DAC TOC

Proceedings of DETC'02
ASME 2002 Design Engineering Technical Conferences
and Computer and Information in Engineering Conference
Montreal, Canada, September 29-October 2, 2002

DETC2002/DAC-34099

PLATFORM SELECTION UNDER PERFORMANCE LOSS CONSTRAINTS IN OPTIMAL DESIGN OF PRODUCT FAMILIES

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ABSTRACT

Designing a family of product variants that share some components usually entails a performance loss relative to the individually optimized variants due to the commonality constraints. Choosing components for sharing may depend on what performance losses can be tolerated. This article presents a methodology for making commonality decisions while controlling individual performance losses. Previous work focused on evaluating individual performance losses due to pre-specified sharing. Trade-offs were identified for different platforms (i.e., the sets of components shared among products) by means of Pareto sets. In the present work an optimal design problem is formulated to choose product components to be shared without exceeding a user-specified performance loss tolerance. This enables the designer to control trade-offs and obtain optimal product family designs for different levels of performance losses in an attempt to maximize commonality. A family of automotive side frames is used to demonstrate the approach.

NOMENCLATURE

$\mathcal{P}$ Set of products.
m Number of products in set $\mathcal{P}$.
p, q Superscripts denoting products.
$\mathcal{C}^p$ Set of components in product p.
c_i^p Component i in product p.
n Number of design variables.
d Number of model responses.
$\mathbf{x}$ Vector of design variables.
η Sharing decision variable.
$\mathbf{R}$ Vector of responses.
f Objective function representing product performance.
f° Optimal objective function value for null platform
f^* Optimal objective function value for a platform-based product.
$\mathbf{g}$ Vector of inequality constraints.
$\mathbf{h}$ Vector of equality constraints.
$\hat{\mathcal{S}}^{pq}$ Set of pairs of indices describing candidate platform between two products.
$\hat{\mathcal{S}}$ Set of all sets $\hat{\mathcal{S}}^{pq}$ with $p,\ q \in \mathcal{P}$.
$\mathcal{S}^{pq}$ Set of pairs of indices describing platform between two products.
$\mathcal{S}$ Set of all sets $\mathcal{S}^{pq}$ with $p,\ q \in \mathcal{P}$.
L Performance loss tolerance.
D Distance function.
ω Preference on sharing component.
b Width of rectangular cross section.
h Height of rectangular cross section.
t Thickness of rectangular cross section.

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1 INTRODUCTION

Sharing components among products is an effective way to cut costs. Expenses are decreased through the reduced time to design and engineer components as well as through savings in manufacturing and inventory. In this regard, a product platform is defined as the set of components and manufacturing and assembly processes that are common among a family of products.

Commonality often causes a penalty with respect to individual product performance (Nelson *et al.* 2001). The design challenge is how to maximize commonality and optimize the family products while satisfying individual constraints and minimizing performance losses. Research in product platform design is characterized by two approaches. One approach tends to be more qualitative, addressing important business issues, common terminology, different schools of thought, and real-world case studies. The second approach uses mathematical formulations of optimal design problems and is more quantitative but is limited by our ability to model a given situation. The present article follows this second approach.

Simpson *et al.* (1999) proposed the product platform concept exploration method for configuring and exploring product platforms concepts that can be scaled to product families by means of so-called parametric and/or configurational factors. They formulated a compromise decision support problem to optimize the design, while Messac *et al.* (2000) adopted a physical programming approach. Conner *et al.* (1999) utilized Simpson's (1998) product variety tradeoff evaluation method and the combination of the above two concepts to evaluate platform portfolios based on commonality and performance losses indices. Nayak *et al.* (2000) employed robust design concepts to formulate a variation-based platform design methodology that consists of two steps: identifying the platform by solving a compromise decision support problem and designing the family around this platform.

Gonzalez-Zugasti *et al.* (1998) presented a method that uses cost gain models as the driving force for designing the product platform while satisfying performance and budget constraints. In this approach, several *a priori* specified platforms are optimized first, while the family variants are designed at the second stage. The performance of the platforms is evaluated to aid the designer to pick the best in an interactive manner. Gonzalez-Zugasti and Otto (2000) formulated a modular product platform-based design optimization problem that can be solved to determine simultaneously module designs and their combination for the variant instantiations. In this work platform and individual goals form a multiobjective function and additional compatibility and sharing constraints are included. Fujita *et al.* (2001) also proposed a method for simultaneous optimization of module attributes and combinations, driven by cost-related functions. In both of the latter papers the modular architecture of the product family is fixed.

Nelson *et al.* (2001) formulated and solved a multiobjective optimal design problem by means of Pareto set theory. Given a fixed platform, a set of optimal points is generated based on the importance of the conflicting variant objectives. The designer can identify trade-offs, evaluate multiple platforms, and then make related decisions. Fellini *et al.* (2000) applied this concept to the design of an automotive product family based on a powertrain platform along with examining the hierarchical structure of the platform design problem. Kokkolaras *et al.* (2002) extended the target cascading formulation to the design of product families for pre-specified platforms. Both common and individual components, subsystems, and/or systems of the family products are designed optimally with respect to family and variant targets.

It can be readily concluded that deciding what can be shared among a set of products is a key process in optimal design of product families. Examining all different platform possibilities is likely to be computationally prohibitive due to the combinatorial nature of the problem. However, one can look for a strategy where a good choice of common components can be made without exhaustive enumeration. In the present work the above mentioned combinatorial problem is relaxed and reformulated to minimize design deviations among the family members while satisfying individual design constraints and observing a designer-specified performance loss tolerance. In this manner, the designer can identify a set of components that can be shared and then obtain optimal designs for a number of scenarios, based on the willingness to sacrifice a certain amount of individual performance.

The article is organized as follows: The platform-based design and commonality decision formulations are presented. The methodology for commonality decisions is then formulated. An example based on a family of automotive vehicle side frames is used to demonstrate the methodology. Results are discussed and conclusions are drawn.

2 TERMINOLOGY AND NOTATION

In this section we review the vocabulary for product platforms and devise a corresponding mathematical notation.

A *component* is defined as a manufactured object that is the smallest (indivisible) element of an assembly, and is represented by a set of design variables. A *product* is an artifact that is made up of components. The *product architecture* is the configuration of components within the product. A *module* is a component or subassembly that can be interchanged within a product architecture to produce variety. A *model* is a mathematical representation of a product that accepts a vector of design variables and returns a vector of responses.

The mathematical notation begins with the set $\mathcal{P} = \{A, B, C, \ldots\}$ used to distinguish each product $p \in \mathcal{P}$. Likewise, the set $\mathcal{C}^p = \{c_1^p, c_2^p, c_3^p, \ldots\}$ is defined to represent the components that form a particular product p. Figure 1 illustrates the above notation. A product p is associated with a vector of design

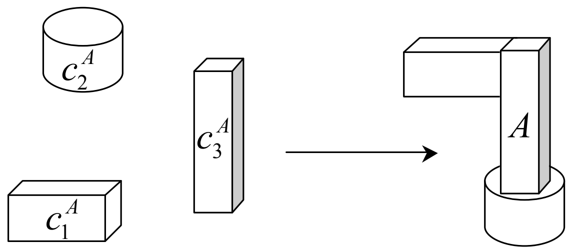

Figure 1. Components and assembled product.

variables $\mathbf{x}^p \in \mathbb{R}^{n^p}$, a vector of responses $\mathbf{R}^p \in \mathbb{R}^{d^p}$, and design inequality and equality constraints $\mathbf{g}^p \in \mathbb{R}^{v^p}$ and $\mathbf{h}^p \in \mathbb{R}^{w^p}$, respectively. The subset of design variables describing component c_i^p is $\mathbf{x}_i^p \subseteq \mathbf{x}^p$.

A *product platform* is the set of all components, manufacturing processes, and/or assembly steps that are common in a set of products. A *product family* is the set of product variants that share a product platform. In terms of platform-based products a *product portfolio* refers to the set of product families offered across market segments over time. A family product derived from a platform is also referred to as a *product variant*.

Two types of sharing can be used when selecting a product platform that is not based on manufacturing processes or assembly steps. In *component sharing*, one or more components are common across a family of products as shown in Figure 2. In

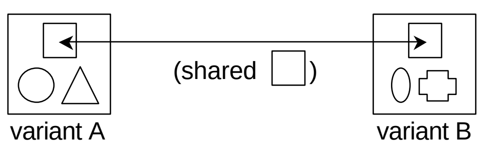

Figure 2. Platform-based products (component sharing).

addition, it is possible to share "scaled" versions of components. Mathematically this can be described as *variable sharing*, where components are based on a platform (of variables) themselves. The example in Figure 3 shows the cross-section of two structural beam elements. While the height and width of both parts are the same, the thickness is different. The manufacturing advantage can be illustrated by this example. By keeping width and height invariant, the same stamping equipment can be used with different gauge steel. In general, manufacturing (and therefore cost) considerations should be taken into account in the design of platforms. We do not address this aspect explicitly but we attempt to recognize the associated design impact.

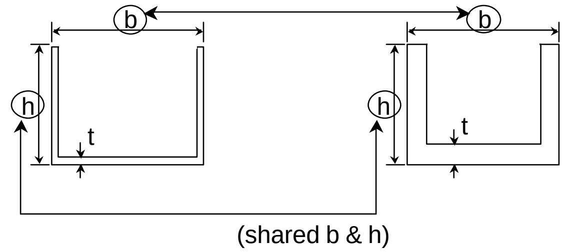

Figure 3. Platform-based components (variable sharing).

The formulation here holds for both component and variable sharing. For simplicity no distinction will be made between components and variables. In fact, a component can be represented by a single variable. The following notation is used to describe a product platform: The set $\mathcal{S}^{pq}$ consists of the pairs of indices of the components that are shared between two products p and q, where $p,q \in \mathcal{P}$ and $p < q$. The set $\mathcal{S} = \{\mathcal{S}^{pq} \mid p,\ q \in P;\ p < q\}$ describes the product platform.

3 PLATFORM-BASED PRODUCT DESIGN

Nelson *et al.* (2001) proposed a multicriteria optimization statement for platforms specified a priori:

$$\begin{array}{rc} \text{maximize} & \textit{product "performance"} \\ \text{with respect to} & \textit{product design variables} \\ \text{subject to} & \textit{product requirements} \\ & \textit{component commonality constraints} \end{array}$$

The mathematical formulation is as follows:

$$\begin{array}{rcl} \text{maximize} & \{f^p(\mathbf{x}^p)\} & p = 1,\ldots,m \\ \text{with respect to} & \mathbf{x} = [\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^m] & \\ \text{subject to} & \mathbf{g}^p(\mathbf{x}^p) \leq \mathbf{0} & p = 1,\ldots,m \\ & \mathbf{h}^p(\mathbf{x}^p) = \mathbf{0} & p = 1,\ldots,m \\ & x_i^p = x_j^q & \\ & (i,j) \in \mathcal{S}^{pq};\ p,q \in \mathcal{P};\ p < q & \end{array} \tag{1}$$

An optimization problem is solved for each product separately to determine the null platform point, shown in Figure 4 as the point $\times(f^{A,\circ}, f^{B,\circ})$. In the product family design problem the objective function is a weighted aggregate of the individual product objectives. The constraint set includes all individual product constraints. Commonality constraints are included as equality constraints $x_i^p = x_j^q$. Solving the family optimization problem for different weights yields the Pareto set. The bounds on the

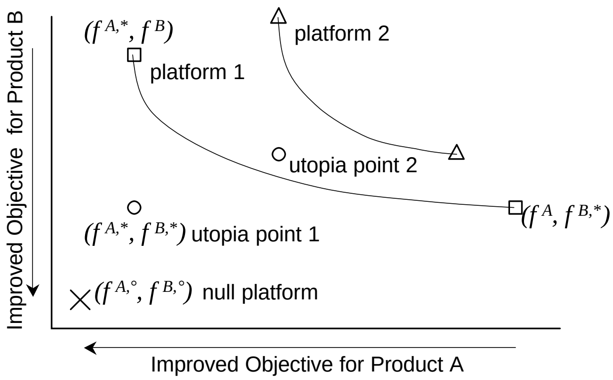

Figure 4. Null platform point and Pareto sets for different platforms.

Pareto set determine the utopia point, the best possible design that can be achieved with the platform. Assuming the design penalty from the null platform to the utopia point is acceptable to the designer, the rest of the Pareto set is computed. Two possible platforms and their associated trade-offs are shown in Figure 4. The preferred design on the Pareto set is selected according to other criteria (cost, customer preference, etc.) not included in the above model.

4 THE COMMONALITY DECISION PROBLEM

The important trade-off that exists when attempting to choose the components which will make up a product platform is one of commonality versus product performance. The product family design problem is stated as:

maximize	*product "performance"*
	component commonality
with respect to	*product design variables*
	sharing decision variables
subject to	*product requirements*
	component commonality constraints

The optimal design problem is now formulated mathematically as a mixed-discrete programming problem due to the presence of binary commonality decision variables:

$$\begin{array}{rll} \text{maximize} & \{f^p(\mathbf{x}^p), \sum_{ijpq}\eta_{ij}^{pq}\} & p=1,\ldots,m \\ \text{with respect to} & \mathbf{x}=[\mathbf{x}^1,\mathbf{x}^2,\ldots,\mathbf{x}^m],\eta & \\ \text{subject to} & \mathbf{g}^p(\mathbf{x}^p)\leq\mathbf{0} & p=1,\ldots,m \\ & \mathbf{h}^p(\mathbf{x}^p)=\mathbf{0} & p=1,\ldots,m \\ & \eta_{ij}^{pq}(x_i^p-x_j^q)=0 & \\ & \eta_{ij}^{pq}\in\{0,1\} & \\ & (i,j)\in\hat{\mathcal{S}}^{pq};\ p,q\in\mathcal{P};\ p<q & \end{array} \tag{2}$$

The set $\hat{\mathcal{S}}^{pq}$ consists of pairs of indices of components of products p and q that are *candidates* for sharing — though will not necessarily be shared. The sharing decision variables η_{ij}^{pq} are set to 1 if variables x_i^p, x_j^q will be shared and 0 otherwise. Our approach to solve this multi-objective optimization problem is to reformulate it by keeping the term $\sum_{ijpq}\eta_{ij}^{pq}$ in the objective function and moving the terms $f^p(\mathbf{x}^p)$, $p=1,\ldots,m$ to the constraints. Using the function

$$D_\circ(x_i^p-x_j^q)=\begin{cases}0 \text{ if } x_i^p=x_j^q\\ 1 \text{ otherwise}\end{cases} \tag{3}$$

the term $\sum_{ijpq}\eta_{ij}^{pq}$ can be computed based on the values of the design variables:

$$\sum_{ijpq}\eta_{ij}^{pq}=\sum_{pq}|\hat{\mathcal{S}}^{pq}|-\sum_{ijpq}D_\circ(x_i^p-x_j^q), \tag{4}$$

where $|\hat{\mathcal{S}}^{pq}|$ is the number of elements in the set $\hat{\mathcal{S}}^{pq}$. Note that the equality constraints $\eta_{ij}^{pq}(x_i^p-x_j^q)=0$ are addressed in Equation (4). Therefore, from Equation (4), the maximization of $\sum_{ijpq}\eta_{ij}^{pq}$ is equivalent to the minimization of $\sum_{ijpq}D_\circ(x_i^p-x_j^q)$. The use of the function $D_\circ$ incorporates the commonality constraint. However, the solution of this formulation still requires a combinatorial approach. In order to make the problem tractable, the function $D_\circ$ is approximated by a function that is continuously differentiable. This also allows the use of gradient-based algorithms. The approximating function D_α is given in Equation (5) and depicted in Figure 5 for $\alpha=0.05$.

$$D_\alpha(x_i^p-x_j^q)=1-\frac{1}{\left(\frac{x_i^p-x_j^q}{\alpha}\right)^2+1} \tag{5}$$

This function is a measure of the distance between designs $\mathbf{x}^p$ and $\mathbf{x}^q$, and approaches the function $D_\circ$ as α goes to zero.

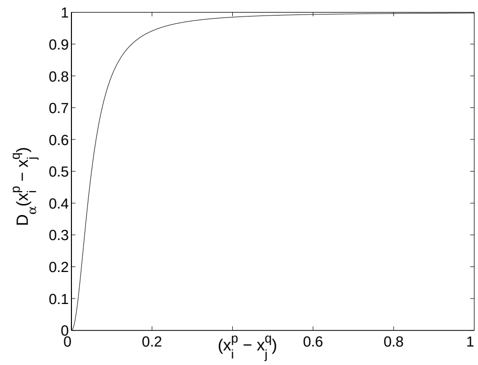

Figure 5. The approximation of the function $D_\circ$.

4.1 Bounds on Performance Loss

As discussed in the previous section, solving the multi-criteria problem can be reformulated by considering the objective terms f^p as constraints. To do this, we define *performance loss factors* L^p that represent the loss in performance of the platform-based products compared to the null platform optima $f^{p,\circ}$. Assuming that the functions f^p, $p = 1,\dots,m$ are minimized, the constraints are defined as:

$$f^p(\mathbf{x}^p) \leq L^p f^{p,\circ} \quad p = 1,\dots,m \tag{6}$$

For example, if a 15% loss in performance for a particular attribute is acceptable, the loss factor is equal to 1.15.

The Pareto set of the multi-criteria problem can be obtained by solving problem (7) for multiple loss factors L^p.

$$\begin{array}{rll} \text{minimize} & \sum_{ijpq} D_\alpha(x_i^p - x_j^q) & \\ \text{with respect to} & \mathbf{x} = [\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^m] & \\ \text{subject to} & \mathbf{g}^p(\mathbf{x}^p) \leq \mathbf{0} & p = 1,\dots,m \\ & \mathbf{h}^p(\mathbf{x}^p) = \mathbf{0} & p = 1,\dots,m \\ & f^p(\mathbf{x}^p) \leq L^p f^{p,\circ} & p = 1,\dots,m \\ & (i,j) \in \hat{\mathcal{S}}^{pq};\ p,q \in \mathcal{P};\ p < q & \end{array} \tag{7}$$

However, it is not necessary to compute the entire Pareto set that represents commonality and performance trade-offs. The loss factors L^p can be used to restrict the loss in performance while designing the platform.

Note that the solution to problem (7) may not be unique. Figure 6 shows the reduced feasible set resulting from the introduction of performance loss constraints. Furthermore, multiple combinations of the same number of shared components can exist; they must be differentiated by their relative performance after solving the product family design problem. It should be emphasized that this step of the methodology is not to find design values of $\mathbf{x}$, but to select the feasible set of commonality constraints.

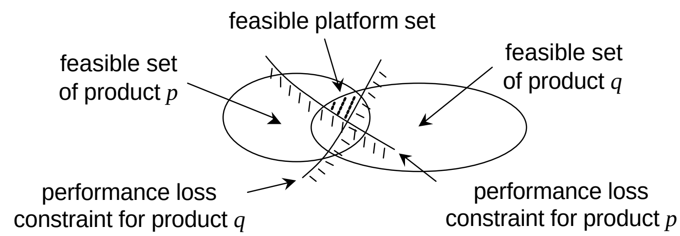

Figure 6. Reduced platform feasible set.

The designer may be able to quantify (e.g., through marketing data or customer preferences) the acceptable performance losses for each variant. The loss factors L^p can thus be considered in the platform selection process as inputs specified by the designer. The optimization problem (7) represents the selection of the platform under performance loss constraints.

4.2 Including Preference for Sharing

The presented commonality decision formulation has multiple terms in the objective function that are equally weighted; there is no bias towards sharing one component over another. However, it is possible to include the designer's preference for sharing in the objective function:

$$\begin{array}{rl} \text{minimize} & \sum_{ijpq} \omega_{ij}^{pq} D_\alpha(x_i^p - x_j^q) \\ & (i,j) \in \hat{\mathcal{S}}^{pq};\ p,q \in \mathcal{P};\ p < q \end{array} \tag{8}$$

The scalar ω_{ij}^{pq} corresponds to the preference on sharing components $\mathbf{x}_i^p$ and $\mathbf{x}_j^q$, which can be chosen using manufacturing or other cost-related criteria in a non-heuristic approach. Including cost may be straightforward by replacing the ω_{ij}^{pq} with the actual cost of the components: If component “1” costs 200 times more to manufacture than component “2”, preference should be placed on sharing the more expensive component. For example, it would most likely be preferable to share the engine over the differential in a family of automobiles.

In this paper we will not examine further the possibility of assigning different weights to the terms of the objective function.

5 MAKING COMMONALITY DECISIONS

The proposed methodology consists of the following steps:

1. Determine the optimal null platform design $f^{p,\circ}$ for each individual product $p \in \mathcal{P}$ by solving the general optimal design

problem (9).

$$\begin{aligned} \text{minimize} \quad & f^p(\mathbf{x}^p) \\ \text{with respect to} \quad & \mathbf{x}^p \\ \text{subject to} \quad & \mathbf{g}^p(\mathbf{x}^p) \leq \mathbf{0} \\ & \mathbf{h}^p(\mathbf{x}^p) = \mathbf{0} \end{aligned} \tag{9}$$

2. Identify the components that could be shared between products, i.e., define the candidate platform set $\hat{\mathcal{S}}^{pq}$ for any two products p and q in the set $\mathcal{P}$. The candidate platform set for the whole product family is then $\hat{\mathcal{S}} = \{\hat{\mathcal{S}}^{pq} \mid p,\ q \in \mathcal{P};\ p < q\}$.
3. Determine the amount of performance loss L^p that is acceptable for each of the products.
4. Formulate and solve the commonality decision problem (7).
5. Based on the results of the commonality decision problem, make a selection of components to be shared, i.e., determine the set $\mathcal{S} = \{\mathcal{S}^{pq} \mid p,\ q \in \mathcal{P};\ p < q\}$ in the following manner: The values of the design variables of the candidate components are compared. These are then assumed to be shared if their relative difference does not exceed an appropriate numerical threshold.
6. Formulate and solve the optimal design problem (10).

$$\begin{aligned} \text{minimize} \quad & ((f^p(\mathbf{x}^p) - f^{p,\circ})/f^{p,\circ})^2 & p = 1,\ldots,m \\ \text{with respect to} \quad & \mathbf{x} = [\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^m] & \\ \text{subject to} \quad & \mathbf{g}^p(\mathbf{x}^p) \leq \mathbf{0} & p = 1,\ldots,m \\ & \mathbf{h}^p(\mathbf{x}^p) = \mathbf{0} & p = 1,\ldots,m \\ & f^p(\mathbf{x}^p) \leq L^p f^{p,\circ} & p = 1,\ldots,m \\ & x_i^p = x_j^q & \\ & (i,j) \in \mathcal{S}^{pq};\ p,q \in \mathcal{P};\ p < q & \end{aligned} \tag{10}$$

Problem (10) is formulated to minimize the distance between the null platform design and the Pareto set associated with the platform determined at Step 4. The performance loss constraints are included in case the Pareto point closest to the null platform lies outside the area of allowable performance losses.

Figure 7 illustrates the above methodology. The conceptual plot shows a null platform point $(f^{A,\circ}, f^{B,\circ})$. The null platform objective function values are multiplied by the performance loss tolerances L^p, and so the region where the associated feasible platforms reside is bounded by the points $(f^{A,\circ}, f^{B,\circ})$, $(L^A f^{A,\circ}, L^B f^{B,\circ})$, $(L^A f^{A,\circ}, f^{B,\circ})$, and $(f^{A,\circ}, L^B f^{B,\circ})$. Solving the commonality decision problem (7) a feasible platform (i.e., common components) is found. The performance loss constraints (6) in (7) will always be active (unless they are dominated by design constraints) and the obtained designs will correspond to the objective function values $L^A f^{A,\circ}$ and $L^B f^{B,\circ}$. Therefore, the product family optimal design problem (10) is solved to obtain a Pareto-optimal design for the platform. Problem (10) is formulated to yield the point closest to the null platform design.

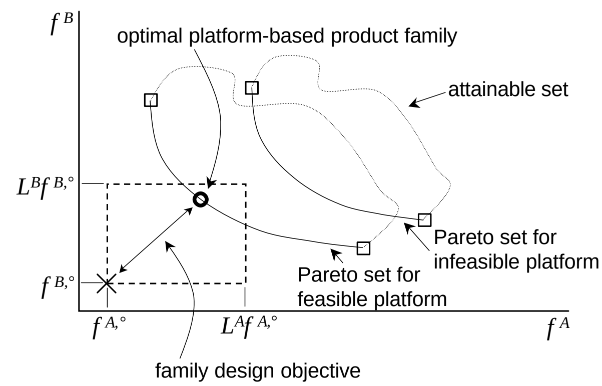

Figure 7. Design Process of Proposed Methodology

6 DESIGNING A FAMILY OF AUTOMOTIVE VEHICLE SIDE FRAMES

We consider a family of side frames for an automotive vehicle with two variants. Recall that a variant can be defined by changing the functional requirements and/or the geometry of the model. In this study variants are based on differing functional requirements for mass and stiffness. Variants A and B are designed for minimum mass and maximum stiffness (minimum deflection), respectively.

6.1 Side Frame Model Description

A simple two-dimensional model was chosen for computational efficiency purposes. The side frame of the automotive body is modeled as an assembly of ten beam elements and seven flexible joints (*cf.* Figure 8). A MATLAB©-based finite element solver is used to compute deflections, stresses, and body mass for different values of the cross-sectional parameters of the beams (width b, height h, and thickness t). Two loading cases (bending and torsion) are considered, as depicted in Figure 8 where $F = 1500$ lbs and $T = 1650$ lbs. The engine and rear compartments are included in the model as reaction forces applied at the centerpoints of the A and C pillars for the bending load case. Torsion is represented by a horizontal force applied at the joint connecting the B pillar and the roof; this force simulates the shear that the structure undergoes under such loading. Displacements are computed at the four locations shown in Figure 8. Total bending displacement is then calculated as $\delta_b = \lambda\Delta_1 + \mu\Delta_2 + \Delta_3$. The roof and the rocker consist of two beam elements each. This re-

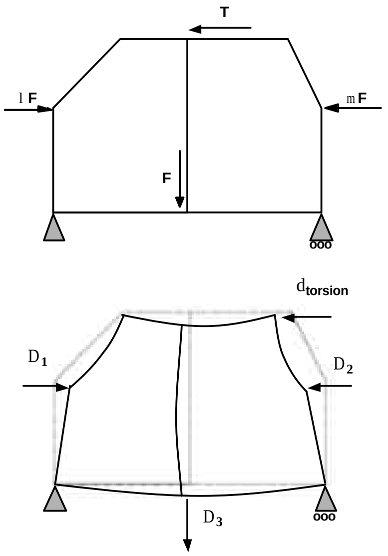

Figure 8. Two-dimensional automotive side frame model.

Table 1. Variant optimal design problems and null platform optima.

Variant	A	B
Objective	min m	min $\delta_b + \delta_t$
Constraints	$\delta_b \leq 0.4$ in $\delta_t \leq 0.8$ in	$m \leq 250$ lbs
Optimum	53.4 lbs	0.587 in

sults in a side frame consisting of eight components. Each component is represented by three design variables. This means that a component can be shared within the family if all three design variables have equal values. However, it may be possible to consider some other form of sharing if only one or two design variables have equal values, for example, from a manufacturing point of view as discussed in Section 2. In this regard, all variables are treated as platform candidates in this study.

6.2 Null platform computation

At first the optimal design problem is solved individually for each variant to obtain null platform optima $f^{p,o}$. Two variants are considered: Variant A is designed with a minimum mass objective subject to a stiffness constraint represented by deflections; variant B is designed with a maximum stiffness objective subject to a mass constraint. Two maximal stress constraints for each beam element are taken into account for all three variants. The computed null platform optima for the considered variants are given in Table 1.

6.3 Commonality Decision and Family Optimization

Having computed the null platform optima $f^{p,o}$, the commonality decision problem (7) can be solved for different values of the performance loss tolerances L^p. Experience to date shows that 0.025 is a good value for the parameter α in Equation (5). Further investigation is necessary to determine a recommended value of α for general use.

In the side frame problem there are 24 design variables representing the cross-sectional variables b (width), h (height), and t (thickness) of the beams. The candidate platform set $\hat{\mathcal{S}}$ is defined by allowing each design variable in variant A to be shared only with the same variable of the corresponding component in variant B. Therefore there are 2^{24} possible sharing combinations (product platforms).

Problem (7) is solved to determine the "maximal" feasible platform under performance loss constraints. The performance loss tolerances L^p are set equal for both variants, i.e., $L^A = L^B = L$. A relative threshold of 0.5% is used to determine which variables will be shared among the two variants. Note that twelve variables were "naturally" shared by inspecting the null platform designs, i.e., they had the same optimal values in both designs. Allowing a performance loss of 1%, 5%, 10%, 20%, and 50% resulted in a commonality decision of sharing 17, 20, 21, 22, and 24 variables, respectively. The trade-off between sharing and performance is shown in Figure 9. This trade-off is

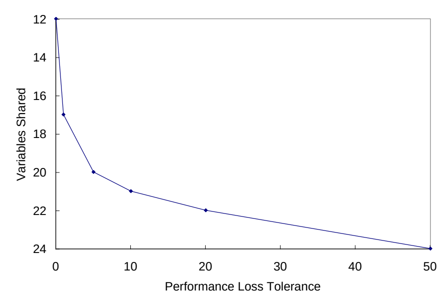

Figure 9. Trade-offs between commonality and performance.

analogous to the Pareto set that could be generated solving the combinatorial optimal design problem (2).

The optimal product family design problem (10) is solved next. The optimal objective function values are presented in Table 2. All platforms that were determined by solving the commonality decision problem (7) are within the specified loss tolerances after solving the product family design problem. The results in Table 2 lead us to believe that solutions with more shared components are not being overlooked. The optimal values of the product family design variables based on the null platform, the platform determined by accepting a 5% performance loss, and the total platform are compared in Table 3.

Table 2. Optimal product family design results and associated performance losses.

Variant	A	B
Null platform	53.4 lbs	0.587 in
Platform of 17 variables	53.9 lbs	0.593 in
Performance loss	0.999%	0.972%
Platform of 20 variables	55.6 lbs	0.616 in
Performance loss	4.17%	5.01%
Platform of 21 variables	56.0 lbs	0.646 in
Performance loss	4.87%	9.96%
Platform of 22 variables	58.3 lbs	0.687 in
Performance loss	9.20%	17.00%
Total platform of 24 variables	73.2 lbs	0.881 in
Performance loss	37.1%	50.0%

6.4 Validation of Platform Solutions

It is too computationally expensive to solve the original combinatorial commonality problem, therefore a reduced version of the (2) is solved to validate the results. It is assumed that "naturally shared" variables, as determined by the individual variant optimizations, will remain shared. It has been observed that twelve out of the sixteen width b and height h variable values of the two variants are always equal. This motivated us to consider all width and height variables shared, and reduce the size of the combinatorial problem by defining the platform to include these sixteen variables. The problem size is thus reduced to 2^8 platform combinations.

A "top-down" algorithm is implemented by starting with sharing all eight component thicknesses (total platform). If the performance loss constraints cannot be satisfied, we move a "level" down by decreasing the number of candidates for sharing from eight to seven; eight different platforms of sharing seven thicknesses are now considered, and so on, until at least one platform is found that satisfies the performance loss constraints for a given "level" . Note that it is possible that more than one platforms may satisfy the performance loss constraints at a given "level".

Using a loss preference of 1%, 5% or 50%, the same unique platform is obtained by solving the commonality decision problem (7). Given a loss preference of 10% and 20%, two and seven platforms were found, respectively. It is encouraging that the platform obtained by solving the commonality decision problem (7) is the one that corresponds to least performance loss for both cases. Intuitively this makes sense: Since problem (7) is solved over a continuous design space, it is natural that components that have less impact (sensitivity) on the performance will be shared first. Note that the number of feasible platforms increases with increasing value of performance loss tolerance. This fact is related to our previous discussion of the feasible platform space shown in Figure 6.

6.5 Practical Considerations

Gradient-based algorithms are recommended to solve the commonality decision problem (7). It is advised to compute gra-

Table 3. Optimal values of the product family design variables for different platforms.

Platform:			null		17 variables		total	
$\mathbf{x}^l$	$\mathbf{x}_0$	$\mathbf{x}^u$	$\mathbf{x}^{A,\circ}$	$\mathbf{x}^{B,\circ}$	$\mathbf{x}^{A,*}$	$\mathbf{x}^{B,*}$	$\mathbf{x}^{A,*}$	$\mathbf{x}^{B,*}$
1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
2.00	4.00	4.40	4.40	4.40	4.40	4.40	4.40	4.40
0.04	0.10	0.50	0.04	0.46	0.04	0.47	0.08	0.08
0.75	1.50	1.65	1.65	1.65	1.65	1.65	1.65	1.65
0.50	1.00	1.10	1.10	1.10	1.10	1.10	1.10	1.10
0.04	0.10	0.50	0.19	0.37	0.19	0.39	0.18	0.18
1.00	2.00	2.20	1.51	2.07	2.08	2.08	2.20	2.20
2.00	4.00	4.40	4.40	4.40	4.40	4.40	4.40	4.40
0.04	0.10	0.50	0.04	0.23	0.04	0.25	0.05	0.05
0.50	1.00	1.10	1.10	1.07	1.09	1.09	1.10	1.10
0.50	1.00	1.10	1.10	1.10	1.10	1.10	1.10	1.10
0.04	0.30	0.50	0.04	0.28	0.06	0.06	0.06	0.06
1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
1.50	3.00	3.30	3.30	3.30	3.30	3.30	3.30	3.30
0.04	0.10	0.50	0.12	0.41	0.12	0.41	0.12	0.12
0.50	1.00	1.10	1.10	1.10	1.10	1.10	1.10	1.10
1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
0.04	0.10	0.50	0.09	0.32	0.09	0.34	0.09	0.09
1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
1.00	2.00	2.20	2.20	2.20	2.20	2.20	2.20	2.20
0.04	0.10	0.50	0.11	0.50	0.11	0.50	0.18	0.18
0.50	1.00	1.10	0.50	1.01	0.96	0.96	1.08	1.08
1.50	3.00	3.30	2.96	3.30	2.95	2.95	3.30	3.30
0.04	0.10	0.50	0.04	0.18	0.04	0.23	0.04	0.04

dients analytically. The use of the function D_α as defined in Equation (5) makes this possible. In the case that simulation-based models are used to evaluate the performance constraints, it is important to use appropriate finite-difference steps. For the application considered in this paper, we used the MATLAB© implementation of the sequential quadratic programming (SQP) algorithm. Design variables, objective, and constraints were scaled to the order of one.

Solving the commonality decision problem (7) may become inefficient as the number of products and/or the number of components that can be shared increases dramatically. Methods based on the use of sensitivity analysis (Fellini *et al.* 2002) can be used to reduce the order of the problem by filtering out components with low impact on sharing.

7 CONCLUSIONS

The proposed methodology integrates platform selection under performance loss constraints with optimal design of product families. The designer can decide (e.g., by using business and marketing data) what performance losses are acceptable relative to individual product variant optimality. Component sharing is determined through the solution of the relaxed and reformulated commonality maximization combinatorial problem subject to these performance loss bounds. The formulation of the commonality decision problem allows the designer to assign different weights to express sharing preferences. The optimal product family design problem is solved to obtain a point on the Pareto set associated with the determined platform that is closest to the null platform point. A family of automotive vehicle side frames has been used to demonstrate the proposed methodology. Results have been validated by comparing them to the ones obtained by solving a reduced version of the original combinatorial commonality problem. It can be concluded that the methodology can be helpful in addressing the platform selection problem, which may be intractable in its original combinatorial form.

ACKNOWLEDGMENTS

This research was partially supported by the Automotive Research Center, a US Army Center of Excellence in Modeling and Simulation of Ground Vehicles, by a Dual-Use Science and Technology Project, and by the General Motors Collaborative Research Laboratory at the University of Michigan. N. Kikuchi provided the automotive side frame model. This support is gratefully acknowledged.

REFERENCES

Conner, C.G., De Kroon, J.P., and Mistree, F., 1999, "A Product Variety Tradeoff Evaluation Method for a Family of Cordless Drill Transmissions," *Proceedings of the 1999 ASME Design Engineering Technical Conferences*, paper DAC-8625, Las Vegas, NV.

Fellini, R., Kokkolaras, M., Michelena, N., Papalambros, P., Saitou, K., Perez-Duarte, A., Fenyes, P., 2002, "A Sensitivity-Based Commonality Strategy for Family Products of Mild Variation, with Application to Automotive Body Structures," *Proceedings of the 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, paper AIAA-2002-5610, Atlanta, GA, to appear.

Fellini, R., Papalambros, P.Y., and Weber, T., 2000, "Application of a Product Platform Design Process to Automotive Powertrains," *Proceedings of the 8th AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, paper AIAA-2000-4849, Long Beach, CA.

Fujita, K., and Yoshida, H., 2001, "Product Variety Optimization: Simultaneous Optimization of Module Combination and Module Attributes," *Proceedings of the 2001 ASME Design Engineering Technical Conferences*, paper DAC-21058, Pittsburgh, PA.

Gonzalez-Zugasti, J.P., Otto, K.N., and Baker, J.D., 1998, "A Method for Architecting Product Platforms with an Application to Interplanetary Mission Design," *Proceedings of the 1998 ASME Design Engineering Technical Conferences*, paper DAC-5608, Atlanta, GA.

Gonzalez-Zugasti, J.P. and Otto, K.N., 2000, "Modular Platform-Based Product Family Design," *Proceedings of the 2000 ASME Design Engineering Technical Conferences*, paper DAC-14238, Baltimore, MD.

Kokkolaras, M., Fellini, R., Kim, H.M., Michelena, N.F., and Papalambros, P.Y., 2002, "Extension of the Target Cascading Formulation to the Design of Product Families," *Journal of Structural and Multidisciplinary Optimization*, to appear.

Messac, A., Martinez, M.P., and Simpson, T.W., 2000, "Effective Product Family Design Using Physical Programming and the Product Platform Concept Exploration Method," *Proceedings of the 2000 ASME Design Engineering Technical Conferences*, paper DAC-14252, Baltimore, MD.

Nayak, R.U., Chen, W., and Simpson, T.W., 2000, "A Variation-Based Methodology for Product Family Design," *Proceedings of the 2000 ASME Design Engineering Technical Conferences*, paper DAC-14264, Baltimore, MD.

Nelson, S.A., Parkinson, M.B., and Papalambros, P.Y., 2001, "Multicriteria Optimization in Product Platform Design," *Journal of Mechanical Design*, 123(2):199-204.

Simpson, T.W., 1998, "A Concept Exploration Method for Product Family Design," PhD thesis, Georgia Instutute of Technology, Atlanta, GA.

Simpson, T.W., Maier, J.R.A., and Mistree, F., 1999, "A Product Platform Concept Exploration Method for Product Family Design," *Proceedings of the 1999 ASME Design Engineering Technical Conferences*, paper DTM-8761, Las Vegas, NV.