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Platform Selection Under Performance Bounds in Optimal Design of Product Families

Designing a family of product variants that share some components usually requires a compromise in performance relative to the individually optimized variants due to the commonality constraints. Choosing components for sharing may depend on what performance losses can be tolerated. In this article an optimal design problem is formulated to choose product components to be shared without exceeding user-specified bounds on performance. This enables the designer to control tradeoffs and obtain optimal product family designs for maximizing commonality at different levels of acceptable performance. A family of automotive body side frames is used to demonstrate the approach.
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1 Introduction

A product platform is defined as the set of components and manufacturing and assembly processes that are common in a family of products. Commonality reduces costs but may require some sacrifice in individually optimized product performance [1]. The design challenge is how to optimize the family of products with maximum commonality while satisfying individual constraints and controlling performance losses.

The literature includes a significant amount of work focusing on developing methodologies for the design of product families. These methods are applied when the product platform has been determined by the designer a priori. Gonzalez-Zugasti et al. [2] used cost gain models for designing the product platform while satisfying performance and budget constraints. Several a priori specified platforms are optimized first, while the family variants are designed at the second stage. Simpson et al. [3] proposed a product platform concept exploration method for configuring and exploring product platforms that are scaled to derive product families. A compromise decision support problem was formulated to optimize the design, while in Messac et al. [4] a physical programming approach was adopted. Conner et al. [5] utilized Simpson's product variety tradeoff evaluation method and the combination of the above two concepts to evaluate platform portfolios based on commonality and performance losses indices. Nelson et al. [1] formulated and solved a multiobjective optimal design problem by means of Pareto set theory. Each platform is shown to correspond to a different Pareto surface. The designer can identify trade-offs, evaluate multiple platforms, and make related decisions. Fellini et al. [6] applied this concept to the design of automotive powertrains and examined the hierarchical structure of the platform design problem. Kokkolaras et al. [7] extended the analytical target cascading formulation to hierarchical models of product families with pre-specified platforms.

At a level above the product family design problem one must also decide what can be shared among a set of products. This is a key ingredient in the complete optimal design of product families. The following methodologies look at not only designing the family of products but aiding the engineer in selecting which components to make common. There have been three groups of approaches taken to address this problem.

The first approach has been simple exhaustive enumeration of the sharing possibilities. Conner Seepersad et al. [8] enumerate a subset of all combinatorial possibilities for assigning fixed platforms to a family of products. These platform alternatives are designed and evaluated by comparing performance losses and cost benefits. This type of method could be applied for relatively small problems; however, these type of methods are severely limited as the problem size moves beyond what can be searched exhaustively.

The second approach to the commonality decision problem has been to develop optimal design formulations which are used to select both the platform and design the product family. This group of methods have been applied both in an all-at-once approach or in a series of intermediate steps. Gonzalez-Zugasti and Otto [9] formulated a modular product platform optimization problem to determine simultaneously module designs and their combination in the variant instantiations. Platform and individual goals form a multiobjective function, and additional compatibility and sharing constraints are included. Fujita and Yoshida [10] also proposed a method for simultaneous optimization of module attributes and combinations, driven by cost-related functions. In both of these papers a modular architecture of the product family is fixed, and a genetic algorithm (GA) is used to solve small-sized combinatorial problems. Other researchers have also adopted GAs for solving the commonality selection and family design problems [11,12].

The main drawbacks of these methods are: (1) the use of heuristic algorithms such as GAs, which require extensive fine tuning, often are extremely expensive to run, and have no formal proof of convergence; (2) the solution of any pure combinatorial problem is quickly bound by the problem size where combinations are increasing exponentially. For example, in the case studies presented in the aforementioned work, a relatively small number of possible sharing combinations are explored, often no more than on the order of 10^3. One benefit of using a combinatorial algorithm, as shown by Gonzalez-Zugasti and Otto, is that distinct module types can be chosen through a catalog selection. This type of problem is however once again limited due to the combinatorial nature. Additional characteristics of these methodologies include: the first two papers allow for "subplatforms" to be chosen, component sharing that is not necessarily across the entire family of products. The latter papers explore component sharing only across the family of products.

The third group of methodologies attempt to approach large problem sizes by developing metrics that sort the components by shareability. Nayak et al. [13] employed robust design concepts to formulate a variation-based platform design methodology that consists of two steps: identifying the platform by solving a compromise decision support problem and designing the family around this platform. Fellini et al. [14] employ sensitivity infor-

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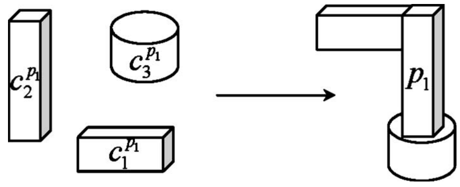

Fig. 1 Components and assembled product

mation at the individually optimized products to produce a sorted vector ranking the shareability of the components, and then optimize the product family on a selected platform. The benefits of these methods are that they can aid in reducing the size of the commonality decision problems, however the actual selection of a platform based on the rankings is ad hoc. The methodologies by D'Souza and Simpson attempt to use design of experiments (DOE) to reduce the problem size, but likely as the problems become large these tools lose much of their accuracy [11,12].

The focus of this paper is to address the problem addressed by the second group of methodologies. In the present work the combinatorial problem is relaxed into a continuous form and reformulated to maximize commonality among the family members (allowing for subplatforms) while satisfying individual design constraints and observing a designer-specified bound on the reduction from the individually optimized performance. The designer can identify a set of components that can be shared, and obtain optimal designs for a number of scenarios based on the willingness to sacrifice a certain amount of individual performance. By relaxing the combinatorial decision into a continuous problem formulation, gradient-based methods such as sequential quadratic programming can be employed. The contribution of the proposed methodology is an approach that not only allows for an enormous increase in the size of the problem that can be solved but also benefits from the use of deterministic methods that have rigorous convergence proofs and do not require tuning parameters.

The paper is organized as follows. After some definitions the product family design and commonality decision problems are posed mathematically. The methodology for solving them is then presented and subsequently demonstrated on a family of automotive body side frames. Results are discussed and conclusions are drawn.

2 Definitions

In this section we review the vocabulary and basic model for a product platform.

A *component* is defined as a manufactured object that is the smallest (indivisible) element of an assembly and is represented by a set of design variables. A *product* is an artifact that is made up of components. The *product architecture* is the configuration (or topology) of components within the product. A *module* is a component or subassembly that can be interchanged within a product architecture to produce a variety of similar products. A *model* is a mathematical representation of a product that accepts a vector of design variables and returns a vector of responses.

The mathematical notation begins with the set $\mathcal{P}=\{p_1,p_2,\ldots\}$ used to distinguish each product $p\in\mathcal{P}$. Likewise, the set $\mathcal{C}^p=\{c_1^p,c_2^p,\ldots\}$ is defined to represent the components that form a particular product p. Thereby the union of all components, $p=\cup_i c_i^p$, constructs the product. Figure 1 illustrates the above notation. A product p is associated with a vector of design variables $\boldsymbol{x}^p$, a vector of responses $\boldsymbol{R}^p$, and design inequality and equality constraints $\boldsymbol{g}^p$ and $\boldsymbol{h}^p$, respectively. The subset of design variables describing component c^p is $\boldsymbol{x}^{c^p}\subseteq\boldsymbol{x}^p$.

A *product platform* is the set of all components, manufacturing processes, and/or assembly steps that are common in a set of products. The following notation is used to describe a product platform: The set $\mathcal{S}^{pq}$ consists of the index pairs of elements that

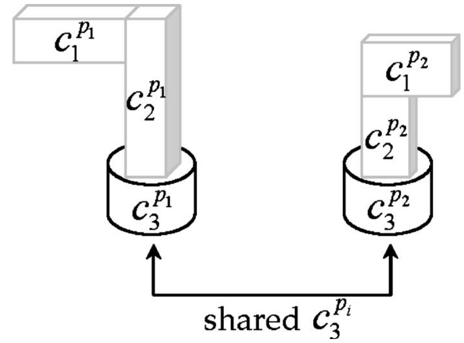

Fig. 2 Component sharing within a family of products

are shared between two products p and q. The set $\mathcal{S}=\{\mathcal{S}^{pq}|p,q\in P;p<q\}$ describes element sharing throughout the family.

Two types of sharing can be used when selecting a product platform that is not based on manufacturing processes or assembly steps. In component sharing, one or more components are common across a family of products as shown in Fig. 2. In addition, it is possible to share "scaled" versions of components. Mathematically this can be described as variable sharing, where components are based on a platform (of variables) themselves. The example in Fig. 3 shows the cross section of two structural beam elements. While the height and width of both parts are the same, the thickness is different. The manufacturing advantage can be illustrated by this example. By keeping width and height invariant, the same stamping equipment can be used with different gauge steel. In general, manufacturing (and therefore cost) considerations should be taken into account in the design of platforms. We do not address this aspect explicitly, but we attempt to recognize the associated design impact. The methodology presented holds for both component and variable sharing. When defining a platform that includes both component and variable sharing the subscripts $\mathcal{C}$ and $\mathcal{P}$ are used to denote the individual platform element types, respectively. The information is then held in the set $\mathcal{S}=\mathcal{S}_{\mathcal{C}}\cup\mathcal{S}_{\mathcal{P}}$. Finally, a *product family* is the set of product variants that share a product platform. A family product derived from a platform is also referred to as a *product variant*.

The formulation to be used in the next section is the multicriteria optimization statement proposed by Nelson et al. [1],

$$\max_{x=[\boldsymbol{x}^{p_1},\boldsymbol{x}^{p_2},\ldots]} \quad \{f^p(\boldsymbol{x}^p)\} \quad \forall\, p,q\in\mathcal{P},(i,j)\in\mathcal{S}^{pq},p<q$$

$$\text{subject to} \quad \boldsymbol{g}^p(\boldsymbol{x}^p)\leq\boldsymbol{0}$$

$$\boldsymbol{h}^p(\boldsymbol{x}^p)=\boldsymbol{0}$$

$$x_i^p=x_j^q. \tag{1}$$

The objective function for the family is a weighted aggregate of the individual product objectives. The constraint set includes all individual design constraints. Commonality constraints are represented as equality constraints $x_i^p=x_j^q$ for each set of shared variables specified in $\mathcal{S}$. An optimization problem is solved for each

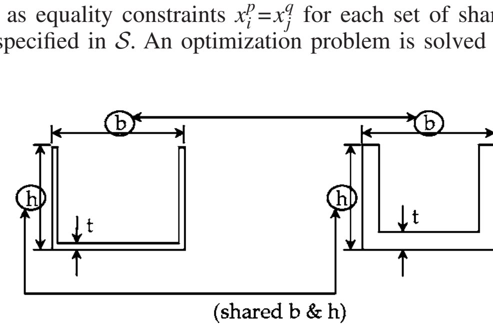

Fig. 3 Variable sharing within a family of products

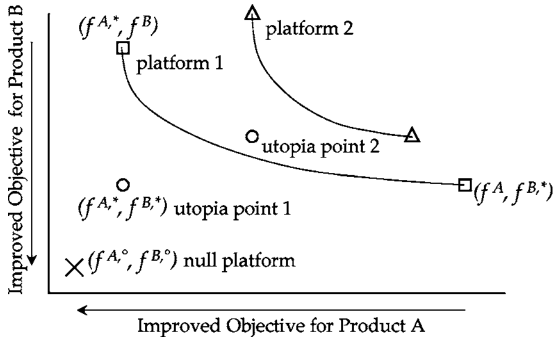

Fig. 4 Null-platform point and Pareto sets for different platforms

product separately to determine the null-platform design. The optimal null-platform objective function and design values are denoted by $f^{p,\circ}$ and $\boldsymbol{x}^{p,\circ}$, respectively. Likewise, $f^{p,*}$ and $\boldsymbol{x}^{p,*}$ are used to represent the optimal objective function and design values for family designed products, respectively. The null-platform objectives from all family products define the null-platform point, shown in Fig. 4 as the point $\times(f^{A,\circ}, f^{B,\circ})$. Solving the multicriteria optimization problem Eq. (1) yields a Pareto set. Two possible platforms (with different equality constraints) are shown in Fig. 4. The bounds of the Pareto set determine the utopia point, the best trade-off design that might be achieved with the platform. In practice, the designer first computes the design penalty from the null-platform point to the utopia point. If that is acceptable, the rest of the Pareto set is computed. The preferred design on the Pareto set is selected according to other criteria (cost, customer preferences, etc.) not included in the above model.

3 The Commonality Decision Problem

The important tradeoff when choosing a platform is commonality versus performance. The product family design problem is now restated to include component commonality in the objective and which variables to share in the decision making, leading to a mixed-discrete programming problem due to the presence of the vector of binary (0-1) sharing decision variables $\boldsymbol{\eta}$

$$\max_{\boldsymbol{\eta}, \boldsymbol{x}=[\boldsymbol{x}^{p_1}, \boldsymbol{x}^{p_2}, \ldots]} \quad \{\{f^p(\boldsymbol{x}^p)\}, \textstyle\sum_{(i,j)pq} \eta_{ij}^{pq}\} \quad \forall\, p,q \in \mathcal{P}, (i,j) \in \mathcal{S}^{pq},$$

$$p < q \tag{2a}$$

$$\text{subject to} \quad \boldsymbol{g}^p(\boldsymbol{x}^p) \leqslant \boldsymbol{0} \tag{2b}$$

$$\boldsymbol{h}^p(\boldsymbol{x}^p) = \boldsymbol{0} \tag{2c}$$

$$\eta_{ij}^{pq}(x_i^p - x_j^q) = 0 \tag{2d}$$

$$\eta_{ij}^{pq} \in \{0,1\}. \tag{2e}$$

The set $\mathcal{S}^{pq}$ in Eq. (2*a*) consists of index pairs of variables in products p and q that are *candidates* for sharing. The sharing decision variables η_{ij}^{pq} are set to 1 if variables x_i^p, x_j^q will be shared and 0 otherwise, hence Eq. (2*d*). Equations (2*b*) and (2*c*) are the individual design constraints. Finally, the objective in Eq. (2*a*) now includes one more term, the "commonality" term that sums up all shared variables.

This multicriteria problem is reformulated keeping the term $\Sigma_{(i,j)pq}\eta_{ij}^{pq}$ as the scalar objective function and treating the terms $f^p(\boldsymbol{x}^p)$, as lower bound constraints (where the constant f_l^p represents the lower bound):

$$\max_{\boldsymbol{\eta}, \boldsymbol{x}=[\boldsymbol{x}^{p_1}, \boldsymbol{x}^{p_2}, \ldots]} \quad \textstyle\sum_{(i,j)pq} \eta_{ij}^{pq} \quad \forall\, p,q \in \mathcal{P}, (i,j) \in \mathcal{S}^{pq}, p < q$$

$$\text{subject to} \quad \boldsymbol{g}^p(\boldsymbol{x}^p) \leqslant \boldsymbol{0}$$

$$\boldsymbol{h}^p(\boldsymbol{x}^p) = \boldsymbol{0}$$

$$f^p(\boldsymbol{x}^p) \geqslant f_l^p$$

$$\eta_{ij}^{pq}(x_i^p - x_j^q) = 0$$

$$\eta_{ij}^{pq} \in \{0,1\}. \tag{3}$$

Simple monotonicity analysis shows that varying bounds systematically will generate the Pareto set so that the two formulations are treated as equivalent: selecting bounds corresponds to a specific set of objective weights.

Using the function

$$D_\circ(x_i^p - x_j^q) = \begin{cases} 0 & \text{if } x_i^p = x_j^q \\ 1 & \text{otherwise} \end{cases}, \tag{4}$$

the term $\Sigma_{(i,j)pq}\eta_{ij}^{pq}$ can be computed based on the values of the design variables:

$$\sum_{(i,j)pq} \eta_{ij}^{pq} = \sum_{pq} |\mathcal{S}^{pq}| - \sum_{(i,j)pq} D_\circ(x_i^p - x_j^q), \tag{5}$$

where $|\mathcal{S}^{pq}|$ is the number of elements in the set $\mathcal{S}^{pq}$, which is constant and can be left out. Note that the equality constraints [Eq. (2*d*)] are included in Eq. (5). Therefore, maximizing $\Sigma_{(i,j)pq}\eta_{ij}^{pq}$ is equivalent to minimizing $\Sigma_{(i,j)pq}D_\circ(x_i^p - x_j^q)$.

To address the combinatorial nature of this problem the function $D_\circ$ is approximated by a function D_α. The function D_α should satisfy two requirements: its range should be [0, 1] and it should be continuously differentiable. The function we have selected for D_α is defined as:

$$D_\alpha(x_i^p - x_j^q) = 1 - \frac{1}{\left(\dfrac{x_i^p - x_j^q}{\alpha}\right)^2 + 1}. \tag{6}$$

This function is constructed as a measure of the distance between designs and approaches the function $D_\circ$ as α goes to zero. Figure 5 shows D_α for α=0.05. Since D_α is continuously differentiable, gradient-based algorithms can be used to solve the approximate commonality problem.

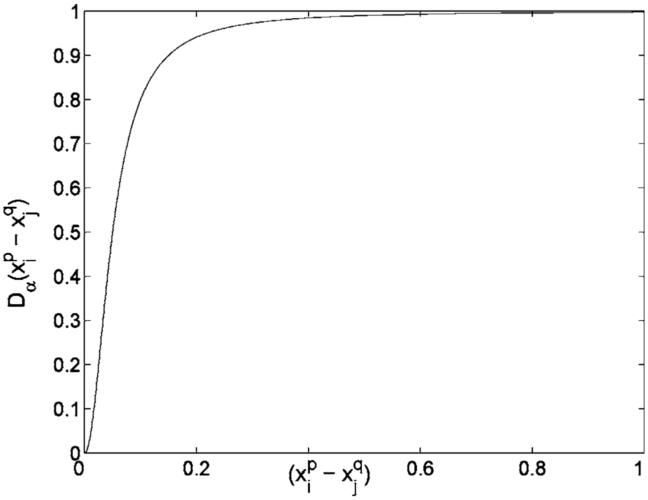

Fig. 5 The approximation of the function $D_\circ$

As discussed previously, solving the multicriteria problem can be reformulated by considering the objective terms f^p as constraints. To do this, we define *performance loss factors* L^p that represent the loss in performance of the family products compared

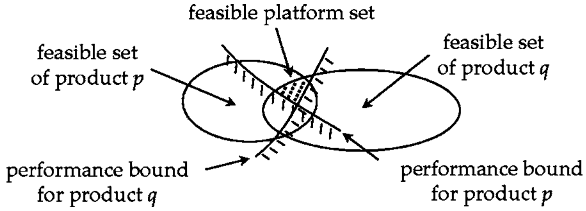

Fig. 6 Reduced platform feasible set

to the null-platform optima $f^{p,\circ}$. The constraints are defined as:

$$f^p(\boldsymbol{x}^p) \geqslant (1-L^p)f^{p,\circ} = f_l^p. \tag{7}$$

The complete formulation of the (approximate) commonality problem is now as follows:

$$\max_{\boldsymbol{x}=[\boldsymbol{x}^{p_1},\boldsymbol{x}^{p_2},\ldots]} \sum_{pq} |\mathcal{S}^{pq}| - \sum_{(i,j)pq} D_\alpha(x_i^p - x_j^q)$$

$$\forall p,q \in \mathcal{P},(i,j) \in \mathcal{S}^{pq}, p<q$$

$$\text{subject to} \quad \boldsymbol{g}^p(\boldsymbol{x}^p) \leqslant \boldsymbol{0}$$

$$\boldsymbol{h}^p(\boldsymbol{x}^p) = \boldsymbol{0}$$

$$f^p(\boldsymbol{x}^p) \geqslant (1-L^p)f^{p,\circ}. \tag{8}$$

Here the performance bounds are placed as lower bounds by defining them as percentage loss from the individually optimized values. For example, if a 15% loss in performance for a particular attribute is acceptable, the loss factor L^p is simply equal to 0.15.

The solution to Eq. (8) may not be unique. Figure 6 shows the reduced feasible set resulting from the introduction of performance bounds. Furthermore, multiple combinations of the same number of shared components can exist; these must be differentiated by their relative performance after solving the family problem, Eq. (1). Recall that this step of the methodology aims at selecting the feasible platform set, not at finding the design values of $\boldsymbol{x}^p$.

The loss factors L^p are considered input parameters specified by the designer. A postoptimal parametric study may be necessary to determine the acceptable tradeoff between performance and commonality, essentially generating the Pareto set of Eq. (2).

Even the commonality decision formulation in Eq. (8) is multiobjective and, as stated, contains no bias towards sharing one variable over another. However, it is possible to include the designer's preference for sharing by modifying the objective function.

$$\max_{\boldsymbol{x}=[\boldsymbol{x}^{p_1},\boldsymbol{x}^{p_2},\ldots]} \sum_{pq} |\mathcal{S}^{pq}| - \sum_{(i,j)pq} \omega_{ij}^{pq} D_\alpha(x_i^p - x_j^q)$$

$$\forall p,q \in \mathcal{P},(i,j) \in \mathcal{S}^{pq}, p<q. \tag{9}$$

The scalar weight ω_{ij}^{pq} corresponds to preference on sharing variables x_i^p and x_j^q, which can be chosen using manufacturing cost, or other criteria not included in the model here. Including cost may be straightforward by replacing the ω_{ij}^{pq} with the actual cost of the components: If component "1" costs 200 times more to manufacture than component "2," preference should be placed on sharing the more expensive component. Exploring these multiple layers of multiobjective decision is beyond the scope of this article. Therefore, in the remainder we assume no preference in component sharing, so that all ω_{ij}^{pq} will be equal.

4 Solving the Commonality Decision Problem

Since the solution to the approximate commonality problem will not yield 0–1 values, a determination must still be made about which variables will be selected as common. This is done after solving the commonality decision problem, Eq. (8), and in the following manner: the values of the design variables of the candidate components are compared, and assumed to be shared if their relative difference does not exceed a numerical tolerance. The designer must choose an appropriate value that ensures accuracy of the solution for the particular problem being solved. The shared variables are included as commonality constraints when solving the family design problem, Eq. (1). Therefore, one might compare the tolerance on sharing to the tolerance on satisfying constraints. A high tolerance might often lead to the suggestion of more sharing; however, this decreases the chances of satisfying the performance deviation constraints when the family design problem is solved.

The multicriteria family design problem, Eq. (1), is reformulated to minimize the distance between the null-platform design and the Pareto set corresponding to the selected platform.

$$\min_{\boldsymbol{x}=[\boldsymbol{x}^{p_1},\boldsymbol{x}^{p_2},\ldots]} \{((f^{p,\circ} - f^p(\mathbf{x}^p))/f^{p,\circ})^2\} \quad \forall\, p,q \in \mathcal{P},(i,j) \in \mathcal{S}^{pq,*}, p<q$$

$$\text{subject to} \quad \boldsymbol{g}^p(\boldsymbol{x}^p) \leqslant \boldsymbol{0}$$

$$\boldsymbol{h}^p(\boldsymbol{x}^p) = \boldsymbol{0}$$

$$f^p(\boldsymbol{x}^p) \geqslant (1-L^p)f^{p,\circ}$$

$$x_i^p = x_j^q. \tag{10}$$

The performance bounds are included in case the Pareto point closest to the null-platform point lies outside the area of allowable performance loss.

The proposed methodology can be stated now with the following steps:

(1) Determine the optimal null-platform design $f^{p,\circ}$, $\boldsymbol{x}^{p,\circ}$ for each individual product $p \in \mathcal{P}$ by solving the individual optimal design problem Eq. (11).

$$\max_{\boldsymbol{x}^p} \quad f^p(\boldsymbol{x}^p)$$

$$\text{subject to} \quad \boldsymbol{g}^p(\boldsymbol{x}^p) \leqslant \boldsymbol{0}$$

$$\boldsymbol{h}^p(\boldsymbol{x}^p) = \boldsymbol{0} \tag{11}$$

(2) Identify the components that could be shared between products, i.e., define the candidate platform set $\mathcal{S}^{pq}$ for any two products p and q in the set $\mathcal{P}$. The candidate platform set for the whole product family is then $\mathcal{S}=\{\mathcal{S}^{pq} | p,q \in \mathcal{P}; p<q\}$.
(3) Determine the performance loss factors L^p acceptable for each of the products.
(4) Solve the approximate commonality decision problem Eq. (8).
(5) Based on the results of the commonality decision problem, make a selection of components to be shared, i.e., determine the set $\mathcal{S}^*=\{\mathcal{S}^{pq,*} | p,q \in \mathcal{P}; p<q\}$.
(6) Solve the family design problem Eq. (10).

Figure 7 illustrates the above methodology. The conceptual plot shows a null-platform point $(f^{A,\circ}, f^{B,\circ})$. The null-platform objective function values are multiplied by the performance loss tolerances in the form $(1-L^p)$, and so the region where the associated feasible platforms reside is bounded by the points $(f^{A,\circ}, f^{B,\circ})$, $(((1-L^A)f^{A,\circ},(1-L^B))f^{B,\circ})$, $((1-L^A)f^{A,\circ},f^{B,\circ})$, and $(f^{A,\circ},(1-L^B)f^{B,\circ})$. Solving the commonality decision problem Eq. (8) a feasible platform (i.e., common components) is found. The performance loss bounds in Eq. (8) will be active, unless they are dominated by design constraints, and the obtained designs will correspond to the objective function values $(1-L^A)f^{A,\circ}$ and $(1-L^B)f^{B,\circ}$. The family

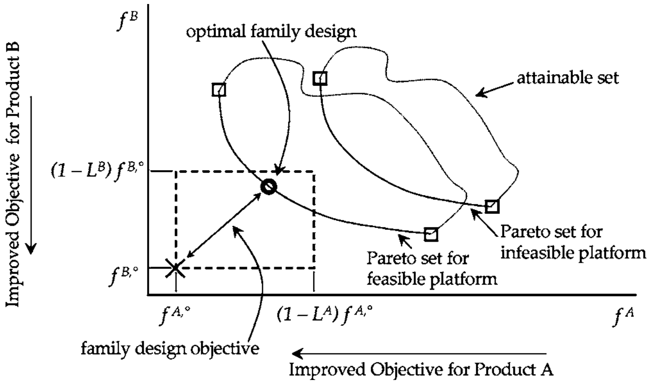

Fig. 7 Design process of proposed methodology

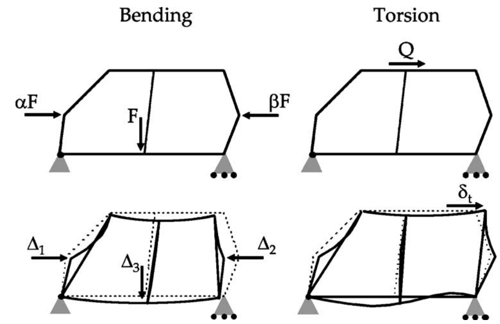

Fig. 9 Two-dimensional automotive side frame model

design problem Eq. (10) is solved next to obtain a Pareto-optimal design, searching for the point closest to the null-platform design.

5 Designing Automotive Body Side Frame Variants

We now consider a family of side frames for an automotive body with two variants. A variant can be defined by changing the functional requirements and/or the geometry of the model. In this study variants A and B are designed for minimum mass and maximum stiffness (minimum deflection), respectively.

The side frame of the automotive body is modeled simply as an assembly of ten beam elements and seven flexible joints (Fig. 8). A finite element solver is used to compute deflections, stresses, and body mass for different values of the rectangular cross section parameters of the beams (width b, height h, and thickness t). Two loading cases (bending and torsion) are considered, as depicted in Fig. 9 where $F=1500$ lbs. and $Q=1650$ lbs. The vehicle dimensions used for this example are $h=30$ in., $a=20$ in., $b=40$ in., $c=40$ in., and $d=10$ in. To compute α and β the following equations are used: $\alpha=(c+d)a/h(a+b+c+d)$ and $\beta=(a+b)d/h(a+b+c+d)$, where for this example $\alpha=10/33$ and $\beta=6/33$. The engine and rear compartments are included in the model as reaction forces applied at the connection of the "A" and hinge pillars and at the centerpoint of the "C" pillar for the bending load case. Torsion is represented by a horizontal force applied at the joint connecting the "B" pillar and the roof; this force simulates the shear that the structure undergoes under such loading and provides a torsional displacement, δ_t. An overall bending displacement is calculated as $\delta_b=\alpha\Delta_1+\beta\Delta_2+\Delta_3$. Each component is represented by three design variables. This means that a component can be shared within the family if all three design variables have equal values. However, it may be possible to consider some other form of sharing if only one or two design variables have equal values, for example, from a manufacturing point of view as discussed in Sec. 2. In this regard, all variables are treated as platform candidates in this study.

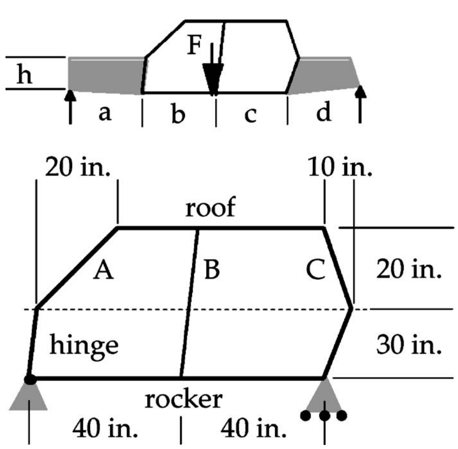

Fig. 8 Two-dimensional automotive side frame model

At first the optimal design problem is solved individually for each variant to obtain null-platform optima $f^{p,\circ}$. Two variants are considered: Variant A is designed with a minimum mass objective subject to a stiffness constraint represented by deflections

$$\min_{x^A} \quad f^A(\boldsymbol{x}^A) = m$$

$$\text{subject to} \quad \boldsymbol{g}^A(\boldsymbol{x}^A) \leqslant \boldsymbol{0}$$

$$\delta_b \leqslant 0.4 \text{ in.}$$

$$\delta_t \leqslant 0.8 \text{ in.} \tag{12}$$

Variant B is designed with a maximum stiffness objective subject to a mass constraint

$$\min_{x^A} \quad f^A(\boldsymbol{x}^A) = \delta_b + \delta_t$$

$$\text{subject to} \quad \boldsymbol{g}^B(\boldsymbol{x}^B) \leqslant \boldsymbol{0}$$

$$m \leqslant 250 \text{ lbs.} \tag{13}$$

Maximal stress constraints, **g**, for each beam element are taken into account for all three variants. A modulus of elasticity for steel of 30 Mpsi. is used along with a safety factor equal to 3.0. The computed null-platform optima for the considered variants are 53.4 lbs. for Variant A and 0.587 in. for Variant B.

Having computed the null-platform optima $f^{p,\circ}$, the commonality decision problem [Eq. (8)] can be solved for different values of the loss factor L^p. Experience to date shows that 0.025 is a good value for the parameter α in Eq. (6). Further investigation is necessary to determine a recommended value of α for general use.

In the side frame problem there are 24 design variables representing the cross-sectional variables b (width), h (height), and t (thickness) of the beams. The candidate platform set $\mathcal{S}$ is defined by allowing each design variable in Variant A to be shared only with the same variable of the corresponding component in Variant B. Therefore there are 2^{24} possible sharing combinations (platforms).

Equation (8) is solved to determine the "maximal" feasible platform under performance bounds set equal for both variants. A tolerance of 0.5% on the commonality constraints in the objective was used to determine which variables will be shared among the two variants. From the null-platform results, twelve variables were found to be "naturally" shared by inspection, i.e., they had the same optimal values in both designs. Allowing a performance loss of 1%, 5%, 10%, 20%, and 50% resulted in a commonality decision of sharing 17, 20, 21, 22, and 24 variables, respectively. The trade-off between sharing and performance is shown in Fig.

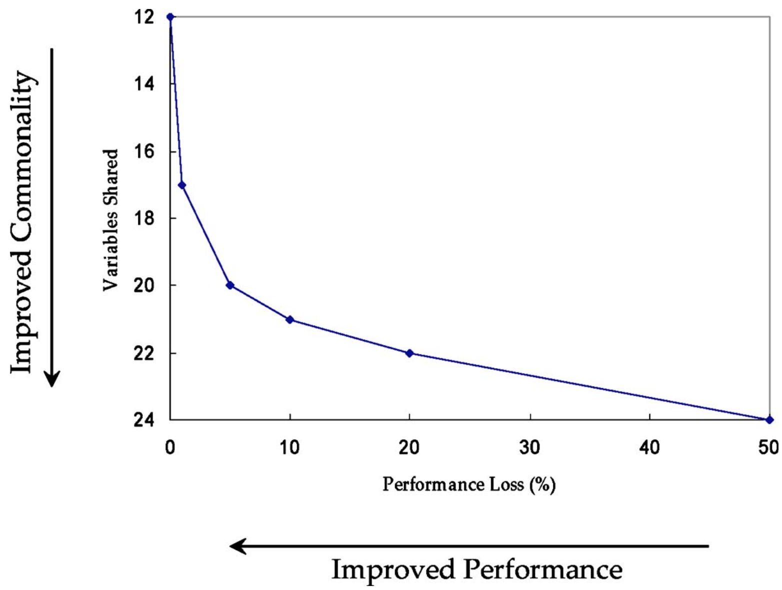

Fig. 10 Trade-offs between commonality and performance

10. This trade-off is analogous to the Pareto set that could be generated solving the combinatorial optimal design problem Eq. (2).

The optimal family problem Eq. (10) is solved next, with results presented in Table 1. The performance results obtained after solving the family problem for all platforms selected is shown. The performance in all cases are within the specified bounds. The optimal values of the product family design variables based on the null platform, the platform determined by accepting a 5% performance loss, and the total platform are compared in Table 2.

In an attempt to validate the results, a reduced version of Eq. (2) is solved. "Naturally shared" variables, as determined by the individual variant optimizations, are shared. Since twelve out of the sixteen width and height variable values of the two variants are always equal, we considered width and height as shared variables, and reduced the size of the combinatorial problem by defining the platform to include these sixteen variables. The problem size is thus reduced to 2^8 platform combinations.

A "top-down" algorithm was implemented by starting with sharing all eight component thicknesses (total platform). If the performance bounds were exceeded, we moved a "level" down by decreasing the number of candidates for sharing from eight to seven; eight different platforms of sharing seven thicknesses were then considered, and so on, until at least one platform was found, for which the performance bounds for a given "level" were not exceeded. Note that it is possible that more than one platform may satisfy the performance bounds at a given "level."

Table 1 Optimal product family design results and associated performance losses

Variant	A	B
Null platform	53.4 lbs.	0.587 in.
Platform of 17 variables	53.9 lbs.	0.593 in.
Performance loss	0.999%	0.972%
Platform of 20 variables	55.6 lbs.	0.616 in.
Performance loss	4.17%	5.01%
Platform of 21 variables	56.0 lbs.	0.646 in.
Performance loss	4.87%	9.96%
Platform of 22 variables	58.3 lbs.	0.687 in.
Performance loss	9.20%	17.00%
Total platform of 24 variables	73.2 lbs.	0.881 in.
Performance loss	37.1%	50.0%

One platform was obtained for each of the loss factors 1%, 5% or 50%, by solving the commonality decision problem Eq. (8). For loss factors of 10% and 20%, two and seven feasible platforms were found, respectively, each once again containing the same number of shared variables as determined by solving Eq. (8). It is encouraging that for both cases the platform obtained by solving the commonality decision problem [Eq. (8)] is the one that corresponds to least performance loss where multiple platforms were found. Intuitively this makes sense: since Eq. (8) is solved over a continuous design space, it is natural that components that have less impact (sensitivity) on the performance will be shared first.

6 Multiple Variants

When product designers study the implementation of a platform the number of products in the family will likely be more than two. The examples provided in this section demonstrate the use of the formulation for families with multiple products.

The size of the problem increases substantially with the number of variants. The number of possible platforms can be calculated as

$$\text{Number of platform combinations} = \left\{ 1 + \sum_{i=2}^{m} \frac{m!}{i!(m-i)!} \right\}^{n}, \quad (14)$$

where m is the number of variants and n is the number of shareable elements. The assumption made for this calculation is that each variant has the same components to be shared. The unity in the equation allows inclusion of the null platform as a possible combination. Likewise, the total platform is included in the summation for $i=m$.

The number of terms in the objective function of Eq. (8) is equal to the number of sharing possibilities among the variants. Assuming one variant with one shareable component, there exist three sharing possibilities among them, for four variants there exist six sharing possibilities, for five variants there exist ten sharing possibilities, and so on. The number of sharing possibilities for n shareable components in m products can be calculated by

Table 2 Optimal values of the product family design variables for different platforms

Component	Variable	Initial guess and bounds			Platform					
					Null		17-variables		Total	
		x_l	x_0	x_u	$x^{A,\circ}$	$x^{B,\circ}$	$x^{A,*}$	$x^{B,*}$	$x^{A,*}$	$x^{B,*}$
Rocker	b (in.)	1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
	h (in.)	2.00	4.00	4.40	4.40	4.40	4.40	4.40	4.40	4.40
	t (in.)	0.04	0.10	0.50	0.04	0.46	0.04	0.47	0.08	0.08
Roof rail	b (in.)	0.75	1.50	1.65	1.65	1.65	1.65	1.65	1.65	1.65
	h (in.)	0.50	1.00	1.10	1.10	1.10	1.10	1.10	1.10	1.10
	t (in.)	0.04	0.10	0.50	0.19	0.37	0.19	0.39	0.18	0.18
Hinge pillar	b (in.)	1.00	2.00	2.20	1.51	2.07	2.08	2.08	2.20	2.20
	h (in.)	2.00	4.00	4.40	4.40	4.40	4.40	4.40	4.40	4.40
	t (in.)	0.04	0.10	0.50	0.04	0.23	0.04	0.25	0.05	0.05
"A" pillar	b (in.)	0.50	1.00	1.10	1.10	1.07	1.09	1.09	1.10	1.10
	h (in.)	0.50	1.00	1.10	1.10	1.10	1.10	1.10	1.10	1.10
	t (in.)	0.04	0.30	0.50	0.04	0.28	0.06	0.06	0.06	0.06
"B" pillar (lower)	b (in.)	1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
	h (in.)	1.50	3.00	3.30	3.30	3.30	3.30	3.30	3.30	3.30
	t (in.)	0.04	0.10	0.50	0.12	0.41	0.12	0.41	0.12	0.12
"B" pillar (upper)	b (in.)	0.50	1.00	1.10	1.10	1.10	1.10	1.10	1.10	1.10
	h (in.)	1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
	t (in.)	0.04	0.10	0.50	0.09	0.32	0.09	0.34	0.09	0.09
"C" pillar (lower)	b (in.)	1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
	h (in.)	1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
	t (in.)	0.04	0.10	0.50	0.11	0.50	0.11	0.50	0.18	0.18
"C" pillar (upper)	b (in.)	0.50	1.00	1.10	0.50	1.01	0.96	0.96	1.08	1.08
	h (in.)	1.50	3.00	3.30	2.96	3.30	2.95	2.95	3.30	3.30
	t (in.)	0.04	0.10	0.50	0.04	0.18	0.04	0.23	0.04	0.04

$$\text{Number of sharing possibilities} = n\sum_{i=1}^{m-1}(m-i). \qquad (15)$$

The number of terms will grow significantly with additional products or components, but not as fast as in the original combinatorial problem. The number of possible platforms is reduced by limiting the number of components that the designer considers as shareable.

Consider the design of three automotive body side-frames (Fig. 11). Variant A is an economy vehicle, where the weight is minimized with constraints on the bending and torsional rigidity. The design problem is the same as the one given in Eq. (12). Variant B is a sporty vehicle, where the torsional rigidity is maximized with respect to constraints on the weight and bending stiffness

$$\min_{x^B} \quad f^B(\boldsymbol{x}^B) = \delta_t$$

$$\text{subject to} \quad \boldsymbol{g}^B(\boldsymbol{x}^B) \leqslant \boldsymbol{0}$$

$$m \leqslant 250 \text{ lbs}$$

$$\delta_b \leqslant 0.4 \text{ in.} \qquad (16)$$

Variant C is a stretched frame, where the lengths of the roof rail and rocker have been extended 20 in. (symmetrically about the "B" pillar); the objective is to minimize the bending displacement subject to weight and torsional rigidity constraints

$$\min_{x^C} \quad f^C(\boldsymbol{x}^C) = \delta_b$$

$$\text{subject to} \quad \boldsymbol{g}^C(\boldsymbol{x}^C) \leqslant \boldsymbol{0}$$

$$m \leqslant 250 \text{ lbs}$$

$$\delta_t \leqslant 0.8 \text{ in.} \qquad (17)$$

All three vehicle models use the same α and β values as in the previous example. The optimal objective function values for the three variants are determined to be 53.4 lbs., 0.373 in., and 0.312 in., respectively.

From Eq. (14) the number of platform combinations is calculated as 60×10^{15}. For the three-variant example, given that the same 24 design variables define each variant as before, Eq. (15) yields 72 terms necessary to capture all sharing combinations.

In the commonality decision problem Eq. (8), the objective for each variable i will have three terms,

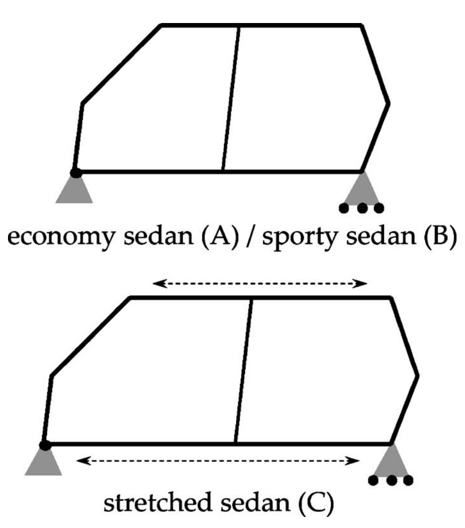

Fig. 11 Side-frames for three different automobiles

Table 3 Commonality decisions and family design for L^p=1%

Component	Variable	Shared among	Family design $\boldsymbol{x}^{A,*}$	$\boldsymbol{x}^{B,*}$	$\boldsymbol{x}^{C,*}$
Rocker	b (in.)	ABC	2.20	2.20	2.20
	h (in.)	ABC	4.40	4.40	4.40
	t (in.)	BC	0.04	0.48	0.48
Roof rail	b (in.)	ABC	1.65	1.65	1.65
	h (in.)	ABC	1.10	1.10	1.10
	t (in.)	BC	0.19	0.33	0.33
Hinge pillar	b (in.)	ABC	1.98	1.98	1.98
	h (in.)	ABC	4.37	4.37	4.37
	t (in.)		0.04	0.25	0.07
"A" pillar	b (in.)	ABC	1.02	1.02	1.02
	h (in.)	ABC	1.10	1.10	1.10
	t (in.)	BC	0.04	0.29	0.29
"B" pillar (lower)	b (in.)	ABC	2.19	2.19	2.19
	h (in.)	AB	3.30	3.30	1.66
	t (in.)	A C	0.12	0.50	0.12
"B" pillar (upper)	b (in.)	ABC	1.06	1.06	1.06
	h (in.)	ABC	2.19	2.19	2.19
	t (in.)	BC	0.09	0.25	0.25
"C" pillar (lower)	b (in.)	ABC	2.20	2.20	2.20
	h (in.)	ABC	2.20	2.20	2.20
	t (in.)	BC	0.11	0.50	0.50
"C" pillar (upper)	b (in.)	ABC	1.01	1.01	1.01
	h (in.)	ABC	3.30	3.30	3.30
	t (in.)	AB	0.04	0.04	0.26

$$\min_{x=[x^A,x^B,x^C]} \sum_{i=1}^{n=24} \left(D_\alpha(x_i^A - x_i^B) + D_\alpha(x_i^A - x_i^C) + D_\alpha(x_i^B - x_i^C)\right). \tag{18}$$

The performance bounds are set at 1%, 5%, and 10% across variants. Once again a tolerance of 0.05% is used for the commonality constraints in the objective. The sharing decisions, and resulting family designs for each frame member cross section are shown in Tables 3–5, respectively. With a loss factor of 1% fifteen variables can be shared among all three products, two variables between A and B, one variable between A and C, and five variables between B and C. Using a loss factor of 5% seventeen variables can be

Table 4 Commonality decision and family design for L^p=5%

Component	Variable	Shared among	Family design $\boldsymbol{x}^{A,*}$	$\boldsymbol{x}^{B,*}$	$\boldsymbol{x}^{C,*}$
Rocker	b (in.)	ABC	2.20	2.20	2.20
	h (in.)	ABC	4.40	4.40	4.40
	t (in.)	BC	0.04	0.46	0.46
Roof rail	b (in.)	ABC	1.65	1.65	1.65
	h (in.)	ABC	1.10	1.10	1.10
	t (in.)	BC	0.20	0.33	0.33
Hinge pillar	b (in.)	ABC	1.90	1.90	1.90
	h (in.)	ABC	4.40	4.40	4.40
	t (in.)	BC	0.04	0.25	0.25
"A" pillar	b (in.)	ABC	1.10	1.10	1.10
	h (in.)	ABC	1.10	1.10	1.10
	t (in.)	BC	0.04	0.35	0.35
"B" pillar (lower)	b (in.)	ABC	2.20	2.20	2.20
	h (in.)	AB	3.30	3.30	1.88
	t (in.)	A C	0.12	0.50	0.12
"B" pillar (upper)	b (in.)	ABC	1.10	1.10	1.10
	h (in.)	ABC	2.20	2.20	2.20
	t (in.)	ABC	0.11	0.11	0.11
"C" pillar (lower)	b (in.)	ABC	2.20	2.20	2.20
	h (in.)	ABC	2.20	2.20	2.20
	t (in.)	BC	0.11	0.50	0.50
"C" pillar (upper)	b (in.)	ABC	1.05	1.05	1.05
	h (in.)	ABC	3.30	3.30	3.30
	t (in.)	ABC	0.04	0.04	0.04

Table 5 Commonality decision and family design for $L^p=10\%$

Component	Variable	Shared among	Family design $\boldsymbol{x}^{A,*}$	$\boldsymbol{x}^{B,*}$	$\boldsymbol{x}^{C,*}$
Rocker	b (in.)	ABC	2.20	2.20	2.20
	h (in.)	ABC	4.40	4.40	4.40
	t (in.)	BC	0.04	0.50	0.50
Roof rail	b (in.)	ABC	1.65	1.65	1.65
	h (in.)	ABC	1.10	1.10	1.10
	t (in.)	ABC	0.18	0.18	0.18
Hinge pillar	b (in.)	ABC	2.17	2.17	2.17
	h (in.)	ABC	4.38	4.38	4.38
	t (in.)	BC	0.04	0.26	0.26
"A" pillar	b (in.)	ABC	1.10	1.10	1.10
	h (in.)	ABC	1.10	1.10	1.10
	t (in.)	ABC	0.09	0.09	0.09
"B" pillar (lower)	b (in.)	ABC	2.20	2.20	2.20
	h (in.)	ABC	3.30	3.30	3.30
	t (in.)	A C	0.12	0.50	0.12
"B" pillar (upper)	b (in.)	ABC	1.10	1.10	1.10
	h (in.)	ABC	2.20	2.20	2.20
	t (in.)	ABC	0.09	0.09	0.09
"C" pillar (lower)	b (in.)	ABC	2.20	2.20	2.20
	h (in.)	ABC	2.20	2.20	2.20
	t (in.)	BC	0.11	0.50	0.50
"C" pillar (upper)	b (in.)	ABC	1.07	1.07	1.07
	h (in.)	ABC	3.30	3.30	3.30
	t (in.)	ABC	0.05	0.05	0.05

shared among all three products, one variable between A and B, one variable between A and C, and five variables between B and C. Finally using a loss factor of 10% twenty variables can be shared among all three products, no variables between A and B, one variable between A and C, and three variables between B and C.

The results from performing the family design optimization can be seen in Table 6. For all three cases performance losses do not exceed the bounds.

7 Aggregating Variables in Shareable Components

So far we examined what design variables can be shared among products. In fact the designer may need to know explicitly whether an entire component can be shared. This section demonstrates how this decision can be reached using the side frame example.

The components for each of the body side frames are included in the component set $\mathcal{C}^p$ (because all three products have an identical topology, the set $\mathcal{C}^p$ is consistent among variants),

$$\mathcal{C}^p=\{\text{Rocker, Roof rail, Hinge pillar, ``A'' pillar, ``B'' pillar}_{(\text{lower})}, \text{``B'' pillar}_{(\text{upper})}, \text{``C'' pillar}_{(\text{lower})}, \text{``C'' pillar}_{(\text{upper})}\}.$$

Table 6 Optimal product family design results and associated performance losses

Variant	A	B	C
Null platform	53.4 lbs.	0.373 in.	0.312 in.
Platform with $L^p=1\%$	53.9 lbs.	0.376 in.	0.314 in.
Performance loss	0.963%	0.830%	0.629%
Platform with $L^p=5\%$	54.9 lbs.	0.385 in.	0.324 in.
Performance loss	2.75%	3.08%	3.73%
Platform with $L^p=10\%$	56.0 lbs.	0.395 in.	0.339 in.
Performance loss	4.87%	5.84%	8.65%

The design variables of the product are then mapped into vectors of design variables which correspond to particular product components as follows:

$$\boldsymbol{x}^{c_1^p}=[x_1^p,x_2^p,x_3^p]^T$$

$$\boldsymbol{x}^{c_2^p}=[x_4^p,x_5^p,x_6^p]^T$$

$$\boldsymbol{x}^{c_3^p}=[x_7^p,x_8^p,x_9^p]^T$$

$$\boldsymbol{x}^{c_4^p}=[x_{10}^p,x_{11}^p,x_{12}^p]^T$$

$$\boldsymbol{x}^{c_5^p}=[x_{13}^p,x_{14}^p,x_{15}^p]^T$$

$$\boldsymbol{x}^{c_6^p}=[x_{16}^p,x_{17}^p,x_{18}^p]^T$$

$$\boldsymbol{x}^{c_7^p}=[x_{19}^p,x_{20}^p,x_{21}^p]^T$$

$$\boldsymbol{x}^{c_8^p}=[x_{22}^p,x_{23}^p,x_{24}^p]^T,$$

where the three vector entries for each component are the width, height, and thickness of the component. With the design vector defined for each component the final step is to modify the objective function of the commonality decision problem to deal with decisions based on vectors rather than scalars. This is accomplished by simply using the l_2 norm to compute the difference in design.

$$\min_{x=[x^A,x^B,x^C]}\sum_{i=1}^{n=8}(D_\alpha(\|\boldsymbol{x}^{c_i^A}-\boldsymbol{x}^{c_i^B}\|_2)+D_\alpha(\|\boldsymbol{x}^{c_i^A}-\boldsymbol{x}^{c_i^C}\|_2)+D_\alpha(\|\boldsymbol{x}^{c_i^B}-\boldsymbol{x}^{c_i^C}\|_2)) \quad (19)$$

Since the focus is now on component selection, the roof and rocker of the stretched vehicle (C) is not shareable with the other

Table 7 Commonality decision (based on components) and family design for $L^p=5\%$

Component	Variable	Shared among	Family design		
			$\boldsymbol{x}^{A,*}$	$\boldsymbol{x}^{B,*}$	$\boldsymbol{x}^{C,*}$
Rocker	b (in.)		2.20	2.20	2.20
	h (in.)		4.40	4.40	4.40
	t (in.)		0.04	0.45	0.50
Roof rail	b (in.)		1.65	1.65	1.65
	h (in.)		1.10	1.10	1.10
	t (in.)		0.19	0.48	0.43
Hinge pillar	b (in.)	A C	2.13	2.14	2.13
	h (in.)		4.40	4.40	4.40
	t (in.)		0.05	0.29	0.05
“A” pillar	b (in.)	BC	0.95	1.10	1.10
	h (in.)		1.10	1.10	1.10
	t (in.)		0.04	0.33	0.33
“B” pillar (lower)	b (in.)		2.20	2.20	2.10
	h (in.)		3.30	3.30	2.41
	t (in.)		0.12	0.50	0.16
“B” pillar (upper)	b (in.)	ABC	1.04	1.04	1.04
	h (in.)		2.20	2.20	2.20
	t (in.)		0.09	0.09	0.09
“C” pillar (lower)	b (in.)	BC	2.20	2.20	2.20
	h (in.)		2.20	2.20	2.20
	t (in.)		0.11	0.50	0.50
“C” pillar (upper)	b (in.)	ABC	1.04	1.04	1.04
	h (in.)		3.30	3.30	3.30
	t (in.)		0.05	0.05	0.05

two vehicles. This is because these are the only two components that do not share a common length with the other two variants. Therefore, when $i=1,2$ the last two terms are removed in Eq. (19). The platform selection process is now performed with a loss factor of 5% and 10%. The tolerance of 0.05% is used again for the commonality constraints in the objective. The platform selection and corresponding family designs for each frame member cross section can be found in Tables 7 and 8.

Figure 12 represents the shared components between pairs of products for each of the two loss factors. The platform among all three products is the intersection of the shared components for each of the sharing pairs. For a loss factor of 5% there are five

Table 8 Commonality decision (based on components) and family design for $L^p=10\%$

Component	Variable	Shared among	Family design		
			$\boldsymbol{x}^{A,*}$	$\boldsymbol{x}^{B,*}$	$\boldsymbol{x}^{C,*}$
Rocker	b (in.)		2.20	2.20	2.20
	h (in.)		4.40	4.40	4.40
	t (in.)		0.04	0.49	0.50
Roof rail	b (in.)	AB	1.65	1.65	1.65
	h (in.)		1.10	1.10	1.10
	t (in.)		0.18	0.18	0.45
Hinge pillar	b (in.)	A C	2.08	2.20	2.08
	h (in.)		4.40	4.40	4.40
	t (in.)		0.04	0.39	0.04
“A” pillar	b (in.)	ABC	1.10	1.10	1.10
	h (in.)		1.10	1.10	1.10
	t (in.)		0.10	0.10	0.10
“B” pillar (lower)	b (in.)	A C	2.05	2.20	2.05
	h (in.)		3.29	3.30	3.29
	t (in.)		0.13	0.50	0.13
“B” pillar (upper)	b (in.)	ABC	1.09	1.09	1.09
	h (in.)		2.20	2.20	2.20
	t (in.)		0.09	0.09	0.09
“C” pillar (lower)	b (in.)	BC	2.20	2.20	2.20
	h (in.)		2.20	2.20	2.20
	t (in.)		0.11	0.50	0.50
“C” pillar (upper)	b (in.)	ABC	1.07	1.07	1.07
	h (in.)		3.30	3.30	3.30
	t (in.)		0.05	0.05	0.05

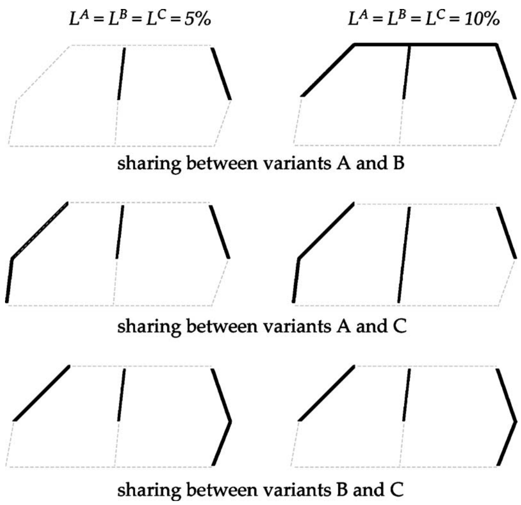

Fig. 12 Platform results for three variants based on component sharing

components shared, with the upper “B” and “C” pillars being shared among all three variants. For a loss factor of 10% there are seven components shared, now including the “A” pillar in the platform between all three products. Actual losses are computed after solving Eq. (10) and are reported in Table 9. They are within the allowable bounds on deviation from the null-platform designs.

Summarizing, the methodology has been applied to family design problems including multiple products. In addition, the formulation was used for performing platform selection based on component selection (all variables of component shared). These results allow the visualization of modularity as well: components or sets of components which produce variety are defined as modules, and are not part of the platform. A thorough discussion of modularity is beyond the scope of this article.

8 Conclusions

The proposed methodology integrates platform selection under performance bounds with optimal design of product families. The designer can decide (potentially by using business and marketing data) what performance losses are acceptable relative to individual product variant optimality. Component sharing is determined through the solution of the relaxed commonality maximization combinatorial problem subject to these performance bounds. The formulation of the commonality decision problem allows the designer to assign different weights to express sharing preferences. The optimal product family design problem is solved to obtain a point on the Pareto set associated with the determined platform that is closest to the null-platform point. A family of automotive body side frames has been used to demonstrate the proposed methodology. Results have been validated by comparing them to those obtained by solving a reduced version of the original combinatorial commonality problem. It is possible to conclude that the methodology can be helpful in addressing the platform selection problem, which may be intractable in its original combinatorial form. Problems have been examined which have a significantly larger number of sharing combinations than has previously been solved in the design literature.

Gradient-based algorithms are recommended to solve the commonality decision problem Eq. (8). It is desirable to compute gradients analytically. The use of the function D_α as defined in Eq. (6) makes this possible. In the case that simulation-based models are used to evaluate the performance constraints, it is important to use appropriate finite-difference steps for evaluating the constraint functions. For the application considered in this paper, we used the MATLAB implementation of the sequential quadratic programming (SQP) algorithm. Design variables, objective, and constraints were scaled to the order of one. On average, for the three-product examples in this paper, on the order of 100 iterations are necessary to solve the commonality decision problem. In examples so far, different starting points have resulted in the same optimal solution. One important assumption that has been made is that the design variables and functions are continuous. This method is not applicable to a catalog selection problem or where design variables are discrete with no underlying continuous trends.

Solving the commonality decision problem [Eq. (8)] may become inefficient as the number of products and/or the number of shareable components increases dramatically beyond the practical limitations of an algorithm such as SQP. Future work will include looking at implementing the ranking methods to reduce the order of the problem by filtering out components with low impact on sharing [13,14]. Metamodels can also be used to decrease computational expense and reduce function noise, which is detrimental when applying gradient-based methods. Lastly, extending the methodology to hierarchical product family problems can prove useful for solving large-scale problems.

Table 9 Optimal product family design results and associated performance losses

Variant	A	B	C
Null platform	53.4 lbs.	0.373 in.	0.312 in.
Platform with $L^p=5\%$	55.3 lbs.	0.385 in.	0.325 in.
Performance loss	3.60%	3.18%	4.12%
Platform with $L^p=10\%$	56.3 lbs.	0.395 in.	0.336 in.
Performance loss	5.49%	5.82%	7.74%

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