_{\tau=1}^{2} \text{Cost}_{\text{CAFE}}^{\tau} = c_{\text{CAFE}}^{1} q_1 + c_{\text{CAFE}}^{2} q_2 \leqslant 0 \end{aligned} \tag{16}$$

For positive quantities of production q^{τ}, Eq. (16) can be solved analytically and the global optimum is

$$q^{\text{PC}^*} = \max\left(\frac{\left(\frac{\theta^{\text{PC}}}{\lambda_P^{\text{PC}}} - \frac{\theta^{\text{SUV}}}{\lambda_P^{\text{SUV}}}\right) + \left(\frac{1}{\lambda_P^{\text{PC}}} (\boldsymbol{\lambda}_{\alpha^{\text{PC}}}^{\text{PC}})^T \boldsymbol{\alpha}^{\text{PC}} - \frac{1}{\lambda_P^{\text{SUV}}} (\boldsymbol{\lambda}_{\alpha^{\text{SUV}}}^{\text{SUV}})^T \boldsymbol{\alpha}^{\text{SUV}}\right) + 2\frac{1}{\lambda_P^{\text{SUV}}} K - (\xi^{\text{PC}} - \xi^{\text{SUV}})}{2\left(\frac{1}{\lambda_P^{\text{PC}}} + \frac{1}{\lambda_P^{\text{SUV}}}\right)} \frac{K c_{\text{CAFE}}^{\text{SUV}}}{c_{\text{CAFE}}^{\text{SUV}} - c_{\text{CAFE}}^{\text{PC}}} \right),$$

$$q^{\text{SUV}^*} = K - q^{\text{PC}^*}, \tag{17}$$

where $((\theta^{\text{PC}}/\lambda_P^{\text{PC}} - \theta^{\text{SUV}}/\lambda_P^{\text{SUV}}) + (1/\lambda_P^{\text{PC}} (\boldsymbol{\lambda}_{\alpha^{\text{PC}}}^{\text{PC}})^T \boldsymbol{\alpha}^{\text{PC}} - 1/\lambda_P^{\text{SUV}} (\boldsymbol{\lambda}_{\alpha^{\text{SUV}}}^{\text{SUV}})^T \boldsymbol{\alpha}^{\text{SUV}}) + 2 1/\lambda_P^{\text{SUV}} K - (\xi^{\text{PC}} - \xi^{\text{SUV}}))(2(1/\lambda_P^{\text{PC}} + 1/\lambda_P^{\text{SUV}}))$ is the interior optimum, and $K c_{\text{CAFE}}^{\text{SUV}} / (c_{\text{CAFE}}^{\text{SUV}} - c_{\text{CAFE}}^{\text{PC}})$ is the boundary optimum.

From the derivation above we see that the enterprise-wide problem Eq. (4) can be partitioned to production and design problems. The production problem is Eq. (16) above, which can be solved separately for $\mathbf{q}^*$. With this partial optimization [28] π^{τ} in Eq. (5) can be replaced by π^{τ^*} computed from Eq. (17), and the problem of Eq. (4) is reduced to Eq. (18) for fixed investment costs. Of course, $\mathbf{q}^*$ depends on the design variables via the quantities $(\boldsymbol{\lambda}_{\alpha}^{\tau})^T \boldsymbol{\alpha}^{\tau}$ and c_{CAFE}^{τ}.

$$\begin{aligned} &\text{maximize } NPV(\mathbf{x}) = -I + \int_0^H \sum_{\tau=1}^{2} \pi^{\tau^*} e^{-(WACC)t} dt \\ &\text{with respect to } \mathbf{x} \\ &\text{subject to } \mathbf{g}^{\tau}(\mathbf{x})^{\tau} \leqslant 0, \quad \tau = 1,2. \end{aligned} \tag{18}$$

4 Incorporating Market Uncertainty

Equation (17) represents the optimal conditions at a specific point in time. Evaluating an investment requires estimation of future cash flows, which will introduce the elements of time and uncertainty in Eq. (18).

4.1 The Demand Side. We assume that future cash flows generated by a product's commercialization are only imperfectly predictable from the current observation. The probability distribu-

Table 2 Market share data of a major US automotive manufacturer for the premium-compact segment

Month	Units	Market Share
Jan-95	135,928	25.43%
Feb-95	154,139	25.63%
Mar-95	176,348	23.74%
⋮	⋮	⋮
Jan-01	161,227	11.78%

tion is determined by the present, but the actual path remains uncertain [1]. We consider product demand and the firm's market share as the two main sources of uncertainty.

To describe future product demand we assume that the automotive product demand X follows geometric Brownian motion. Seasonality [29] and life-cycle considerations [30] can be also taken into account.

$$\Delta X_t = \mu X_t \Delta t + \sigma X_t \Delta z$$

$$\Delta z = \varepsilon \sqrt{\Delta t} \tag{19}$$

$$\varepsilon \sim N(0,1)$$

Here $\mu\Delta t$ and $\sigma\sqrt{\Delta t}$ are the expected value and the standard deviation, respectively, of $\Delta X_t / X_t$ in Δt. To simulate the path followed by X we divide the life of the source of uncertainty into 73 monthly intervals from January 1995 until January 2001. The value of X at time Δt (i.e., February 2001) is calculated from the initial value of X (i.e., January 2001), the value at time $2\Delta t$ is calculated from the value at time Δt and so on (Hull 2000). One simulation trial involves constructing a complete path for X using 73 random samples from a normal distribution. Data provided by J.D. Power & Associates for the period between January 1995 and January 2001 (see Table 2) have been employed for the estimation of the expected growth rate μ and volatility σ.

The Ornstein–Uhlenbeck or mean-reverting process is used to model market share uncertainty Y.

$$\Delta Y_t = \alpha(\overline{Y_t} - Y_t)\Delta t + \sigma' \Delta z$$

$$\Delta z = \varepsilon \sqrt{\Delta t} \tag{20}$$

$$\varepsilon \sim N(0,1).$$

Here α is the speed of mean reversion, $\overline{Y}$ is the "normal" level of Y, i.e., the level to which $\overline{Y}$ tends to revert. By running the nonlinear regression [31]

$$\Delta Y = p + qY_{t-1} + \epsilon_t, \tag{21}$$

from data provided by J.D. Power and Associates (see Table 2) for the period between January 1995 and January 2001 we estimate

$$\overline{Y} = \hat{p}/\hat{q},$$

$$\hat{\alpha} = -\log(1+\hat{q}), \tag{22}$$

$$\hat{\sigma}' = \hat{\sigma}'_{\epsilon} \sqrt{\frac{\log(1+\hat{q})}{(1+\hat{q})^2 - 1}},$$

where $\hat{\sigma}_{\epsilon}$ is the standard error of the regression.

The firm's product demand $(q_t^\tau)^D$ for product τ at time t is expressed as a product of the two sources of uncertainty defined above, namely, market product demand and market share:

$$(q_t^\tau)^D = \begin{cases} 0 & 0 \leqslant \text{month} \leqslant 24 \\ X_t^\tau Y_t^\tau & 25 \leqslant \text{month} \leqslant 84 \end{cases} \tag{23}$$

$(q_t^\tau)^D$ is a 1×84 vector and represents a random walk in the future (see Fig. 2). During the first 24 months of product development and production start-up time we have null sales.

The decision-maker is making an investment decision not a procurement or a competitive response decision. Therefore, Eqs. (19) and (20) aim to incorporate uncertainty in the decision model of Eq. (4) and not to predict quantity sold in the next time period.

4.2 Supply and Demand. If the actual demand exceeds the optimal capacity of the plant the firm will sell only at capacity. Thus, if the actual demand $(q_t^\tau)^D$ is less than the optimal capacity $q^{\tau*}$ of the plant then the firm will supply only as much as the demand permits. That is, the actual supply $(q_t^\tau)^S$ is given by

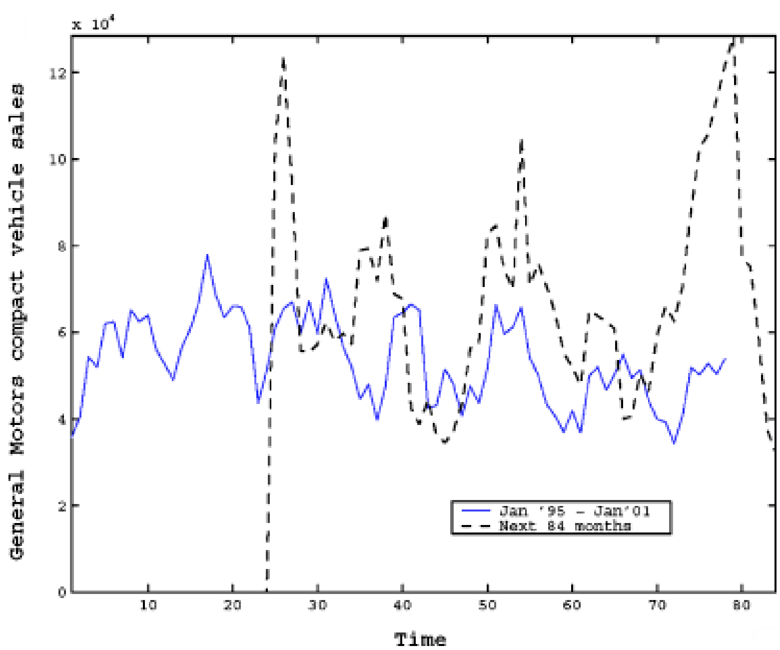

Fig. 2 A random walk in the future

$$(q_t^\tau)^S = \begin{cases} (q_t^\tau)^D & \text{if } (q_t^\tau)^D < q^{\tau^*} \\ q^{\tau^*} & \text{otherwise} \end{cases} \tag{24}$$

The assumption is that the firm does not possess flexibility in adjusting capacity. If that were not the case then an appropriate theory [15] would have been needed to model the resource allocation decision.

We would like to note here that in the mathematical finance and economics literature [30,32] product demand uncertainty is also described by the intercept θ of Eq. (8). That essentially describes random shifts of the demand curve—with the same $\boldsymbol{\lambda}$. This approach would require in addition to historical demand data, pricing data as well, which were unavailable. In our effort to trigger the intuition of designers and managers we decided not to use hypothetical data, thus maintaining the credibility of the actual results in the context of the assumptions made.

5 Design Model

5.1 Net Present Value Model. Collecting Eqs. (11), (14), (15), (17), and (24) for the PC segment we get the complete calculation of the monthly profit. A similar set of equations holds for the SUV segment.

$$\pi_t^{\mathrm{PC}^*} = P^{\mathrm{PC}}(q_t^{\mathrm{PC}})^S - \xi(q_t^{\mathrm{PC}})^S$$

$$q_t^{\mathrm{PC}^S} = \min\{(q_t^{\mathrm{PC}})^D, q^{\mathrm{PC}^*}\}$$

$$q_t^{\mathrm{PC}^*} = \max\left(\frac{\left(\frac{\theta^{\mathrm{PC}}}{\lambda^{\mathrm{PC}}} - \frac{\theta^{\mathrm{SUV}}}{\lambda^{\mathrm{SUV}}}\right) + 2\frac{1}{\lambda^{\mathrm{SUV}}}K + \left(\frac{1}{\lambda^{\mathrm{PC}}}\boldsymbol{\lambda}_\alpha^{\mathrm{PC}^T}\boldsymbol{\alpha}^{\mathrm{PC}} + \frac{1}{\lambda^{\mathrm{SUV}}}\boldsymbol{\lambda}_\alpha^{\mathrm{SUV}^T}\boldsymbol{\alpha}^{\mathrm{SUV}}\right) - (\xi^{\mathrm{PC}} - \xi^{\mathrm{SUV}})}{2\left(\frac{1}{\lambda^{\mathrm{PC}}} + \frac{1}{\lambda^{\mathrm{SUV}}}\right)} \frac{Kc_{\mathrm{CAFE}}^{\mathrm{SUV}}(\mathbf{x}^{\mathrm{SUV}})}{c_{\mathrm{CAFE}}^{\mathrm{SUV}} - c_{\mathrm{CAFE}}^{\mathrm{PC}}} \right) \tag{25}$$

$$\xi^{\mathrm{PC}} = c_0^{\mathrm{PC}} + c_{\mathrm{CAFE}}^{\mathrm{PC}}$$

$$c_{\mathrm{CAFE}}^\tau q^\tau = \begin{cases} c_{\mathrm{CAFE}}^\tau q^\tau & \sum_1^\tau c_{\mathrm{CAFE}}^\tau q^\tau \geq 0 \\ c_{\mathrm{CAFE}}^\tau q^\tau + \dfrac{\mathcal{U}(c_{\mathrm{CAFE}}^\tau) c_{\mathrm{CAFE}}^\tau q^\tau}{\sum_1^\tau \mathcal{U}(c_{\mathrm{CAFE}}^\tau)|c_{\mathrm{CAFE}}^\tau| q^\tau} \sum_1^\tau c_{\mathrm{CAFE}}^\tau q^\tau & \sum_1^\tau c_{\mathrm{CAFE}}^\tau q^\tau < 0 \end{cases}$$

From Eq. (11) after substituting $(q_t^\tau)^S$ for q_t^τ using Eq. (24) and C^τ using Eq. (14) instead of Eq. (10) we get the monthly profit $\pi_t^{\tau^*}$ over the eighty-four month sales period for product τ.

$$\pi_t^{\tau^*} = P^\tau(q_t^\tau)^S - (\xi^\tau)(q_t^\tau)^S. \tag{26}$$

During the first 24 months we have null profits.

Recall that we assume the decision to develop the new engines and transmissions has already been made, and so the decision facing the firm now is one of resource allocation. Upon determining the optimal production ratio, this decision will be implemented immediately. Hence, the decision contains no embedded real option [15], which simplifies the net present value calculation. The time period T for both products is estimated to be seven years or eighty-four months and includes the product development, production start-up, and sales periods. The present value, PV, of discounted future payoffs $\pi_t^{\tau^*}$ is represented as an integral over the space of sample paths of the underlying stochastic processes Q and M

$$PV \approx \frac{\sum_1^n \left[\int_0^T \left(\sum_{i=1}^2 \pi_{t_n}^{\tau^*} \right) e^{-WACCt} dt \right]}{n} \tag{27}$$

where $n=100{,}000$. The exponent $WACC$, the weighted average cost of capital, is estimated as

$$WACC = r_d(1-t_c)\frac{D}{E+D} + r_e\frac{E}{E+D} \tag{28}$$

where r_d is firm's cost of debt, t_c tax rate, D market value of debt, E equity, and r_e is the cost of equity estimated by the Capital Asset Pricing Model (CAPM) as in [13]

$$r_e = r_f + \beta(r_m - r_f), \tag{29}$$

where r_f is the risk-free rate, $(r_m - r_f)$ the market risk premium, and β is the firm's stock sensitivity to fluctuation of the market as a whole. Using the values in Table 3 we estimate the weighted average cost of capital of a publicly traded US automotive manufacturer to be 9.4%. By using a single $WACC$ for all 84 months we are making the assumption that the firm's capital structure (the debt to equity ratio D/E) and the risk of the firm to its shareholders would remain the same for all 84 months. We do not consider cost of capital a major source of uncertainty and, therefore, use of

Table 3 Parameters values in Eqs. (28) and (29)

t_c	36%
D	$81.3 B
r_f	6.7%
β	1.1
Long term interest	$2.8 B
Long term Debt	$31.3 B
Number of Shares	756 M
Stock price	$58
Market risk premium	8.4%

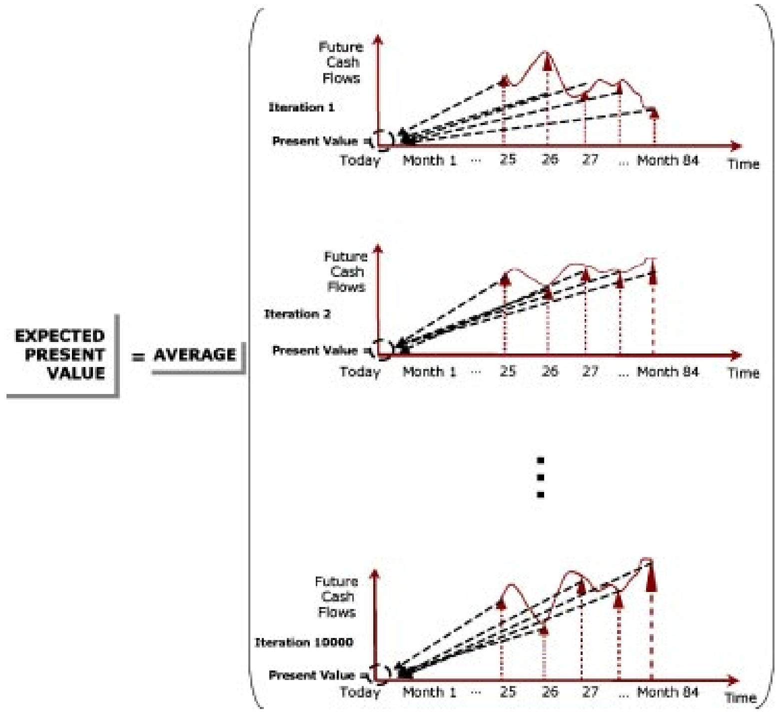

Fig. 3 Expected present value

dynamic models [33] is considered a subtlety in the context of the specific example.

To estimate the PV we generate 100,000 random walks resulting in a $100{,}000 \times 84$ matrix. Discounting all future payoffs across the probability space we get a $100{,}000 \times 1$ matrix. The present value is the average of those 100,000 numbers (see also Fig. 3). Note that Eq. (27) does not take into account working capital.

Subtracting the fixed capital investment needed ($I = \$3$ B) we calculate the net present value

$$NPV = PV - I \approx -I + \frac{\sum_{1}^{n} \left[\int_{0}^{T} \left(\sum_{i=1}^{2} \pi_{t_n}^{\tau^*} \right) e^{-WACCt} dt \right]}{n} \quad (30)$$

Other investment costs are ignored; for example, the cost of building the production facility plant is considered a sunk cost because we assume the plant has already been built.

The NPV expression in Eq. (30) is the stochastic calculation of the objective in the model of Eq. (18). This NPV criterion is based on the optimal economic conditions [15,30] computed in Eq. (17), the uncertainty of future cash flows Eqs. (19) and (20), and the engineering performance Eq. (12).

5.2 Engineering Constraints and Model Summary. Bounds on vehicle performance attributes define the constraints for each product and its corresponding market segment. These "engineering" constraints are expressed in terms of the design variables using the ADVISOR program, cited earlier.

For each segment the engineering constraints are

$$\begin{aligned} (fe) &\geq b_1 \\ t_{0-60} &\leq b_2 \\ t_{0-80} &\leq b_3 \\ t_{40-60} &\leq b_4 \\ 5\text{ s distance} &\geq b_5 \\ \text{max acceleration} &\geq b_6 \\ \text{max speed} &\geq b_7 \\ \text{max grade at 55 mph} &\geq b_8 \end{aligned} \quad (31)$$

where the b's are upper or lower bound parameters. The constraint bounds are set at values that are 20% beyond the current vehicle nominal performance values to allow for new design possibilities. The parameter values used for computation are shown in the summary model Eq. (32) below.

The model in Eq. (32) involves four variables and sixteen constraints (eight each for the PC and SUV segments of the engineering design model). The complete model of Eq. (32) is now assembled using the expressions derived in the preceding sections.

$$\text{maximize } NPV$$

$$\begin{aligned} &\text{with respect to } \{(\text{engine size})^{PC}, (\text{final drive ratio})^{PC}, \\ &(\text{engine size})^{SUV}, (\text{final drive ratio})^{SUV}\} \\ &\text{subject to } (\text{fuel economy})^{PC} \geq 27.3(\text{mpg}) \\ &(\text{acceleration 0 to 60})^{PC} \leq 12.5(\text{s}) \\ &(\text{acceleration 0 to 80})^{PC} \leq 26.3(\text{s}) \\ &(\text{acceleration 40 to 60})^{PC} \leq 5.9(\text{s}) \\ &(5\text{ s distance})^{PC} \geq 123.5(\text{ft}) \\ &(\text{max acceleration})^{PC} \geq 13(\text{ft/s}^2) \\ &(\text{max speed})^{PC} \geq 97.3(\text{mph}) \\ &(\text{max grade at 55 mph})^{PC} \geq 18.1(\%) \\ &(\text{fuel economy})^{SUV} \geq 12.8(\text{mpg}) \end{aligned}$$

$$(\text{acceleration 0 to 60})^{SUV} \leqslant 9.8(\text{s})$$

$$(\text{acceleration 0 to 80})^{SUV} \leqslant 22.8(\text{s})$$

$$(\text{acceleration 40 to 60})^{SUV} \leqslant 5.0(\text{s})$$

$$(\text{5 s distance})^{SUV} \geqslant 154.5(\text{ft})$$

$$(\text{max acceleration})^{SUV} \geqslant 15.4(\text{ft/s}^2)$$

$$(\text{max speed})^{SUV} \geqslant 100.4(\text{mph})$$

$$(\text{max grade at 55 mph})^{SUV} \geqslant 18.6(\%)$$

$$2.5 \leqslant (\text{final drive ratio})^{PC} \leqslant 4.5$$

$$2.5 \leqslant (\text{final drive ratio})^{SUV} \leqslant 4.5$$

$$50(\text{kW}) \leqslant (\text{engine size})^{PC} \leqslant 150(\text{kW})$$

$$150 \leqslant (\text{engine size})^{SUV} \leqslant 250(\text{kW}) \quad (32)$$

We proceed with solution of Eq. (32) and discussion of results in the next section.

6 Results

The divided rectangles (DIRECT) optimization algorithm [34] was used to solve the valuation problem posed in Eq. (32). DIRECT can locate global minima efficiently without derivative information, when the number of variables is small, as in this case. It is often inefficient at refining local minima, and so a sequential quadratic programming algorithm was combined with DIRECT to local final solutions.

Recall that Eq. (32) is a resource allocation problem with scarce resources. That is, the sum of the optimal quantities of each segment is greater or equal to the manufacturing resources of the firm. In our effort to understand how the extend of this difference affects design decisions we solve the optimization problems for two different production capacities: 50,000 and 20,000. The optimal solutions found are shown in Table 4.

One can interpret the results by paying attention to production quantity q, the regulatory penalty (or credit) per vehicle, and the vector of design variables (i.e., engine size, final drive). Specifically:

- At the small capacity level of 20,000, the small quantity of produced compact vehicles calls for high fuel efficiency of the compact car and average performance of the sport utility vehicle.
- At the high capacity level of 50,000, the quantity of compact vehicles has enough scale to mitigate the regulatory penalty, while improving the performance of the sport utility vehicle by a 28% increase in engine size.

From a public policy point of view, the CAFE regulation can be interpreted as an active constraint to consumer preferences. As long as the consumer asks for more horsepower, and the producer can materialize this preference in terms of profit, the fuel efficiency regulatory constraint would be active. A question of interest is whether or not new technologies can make the CAFE constraint inactive and at what cost.

Finally, an analysis that would be of interest to decision-makers is the engine choice versus the capacity level. For example, the optimal engine size at 20,000 units of capacity is 219 (kW). If an expansion of capacity is under consideration which would be the most robust design with respect to the level of expansion? A simple parametric study using the optimization model here would be a good starting point for addressing this question.

Table 4 Solutions of the enterprise model for different capacities

Variable	Solution for K=20,000	Solution for K=50,000
Quantity^{PC}	**5286**	**28675**
w^{PC}	26 (%)	58 (%)
CAFE^{PC}	−$508	−$510
(**engine size**)^{PC}	**72.85** (kW)	**72.50** (kW)
(**final drive ratio**)^{PC}	**3.49**	**3.53**
(fuel economy)^{PC}	37.67 (mpg)	37.70 (mpg)
(acceleration 0 to 60)^{PC}	12.43 (s)	12.4 (s)
(acceleration 0 to 80)^{PC}	26.17 (s)	26.23 (s)
(acceleration 40 to 60)^{PC}	5.7 (s)	5.62 (s)
(5 s distance)^{PC}	130.46 (ft)	130.62 (ft)
(max acceleration)^{PC}	16.05 (ft/s^2)	16.05 (ft/s^2)
(max speed)^{PC}	110.7 (mph)	111.09 (mph)
(max grade at 55 mph)^{PC}	18.37 (%)	18.52 (%)
Quantity^{SUV}	**14714**	**21325**
w^{SUV}	74 (%)	42 (%)
CAFE^{SUV}	$183	$314
(**engine size**)^{SUV}	**194** (kW)	**250** (kW)
(**final drive ratio**)^{SUV}	**3.97**	**2.58**
(fuel economy)^{SUV}	16.65 (mpg)	14 (mpg)
(acceleration 0 to 60)^{SUV}	8.24 (s)	7.54 (s)
(acceleration 0 to 80)^{SUV}	19.09 (s)	18.3 (s)
(acceleration 40 to 60)^{SUV}	4.06 (s)	3.55 (s)
(5 s distance)^{SUV}	193.29 (ft)	196.7 (ft)
(max acceleration)^{SUV}	19.24 (ft/s^2)	19.24 (ft/s^2)
(max speed)^{SUV}	132.07 (mph)	119.18 (mph)
(max grade at 55 mph)^{SUV}	26.57 (%)	36.93 (%)
Fleet CAFE	0	−$7.9M
NPV(7 year period)	$7.3 B	$9.36 B

7 Conclusion

We have demonstrated a model-based methodology that can quantify the impact of engineering design decisions on investment decision-making. The methodology has been generalized in [22]. Using a simplified example we showed the effect of production capacity resources on design decisions. The decision-maker should optimally allocate resources and supply product differentiation to yield maximum profitability.

We synthesized concepts that have broad acceptance in their respective fields. The main assumption behind this synthesis is that economic, investment and engineering design decisions do not take place in a vacuum. They take place simultaneously and affect each other, either implicitly or explicitly. In future work it would be interesting to examine the conditions under which engineering and business decision-making processes are independent from each other.

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Nomenclature

B = firm's market value of debt
C = total cost
c_0 = average total cost
CAFE = corporate average fuel economy
D = demand
E = firm's equity
E_p = price elasticity of demand
fe = fuel economy
$\mathbf{g}(\mathbf{x})$ = engineering constraint set
I = capital investment
K = units of capacity currently installed
L = vehicle's segment CAFE limit
M = firm's market share
n = number of random sequences
NPV = net present value
P = vehicle's price
PV = present value of future cash flows
PC = premium compact vehicle
$\mathbf{q}$ = vector of production quantities
q^D = firm's quantity demanded
q^S = firm's quantity supplied
Q = market product demand
r_e = cost of equity
r_d = cost of debt
r_f = risk-free rate of return
r_m = expected return on a market portfolio
S = supply
SUV = sport utility vehicle
$t_{0\text{–}60}$ = 0 to 60 acceleration time
t_c = corporate tax rate
T = end of life-cycle
$WACC$ = weighted average cost of capital
Δt = monthly period
$\mathbf{x}$ = engineering design variables
w = product portfolio weight
α = speed of reversion
β = firm's beta
θ = intercept of the demand curve
λ = slope of the demand curve
μ = expected growth rate of product demand
ξ = average total cost including CAFE cost
π = profit
σ = volatility of product demand
τ = vehicle index

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