Target Cascading for Design of Product Families

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Abstract
The target cascading methodology for product development is extended to product families with predefined platforms. The single-product formulation is modified to accommodate the presence of shared systems, subsystems, and/or components. Hierarchical optimization problems associated with each product variant are combined to formulate the product family multicriteria design problem, and common subproblems are identified based on the shared elements (i.e., the platform). The solution of the overall design problem is coordinated so that the shared elements are consistent with the performance and behavior of the product variants. A simple automotive design example is used to demonstrate the proposed methodology.
Keywords: Product platforms, product families, commonality, target cascading, systems design optimization.

1. Introduction

Product platforms enable rapid enrichment of a product portfolio to meet changing market needs while keeping design and manufacturing cycle times and costs low [1,2]. Product families share by definition one or more subsystems or components; therefore, the development of a product family depends on an appropriate design strategy that addresses the commonality of parts (i.e., the platform). Different product variants are typically characterized by conflicting performance criteria. A multicriteria optimization formulation can identify the associated tradeoffs by means of Pareto surfaces, with commonality constraints used to define the common elements of the various product variants. Design decisions can then be made based on subjective engineering, marketing, and manufacturing criteria.

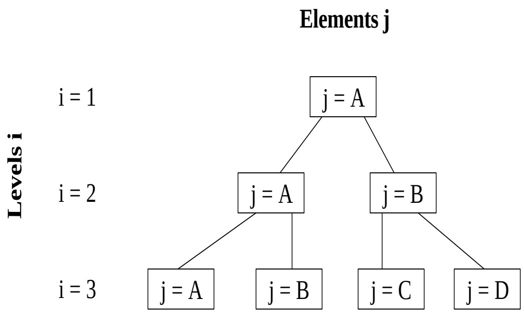
The development of any complex product is strongly associated with efforts of matching specifications for each of the product's attributes. Analytical target cascading [3,4] is a novel methodology for the design of large engineering systems at the early product development stages. First, the design problem is partitioned into a hierarchical set of subproblems associated with systems, subsystems, and components. The desired design specifications (or targets) are then defined at the top level of a multilevel design formulation and "cascaded down" to lower levels. Design subproblems are formulated at each level so that the components, subsystems, and systems are designed to match the cascaded targets consistent with the overall system targets. The main benefits of target cascading are reduction in design-cycle time, avoidance of design iterations late in the development process, and increased likelihood that physical prototypes will be closer to production quality. Target cascading also facilitates concurrency in system design: Once targets are identified for systems, subsystems, and components, the latter can be isolated and designed in detail independently, allowing the outsourcing of subsystems and components to suppliers. Target cascading offers a robust framework for multilevel design and has been demonstrated to be convergent, whereas other similar problem formulations exhibit convergence difficulties.

Previous work had addressed product family design [5,6] and target cascading separately. In this work, the target cascading methodology is extended to product family design.

2. Target Cascading in Product Development

The analytical target cascading process was presented in [3,4] in the context of automotive engineering systems. In this paper a more general notation is introduced, from which the design problem for each element (i.e., system, subsystem, or component) can be recovered as a special case. To represent the hierarchy of the partitioned design problem, the set $\mathcal{E}_i$ is defined at each level i, in which all the elements of the level are included. For each element j in the set $\mathcal{E}_i$, the set of children $\mathcal{C}_{ij}$ is defined, which includes the elements of the set $\mathcal{E}_{i+1}$ that are children of the element. An illustrative example is presented on the left side of Figure 1: At level $i = 2$ of the partitioned problem we have $\mathcal{E}_2 = \{\text{A, B}\}$, and for element "B" on that level we have $\mathcal{C}_{2\text{B}} = \{\text{C, D}\}$.

There are two types of responses: responses $\tilde{R}$ linked to "local" targets (e.g., at the top level), and responses R linked to "cascaded" targets, i.e., linking two successive levels in the problem hierarchy. The design problem P_{ij} corresponding to the j-th element at the i-th level is formulated as follows:



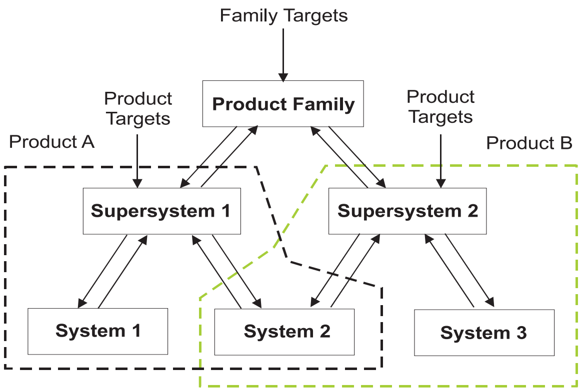


Figure 1: Examples of hierarchically partitioned optimal design problems: single product (left) and product family (right).

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\begin{aligned}
\min_{\bar{x}_{ij}} \; & \|\tilde{R}_{ij} - T_{ij}\| + \|R_{ij} - R^U_{ij}\| + \|y_{ij} - y^U_{ij}\| + \epsilon^R_{ij} + \epsilon^y_{ij} \\
\text{subject to} \; & \sum_{k \in \mathcal{C}_{ij}} \|R_{(i+1)k} - R^L_{(i+1)k}\| \le \epsilon^R_{ij} \\
& \sum_{k \in \mathcal{C}_{ij}} \|y_{(i+1)k} - y^L_{(i+1)k}\| \le \epsilon^y_{ij} \\
& g_{ij}(\hat{R}_{ij}, x_{ij}, y_{ij}) \le 0, \\
& h_{ij}(\hat{R}_{ij}, x_{ij}, y_{ij}) = 0,
\end{aligned}
\tag{1}
$$

where $\hat{R}_{ij} = \left[\tilde{R}_{ij}, R_{ij}\right]^T = r_{ij}(R_{(i+1)k_1}, \ldots, R_{(i+1)k_{c_{ij}}}, x_{ij}, y_{ij})$, $\mathcal{C}_{ij} = \{k_1, \ldots k_{c_{ij}}\}$, and c_{ij} is the number of child elements. In the above problem formulation,

- $\bar{x}_{ij} = \left[x_{ij}, y_{ij}, y_{(i+1)k_1}, \ldots, y_{(i+1)k_{c_{ij}}}, R_{(i+1)k_1}, \ldots, R_{(i+1)k_{c_{ij}}}, \epsilon^R_{ij}, \epsilon^y_{ij}\right]^T \in \mathbb{R}^{z_{ij}}$ is the vector of all optimization variables where $z_{ij} := n_{ij} + l_{ij} + 2 + \sum_{k \in \mathcal{C}_{ij}} l_{(i+1)k} + \sum_{k \in \mathcal{C}_{ij}} d_{(i+1)k}$,
- $x_{ij} \in \mathbb{R}^{n_{ij}}$ is the vector of local design variables, that is, variables exclusively associated with the element,
- $y_{ij} \in \mathbb{R}^{l_{ij}}$ is the vector of linking design variables, that is, variables associated with two or more elements that share the same parent,
- $\hat{R}_{ij} \in \mathbb{R}^{\hat{d}_{ij}}$ is the vector of responses, where $\tilde{R}_{ij}$ corresponds to responses linked to local targets, while R_{ij} corresponds to responses linked with cascaded targets ($\hat{d}_{ij} := \tilde{d}_{ij} + d_{ij}$). As shown above, $\hat{R}_{ij}$ is a function of local and linking design variables and responses of child elements: $r_{ij} : \mathbb{R}^{n_{ij} + l_{ij} + \sum_{k \in \mathcal{C}_{ij}} d_{(i+1)k}} \to \mathbb{R}^{\hat{d}_{ij}}$,
- $\epsilon^R_{ij} \in \mathbb{R}_+ \bigcup \{0\}$ is the tolerance optimization variable for coordinating the responses of the children of the element,
- $\epsilon^y_{ij} \in \mathbb{R}_+ \bigcup \{0\}$ is the tolerance optimization variable for coordinating the linking design variables of the children of the element,
- $T_{ij} \in \mathbb{R}^{\tilde{d}_{ij}}$ is the vector of local target values,
- $R^U_{ij} \in \mathbb{R}^{d_{ij}}$ is the vector of response values cascaded down to the element from its parent,
- $y^U_{ij} \in \mathbb{R}^{l_{ij}}$ is the vector of linking design variable values cascaded down to the element from its parent,
- $R^L_{(i+1)k} \in \mathbb{R}^{d_{(i+1)k}}$ is the vector of response values cascaded up to the element from its k-th child,
- $y^L_{(i+1)k} \in \mathbb{R}^{l_{(i+1)k}}$ is the vector of linking design variable values cascaded up to the element from its k-th child,

- $g_{ij} : \mathbb{R}^{\hat{d}_{ij}+n_{ij}+l_{ij}} \rightarrow \mathbb{R}^{v_{ij}}$ and $h_{ij} : \mathbb{R}^{\hat{d}_{ij}+n_{ij}+l_{ij}} \rightarrow \mathbb{R}^{w_{ij}}$ are vector functions representing inequality and equality design constraints, respectively, and

- $\| \cdot \|$ is some norm; typically, some weighted norm is used for the metrics involving local targets T_{ij} in order to enable trade-off evaluation studies, while the l_2-norm is used in all other cases.

In the above definitions it is assumed for simplicity that all problems are continuous, but the formulation holds if some optimization variables are discrete. In the latter case, suitable optimization algorithms are necessary for the solution of the associated mixed-integer programming problems.

Non-ascent properties for the analytical target cascading process based on hierarchical overlapping coordination convergence theory [7] are presented in [3] under the assumption that Problem (1) is convex.

3. Target Cascading in Product Family Development

When designing a family of platform-based products, one must identify the tradeoffs that are a consequence of shared attributes. These tradeoffs exist because the products are no longer optimized for their individual performance, but for the family as a whole. The shared elements of the product family influence the performance of the individual products.

A Pareto-based approach that quantifies the aforementioned tradeoffs was proposed in [5]. The first step in this method is to formulate individual design problems for each product. The problems are then combined into one family problem using a multicriteria optimization formulation that includes the individual product requirements. By introducing an equality commonality constraint one can specify the product elements to be shared. By solving the multicriteria problem for the Pareto set one can visualize the design tradeoffs, and choose a suitable design.

In order to exploit a product family structure, the design problem is formulated as a hierarchical optimization problem [6]. The top level problem addresses family attributes, while lower levels (including the product level) address attributes associated with particular elements. Given a set of family and individual product targets, analysis models for all design elements, and a predefined platform, targets are cascaded to elements below in the hierarchy (i.e., system, subsystem, and component targets are determined).

The rest of this section discusses modifications to the single product target cascading formulation necessary for the design of product families. These modifications enable subproblems to return design response and linking variable values to multiple parents. The right side of Figure 1 illustrates a simple example of a product family with two product variants; each variant is partitioned into two systems, and the two variants share one system. Desired product family design specifications are defined as targets at the top level and cascaded to lower levels. Product variant targets are introduced at the product level. Design subproblems are formulated at each level so that components, subsystems, and systems are designed to match the cascaded targets while the overall system is consistent. In order to allow for elements to be shared (i.e., to have multiple parents), the set of parents $\mathcal{P}_{ij}$ is defined for each element j of the set $\mathcal{E}_i$ at every level i; this set includes the elements of the set $\mathcal{E}_{i-1}$ that are parents of this element. The design problem P_{ij} for the j-th element at the i-th level is reformulated as follows:

$$
\begin{aligned}
\min_{\bar{x}_{ij}} \; & \|\tilde{R}_{ij} - T_{ij}\| + \sum_{q \in \mathcal{P}_{ij}} \|R_{ij} - R^U_{ijq}\| + \sum_{q \in \mathcal{P}_{ij}} \|y_{ij} - y^U_{ijq}\| + \epsilon^R_{ij} + \epsilon^y_{ij} \\
\text{subject to} \; & \sum_{k \in \mathcal{C}_{ij}} \|R_{(i+1)k} - R^L_{(i+1)k}\| \leq \epsilon^R_{ij} \\
& \sum_{k \in \mathcal{C}_{ij}} \|y_{(i+1)k} - y^L_{(i+1)k}\| \leq \epsilon^y_{ij} \\
& g_{ij}(\hat{R}_{ij}, x_{ij}, y_{ij}) \leq 0, \\
& h_{ij}(\hat{R}_{ij}, x_{ij}, y_{ij}) = 0,
\end{aligned}
\tag{2}
$$

with $\mathcal{P}_{ij} = \{q_1, \ldots, q_{p_{ij}}\}$, where p_{ij} is the number of parent elements, and where

- $R^U_{ijq} \in \mathbb{R}^{d_{ij}}$ is the vector of response values cascaded to the element from its q-th parent,

- $y^U_{ijq} \in \mathbb{R}^{l_{ij}}$ is the vector of linking design variable values cascaded to the element from its q-th parent.

It can be readily shown that the target cascading formulation for optimal single product design shown in Problem (1) is recovered if all the sets of parents $\mathcal{P}_{ij}$ consist of only one element.

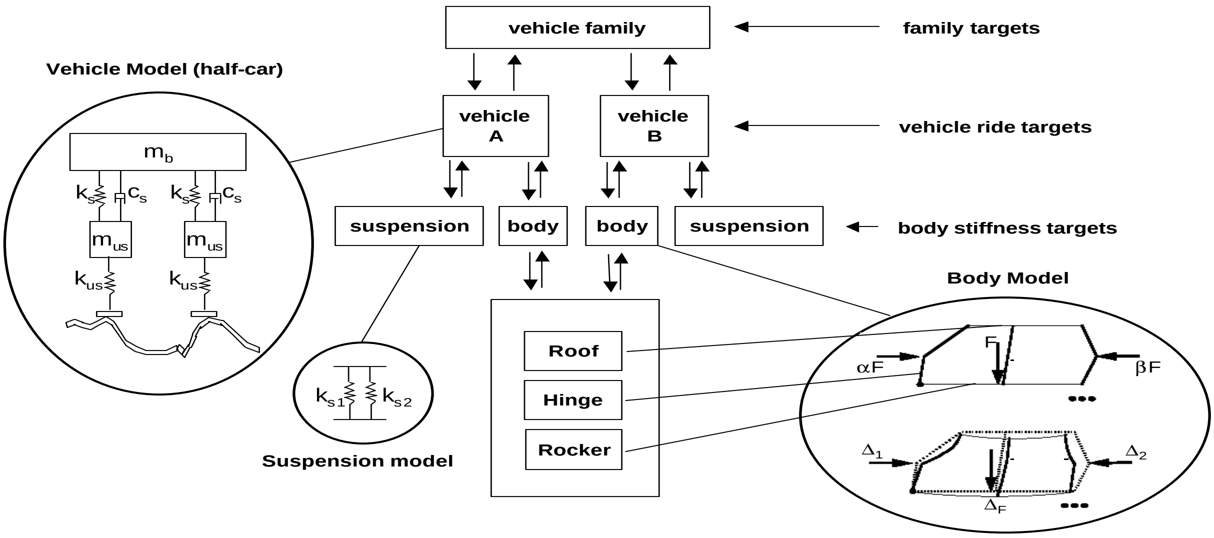

Figure 2: Schematic of the case study.

4. Case Study

To illustrate the use of target cascading to design a family of products, a multi-vehicle design problem has been formulated. The individual vehicles are modelled as half-cars (cf. Figure 2). Body and suspension models at the subassembly level provide responses to the half-car models. The body is represented by a finite element model consisting of ten elements. These ten elements model eight members of a two-dimensional body including the A pillar, B pillar, hinge pillar, and so on. Each member is described by its cross-sectional properties, area, and moment of inertia, which are functions of sheet metal thicknesses. The joints linking elements are modelled as radial springs. The suspension model computes sprung stiffness for the half-car model based on the stiffness of two individual springs.

The product platform for the family consists of three body members: namely, the roof, rocker, and hinge pillar. At the vehicle level, targets are set for the ride quality of each variant. Ride quality is defined by the following six responses: front and rear ride frequency, front and rear wheel hop frequency, pitch frequency, and under-steer gradient. Vehicle A should have a stiffer ride, whereas vehicle B should have a softer ride. The local target for the body is to maximize the stiffness of the structure by minimizing deflection. At the family level an objective is defined that combines the masses of both vehicle variants. More emphasis is given to minimizing the mass of vehicle A.

Figure 3 illustrates the overall target cascading formulation and coordination. The shared pillars return a common response to the body models of both vehicle variants. To solve the multilevel problem a generic coordination strategy is implemented that starts at the top-most level. Each level is solved in sequence, and then the problems are solved once again by returning to the top level problem. This process is counted as one iteration, and convergence is tested by checking if deviation terms are sufficiently reduced.

The obtained results are presented in Table 1, and include the optimal design values for the shared body components. By comparing the target and optimal values for the relevant metrics, it can be seen that a) the combined mass is minimized at the family level (of course, the target cannot be achieved), b) the ride quality and handling targets for the two variants at the product level are matched quite well with the exception of pitch frequency, and c) the body deflections at the system level are also minimized (although a zero deflection cannot be achieved), maximizing thus the stiffness of the body. Ellipses facilitate the comparison of the values for the cascaded responses. It can be seen how they are matched at the optimal design. For simplicity the design variables for the pillars are shown in the table as being cascaded up as responses to the body instead of the mass moments of inertia, which are explicit functions of the thicknesses. Specifications for mass and ride quality for the vehicles, mass and maximum deflection for the bodies, stiffness for the suspension, and cross-sectional properties for the pillars are the main outcome of the target cascading process.

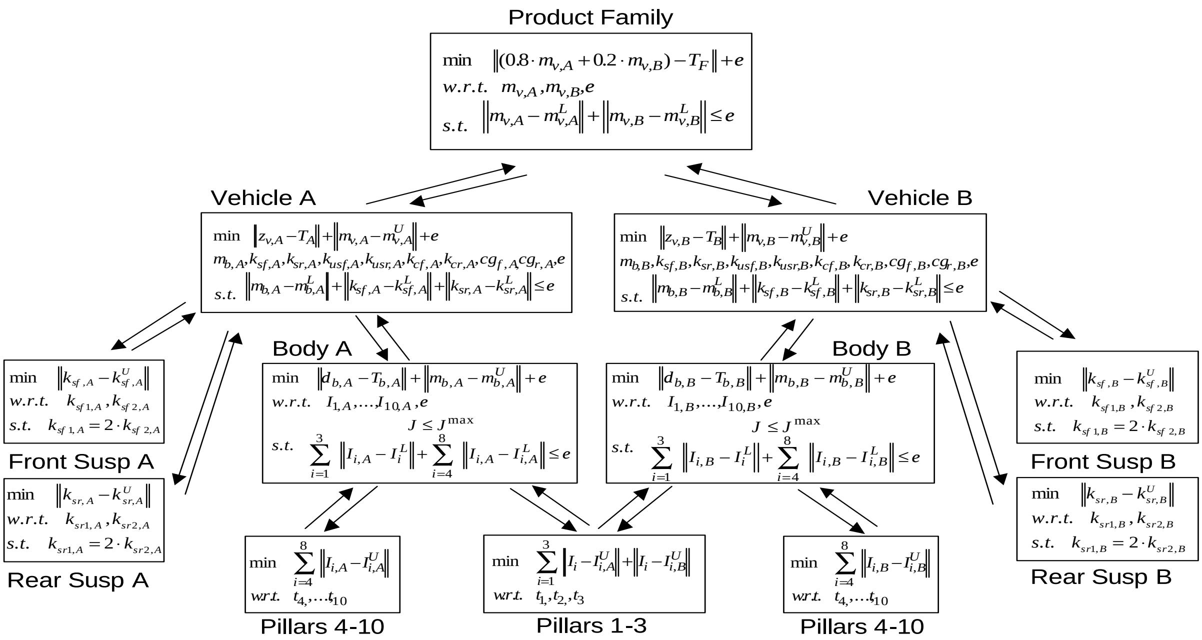

Figure 3: Target cascading formulation and coordination.

5. Conclusions

The target cascading methodology is proposed for design of product families. Given a platform, hierarchical partitions of the individual product design problems, and the necessary analysis models, family and product targets are cascaded down to systems, subsystems, and components. In this manner design specifications are determined for all elements, including the product platform. The information flow within the coordination strategy is based on the non-hierarchical multilevel structure underlying the family design problem. The technique is successfully applied to a simple automotive example of two vehicles that share body components.

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Table 1: Target cascading results.

	Initial Values		Targets		Optimal Values	
Family Responses						
combined mass (lbs)	2150		0		2124	
Family Variables	Vehicle A	Vehicle B	Vehicle A	Vehicle B	Vehicle A	Vehicle B
vehicle mass (lbs)	2150	2150			2140	2060
Vehicle Responses						
vehicle mass (lbs)	2150	2150			2140	2060
front ride frequency (Hz)	1.210	1.210	1.273	0.955	1.147	0.948
rear ride frequency (Hz)	1.625	1.625	1.592	1.592	1.539	1.565
front wheel hop frequency (Hz)	9.428	9.428	10.345	10.345	10.113	10.237
rear wheel hop frequency (Hz)	9.428	9.428	10.345	9.549	10.229	9.474
pitch frequency (Hz)	0.884	0.884	0.500	0.477	0.817	0.718
under-streer gradient (rad/m/s^2)	0.001	0.001	0.007	0.007	0.007	0.007
Vehicle Variables						
cg distance to front (m)	1.320	1.320			1.250	1.250
cg distance to rear (m)	2.380	2.380			2.310	2.310
front suspension stiffness (N/mm)	40.000	40.000			36.076	23.736
rear suspension stiffness (N/mm)	40.000	40.000			35.131	34.990
front tire stiffness (N/mm)	20.000	20.000			23.015	23.580
rear tire stiffness (N/mm)	20.000	20.000			23.545	20.196
front cornering stiffness (N/rad/10e-4)	10.000	10.000			11.095	10.218
rear cornering stiffness (N/rad/10e-4)	10.000	10.000			12.484	10.943
body-in-white mass (lbs)	250	250			240	160
Body Responses						
body-in-white mass (lbs)	143	143			240	161
deflection by vertical loading (in)	0.353	0.353	0	0	0.306	0.335
deflection by horizontal loading (in)	0.728	0.728	0	0	0.520	0.640
Body Variables						
rocker (in)	0.100	0.100			0.122	0.119
roof rail (in)	0.100	0.100			0.240	0.239
hinge pillar (in)	0.100	0.100			0.083	0.083
A pillar (in)	0.300	0.300			0.186	0.109
B pillar below belt (in)	0.100	0.100			0.381	0.150
B pillar above belt (in)	0.100	0.100			0.284	0.103
C pillar below belt (in)	0.100	0.100			0.400	0.114
C pillar above belt (in)	0.100	0.100			0.102	0.050
Front Suspension Responses						
suspension stiffness(N/mm)	40.000	40.000			36.045	24.000
Front Suspension Variables						
spring stiffness 1 (N/mm)	9.000	9.000			24.030	16.000
spring stiffness 2 (N/mm)	9.000	9.000			12.015	8.000
Rear Suspension Responses						
suspension stiffness(N/mm)	40.000	40.000			35.154	34.989
Rear Suspension Variables						
spring stiffness 1 (N/mm)	9.000	9.000			23.436	23.326
spring stiffness 2 (N/mm)	9.000	9.000			11.718	11.663
Pillar Variables						
rocker (in)	0.100				0.119	
roof rail (in)	0.100				0.240	
hinge pillar (in)	0.100				0.084	
A pillar (in)	0.300	0.300			0.186	0.109
B pillar below belt (in)	0.100	0.100			0.381	0.150
B pillar above belt (in)	0.100	0.100			0.284	0.103
C pillar below belt (in)	0.100	0.100			0.400	0.114
C pillar above belt (in)	0.100	0.100			0.102	0.050