**Hyung Min Kim**
Research Fellow
e-mail: kimhm@umich.edu

**Nestor F. Michelena**
Research Scientist
e-mail: nestorm@umich.edu

**Panos Y. Papalambros**
Professor
e-mail: pyp@umich.edu

Optimal Design Laboratory,
Department of Mechanical Engineering,
University of Michigan,
Ann Arbor, MI 48109

**Tao Jiang**
Supervisor
e-mail: tjiang@ford.com
Core Vehicle Integration Section,
North American Family Cars,
Ford Motor Company,
Dearborn, MI 48121

# Target Cascading in Optimal System Design

*Target cascading is a key challenge in the early product development stages of large complex artifacts: how to propagate the desirable top level design specifications (or targets) to appropriate specifications for the various subsystems and components in a consistent and efficient manner. Consistency means that all parts of the designed system should work well together, while efficiency means that the process itself should avoid iterations at later stages, which are costly in time and resources. In the present article target cascading is formalized by a process modeled as a multilevel optimal design problem. Design targets are cascaded down to lower levels using partitioning of the original problem into a hierarchical set of subproblems. For each design problem at a given level, an optimization problem is formulated to minimize deviations from the propagated targets and thus achieve intersystem compatibility. A coordination strategy links all subproblem decisions so that the overall system performance targets are met. The process is illustrated with an explicit analytical problem and a simple automotive chassis design model that demonstrates how the process can be applied in practice.* [DOI: 10.1115/1.1582501]

## Introduction

Much of the motivation for the work described in this article comes from recent efforts in the automotive industry to formalize the product development process and to take full advantage of computer-aided engineering (CAE) tools. A complex product development process is most efficient when the various required design tasks can be accomplished in a truly ''concurrent'' manner. In some sense, concurrency means that tasks should be conducted in parallel without isolation from each other, so that decisions are made taking all product requirements into account. Product teams of diverse specialists are formed and communicate regularly. Ironically, experience shows that such efforts also have serious adverse effects because decisions are not made with sufficient speed when trying to satisfy everyone's viewpoint. In another sense, then, concurrency means that each task should be conducted in isolation or with the least amount of interaction with other tasks, so that each specialist or team can concentrate on their own specialized task. Yet, clearly, interactions do exist, and task isolation could lead to costly downstream iterations.

The solution to this dilemma is to identify the key links among design tasks, reach the associated trade-off decisions early, and then let the individual tasks be conducted separately. The target cascading process attempts to achieve just that. Subsequent interactions can take place when the specifications given to each task are difficult or impossible to meet and another round of joint decisions is necessary. Important specifications for the entire system as well as for each system element (subsystems and components) are first identified, particularly those that will have influence on other parts of the system. Then, specification target values are assigned at the top level of the system, usually based on management criteria. These targets are propagated to the rest of the system, and appropriate values are determined for the expected performance of each element of the system. Design tasks are executed for each individual element, and interaction with the rest of the system is revisited only when a target cannot be met.

Contributed by the Design Automation Committee for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received July 2001; rev. October 2002. Associate Editor: J. Renaud.

Target cascading in automotive vehicle design can be viewed as a four-step process: (i) specify overall vehicle mission targets, (ii) propagate vehicle targets to subsystem and component sub-targets, (iii) design vehicle systems, subsystems and components to achieve their respective sub-targets, and (iv) verify that the resulting design meets overall vehicle mission targets. This process is possible if appropriate CAE models are available to analyze design decisions that can be made through a formal decision-making process. The results, of course, will be as good as the available analysis models, and this is a practical challenge: the models must capture the salient characteristics of the system interactions without being burdened by cumbersome details. Typical CAE tools tend to be used for very sophisticated analysis and are expensive to develop and to compute, while ''back-of-the-envelope'' calculations do not capture the complexity of the interactions. In our presentation of the target cascading formalism below, availability of the appropriate models is assumed.

Multilevel optimization methods have been well studied [1–3]. Collaborative optimization [4–6] is particularly interesting in the present context. In this formulation design objectives in the subproblems attempt to minimize the discrepancy between interaction variables and should become zero at the optimum. Constraints in the original optimization problem are distributed in the subsystem optimization problems, and subproblem objectives become equality constraints at the system level. The equality constraints are reformulated in two ways: ''$CO_1$'' and ''$CO_2$'' [7]. In $CO_1$ system level variables are directly matched with their subsystem counterparts, and in $CO_2$ system level constraints are square sums of deviations. During iterations, subproblems may return different values for an interdisciplinary variable, which can cause convergence difficulties in that equality constraints at the system level are not satisfied in the $CO_1$ formulation. In $CO_2$, the system level constraints do not satisfy the regularity condition, which is a form of constraint qualification [8]. Convergence difficulties are not uncommon for coordination strategies needed to solve multilevel optimization problems.

Though different from collaborative optimization, target cascading shares the idea of minimizing deviations between design problems to achieve consistency but can be shown to satisfy constraint qualifications [9]. In collaborative optimization, analysis models are decomposed at the same level, and a coordination problem is defined for a bilevel optimization problem. Without a convergent coordination strategy, it is not clear how to extend collaborative optimization to a multilevel hierarchy. In target cascading, the multilevel hierarchical problem is shown to be equivalent to a special case of hierarchical overlapping coordination [10–12] and this allows convergence proofs [9,11]. In the present study, models are checked for feasibility and boundedness [13] and for constraint qualifications of the additional deviation constraints [8,9].

The next sections present a general description of the steps involved in the target cascading process, followed by the mathematical statement of the resulting multilevel hierarchical problem. An analytical example helps fix the mathematical ideas, and an automotive chassis system design problem shows how the formulation can be used in practice. We will assume that a vehicle can be hierarchically decomposed into levels: supersystem, systems, subsystems, components and so on. For example, in a vehicle design, supersystem is the (entire) vehicle; systems are powertrain, chassis, and body; and subsystems are engine, body-in-white, suspension, etc. However, the target cascading process is applicable to any hierarchical decomposition of a system.

## Target Cascading Process Description

Target cascading is a system design approach enabling top level design targets to be cascaded down to lower levels of the modeling hierarchy. The steps are: (i) development of appropriate analysis models, (ii) partitioning the system, (iii) formulating the target problems for each element of the partition, and (iv) solving the partitioned problem through a coordination strategy to compute all stated targets. In this section we discuss the issues involved in these steps.

**Development of Models.** Existence of appropriate analytical or experimental, but quantitative, models for the performance of system elements (systems, subsystems and components) is assumed. If a model is not available, development of a low-fidelity model by experiment, simulation, and/or a response surface method is necessary. Since computationally expensive models are generally not appropriate for target cascading, inexpensive models should replace expensive ones through surrogate modeling. Given the analysis models, verification of the individual optimal design models for all system elements for feasibility and boundedness is necessary (see, e.g., Papalambros and Wilde [13]). This model development stage is time-consuming, but without good models subsequent design decisions may be of little value.

**System Partitioning.** In general, system and model partitioning can be done in several ways: namely, object, aspect (or discipline), and model-based partitioning [14]. Object and aspect partitioning are ‘‘natural’’ partitions, and some large companies employ both partitions simultaneously in matrix organizations. For example, an automotive manufacturer partitions its organization into powertrain, body, chassis, and electronics divisions but also dedicated groups for durability, packaging, dynamics, safety, or noise-vibration-harshness (NVH). Model-based partitioning uses matrix or graph representations derived from the actual models to find a properly ‘‘balanced’’ partition [15,16]. After partitioning, design variables are categorized into linking variables, common to more than one subproblem, and local variables belonging only to one subproblem.

For target cascading the easiest way is to start with an object partition and recognize that each design problem at a given level is likely to be multidisciplinary. The exact partitioning choice will also depend on the availability of models, so this task should be done carefully and considered as subject to revisions during process implementation.

**Target Cascading.** After partitioning the original problem into subproblems at multiple levels, the linking variables between subproblems at the same level and the response connecting subproblems at different levels must be identified. The definition of ‘‘response’’ will be elucidated in the next section, but the term is used here to indicate the output of an analysis model (e.g., a simulation). At each subproblem element, an optimization problem is formulated. A coordination strategy is required to ensure convergence of the solutions generated by subproblem optimizations to the original design problem solution.

In a typical hierarchical coordination strategy, there is a master problem and one or more subproblems. The master problem is solved for the linking variables that are then input as parameters to the subproblems, and the subproblems are solved for local variables that are input as parameters to the master problem.

In target cascading a hierarchical coordination strategy is used. However, unlike the typical hierarchical coordination, *linking variables are transferred to lower level problems as targets after solving the top-level problem*. Furthermore, *some of the top-level optimization variables are also transferred to the lower level as response targets*. In the low-level problem, the objective function is to minimize the discrepancy between the target values determined at the top level and the linking variables and responses. These ideas will become more clear in the mathematical formulation of the next section.

**Embodiment Design.** Once targets are set for the individual design problems after a successful target cascading process, all interactions among design problems, such as linking variables and responses, are specified. Maintaining these values as fixed parameters, the simple models used in the target cascading process are replaced by more detailed models for embodiment design, which will have many more variables and design degrees of freedom. Local design problems can be formulated and solved. If current design targets cannot be realized using more detailed models (e.g., the new problem is infeasible), the designer must either explore the local constraints for relaxation or return to the target cascading process and request adjustments there.

The steps described above are not easy but are systematic. Forcing design teams to create the correct models and to negotiate the selection of targets goes a long way toward a successful process. The optimization formalism and attendant numerical solutions help put any further needed negotiations on a rational basis.

## Mathematical Formulation

In this section, the mathematical statement of the target cascading process is given for a product design example composed of supersystem, system, and subsystem levels, which correspond to top, middle and bottom levels of the modeling hierarchy. However, the target cascading formulation is not limited to a three-level modeling hierarchy, but can be further expanded to more levels depending on the complexity of the original product design problem. The general formulation follows at the end of the section.

**Designing With Targets.** The original design problem can be stated as follows: find a design that minimizes the deviations between design targets and actual responses while satisfying design constraints. Alternatively, determine the values of supersystem, system and subsystem design variables that minimize the deviation of supersystem responses from supersystem targets. The original design problem $\mathbf{P}_0$ is formally stated as follows:

$$\mathbf{P}_0: \ \text{Minimize} \ \|\mathbf{T}-\mathbf{R}\|$$

$$\mathbf{x}$$

$$\text{where} \ \ \mathbf{R}=r(\mathbf{x})$$

$$\text{subject to}$$

$$g_i(\mathbf{x}) \leqslant 0 \quad i=1,\ldots,m_i$$

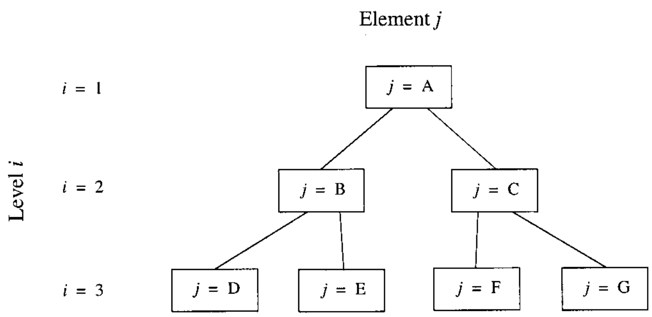


**Fig. 1 Example of hierarchically partitioned optimal design problem**

$$h_j(\mathbf{x})=0 \quad j=1,\ldots,m_e$$

$$x_k^{\min}\leqslant x_k\leqslant x_k^{\max} \quad k=1,\ldots,n \tag{1}$$

The objective is defined as the discrepancy between the target vector $\mathbf{T}$ and the response vector $\mathbf{R}$ obtained from the analysis model $r(\mathbf{x})$; $\mathbf{g}$ and $\mathbf{h}$ are inequality and equality design constraint vectors, and the design variable $\mathbf{x}$ is defined within lower and upper bounds, $\mathbf{x}^{\min}$ and $\mathbf{x}^{\max}$.

In theory, given models for the supersystem, systems and subsystems, formulating and solving the above design problem as a single optimization problem is possible using classical optimization techniques. However, this single problem approach is often impractical and even computationally impossible. An alternative is to solve the problem in a systematic way utilizing a multilevel formulation, namely, the target cascading process. The target cascading problem can be stated then as follows: given a set of supersystem targets and models for all design elements (namely, systems, subsystems and components) determine element targets by partitioning the overall design problem, while satisfying feasibility of element designs and achieving top-level targets.

**Modeling Hierarchy.** Large-scale design problems usually possess a multilevel hierarchical structure. Two types of models exist in the modeling hierarchy of the target cascading process: *optimal design models* $\mathbf{P}$ and *analysis models* $r$ (Fig. 2). Optimal design models use analysis models to evaluate supersystem, system and subsystem responses. Thus, analysis models take design variables, parameters and lower-level responses as inputs and return responses for upper-level design problems as output. A response is an output from an analysis model, and a linking variable is a common design variable between two or more design problems sharing the same parent design problem.

To represent the hierarchy of the partitioned design problem the set of elements $E_i$ is defined at each level $i$. For each element $j$ in the set $E_i$, the set of children $C_{ij}$ is defined, which includes the elements of the set $E_{i+1}$ that are children of the element. An illustrative example is presented in Figure 1: at level of the partitioned problem we have $E_2=\{B,C\}$, and for element "$B$" at that level we have $C_{2B}=\{D,E\}$.

Figure 2 shows interactions between analysis models and design models at the system level. Targets for system responses $\mathbf{R}_{s1}^U$ and system linking variables $\mathbf{y}_{s1}^U$ are passed down from the supersystem level. After solving the system design problem, target values for system responses $\mathbf{R}_{s1}^L$ and system linking variables $\mathbf{y}_{s1}^L$ are passed up to the supersystem level. Likewise, for subsystem $ss1$, $\mathbf{R}_{ss1}^U$ and $\mathbf{y}_{ss}^U$ are passed down as targets from the system-level design problem, whereas $\mathbf{R}_{ss1}^L$ and $\mathbf{y}_{ss1}^L$ are returned to the system level. Responses from subsystem $ss1$ $\mathbf{R}_{ss1}$, responses from subsystem $ss2$ $\mathbf{R}_{ss2}$, system local design variables $\tilde{\mathbf{x}}_{s1}$, and system linking variables $\mathbf{y}_s$ are input to the analysis model $r_{s1}$, whereas system responses $\mathbf{R}_{s1}$ are returned as output.

**Target Cascading at the Top Level (Supersystem Level).** At the top level of the hierarchy the problem is stated as follows: minimize the deviations between top level responses and targets subject to supersystem design constraints and tolerance constraints that coordinate system responses and system linking variables. Formally,

$$\mathbf{P}_{\text{sup}}:\ \text{Minimize}\ \ \|\mathbf{R}_{\text{sup}}-\mathbf{T}_{\text{sup}}\|+\boldsymbol{\varepsilon}_{\mathbf{R}}+\boldsymbol{\varepsilon}_{\mathbf{y}}$$

$$\text{with respect to}\quad (\tilde{\mathbf{x}}_{\text{sup}},\mathbf{y}_s,\mathbf{R}_s,\boldsymbol{\varepsilon}_{\mathbf{R}},\boldsymbol{\varepsilon}_{\mathbf{y}})$$

$$\text{where}\quad \mathbf{R}_{\text{sup}}=r_{\text{sup}}(\mathbf{R}_s,\tilde{\mathbf{x}}_{\text{sup}})$$

$$\text{subject to}$$

$$\sum_{k\in C_{\text{sup}}}\|\mathbf{R}_{s,k}-\mathbf{R}_{s,k}^L\|\leqslant\boldsymbol{\varepsilon}_{\mathbf{R}} \tag{2}$$

$$\sum_{k\in C_{\text{sup}}}\|\mathbf{y}_{s,k}-\mathbf{y}_{s,k}^L\|\leqslant\boldsymbol{\varepsilon}_{\mathbf{y}}$$

$$\mathbf{g}_{\text{sup}}(\mathbf{R}_{\text{sup}},\tilde{\mathbf{x}}_{\text{sup}})\leqslant\mathbf{0},\qquad \mathbf{h}_{\text{sup}}(\mathbf{R}_{\text{sup}},\tilde{\mathbf{x}}_{\text{sup}})=\mathbf{0}$$

$$\tilde{\mathbf{x}}_{\text{sup}}^{\min}\leqslant\tilde{\mathbf{x}}_{\text{sup}}\leqslant\tilde{\mathbf{x}}_{\text{sup}}^{\max}$$

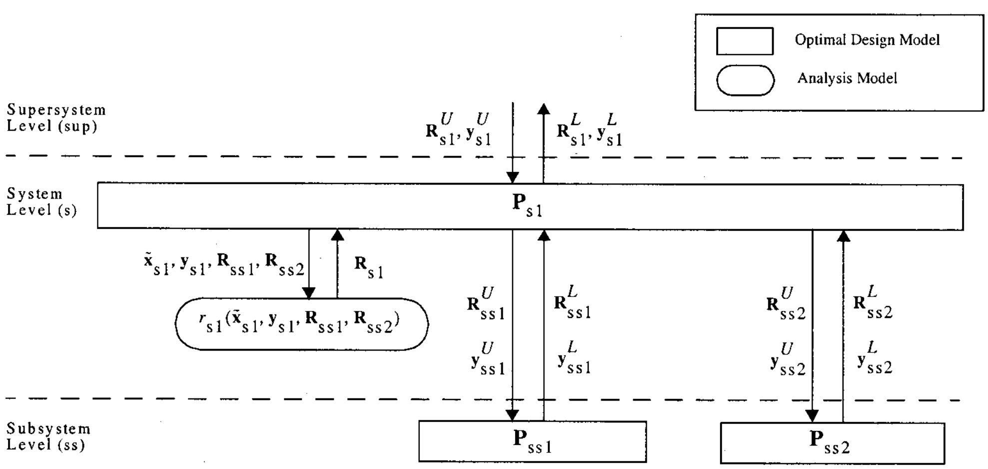


**Fig. 2 Data flows from and into the system-level design problem**

where $C_{\sup}=\{k_1,\ldots,k_{c_{\sup}}\}$, $c_{\sup}$ is the number of child elements of the supersystem-level problem and $\mathbf{R}_s=(\mathbf{R}_{s,1},\ldots,\mathbf{R}_{s,c_{\sup}})$, $\mathbf{R}_s=\mathbf{R}_{s,1}\cup\ldots\cup\mathbf{R}_{s,c_{\sup}}$ and $\mathbf{R}_{s,i}\cap\mathbf{R}_{s,j}=\emptyset$ for $i\neq j$. The objective that minimizes deviations between design targets $\mathbf{T}_{\sup}$ and supersystem responses $\mathbf{R}_{\sup}$ is modified by adding deviation tolerances $\varepsilon_{\mathbf{R}}$ and $\varepsilon_{\mathbf{y}}$ to coordinate values of the responses from the system, $\mathbf{R}_s$, and the system linking variables, $\mathbf{y}_s$. At convergence, deviation tolerances become zero as the system linking variables converge to the same values for the different system design problems. To account for large target magnitude differences, the deviation terms are multiplied by penalty constants. The penalty constants are selected so that the magnitude of the deviation terms are of the same order. The values of the system responses match $\mathbf{R}_s^L$, where $\mathbf{R}_s^L$ is the target response calculated at the system optimal design problem. Finally, $\mathbf{g}_{\sup}$ and $\mathbf{h}_{\sup}$ are inequality and equality design constraints at the supersystem level, subsets of the original constraints $\mathbf{g}$ and $\mathbf{h}$.

**Target Cascading at the Middle Level (System Level).** Similarly at the $j$th system level, the problem is stated as in Eq. (3): Minimize the deviations for system responses and system linking variables, subject to system design constraints and deviation constraints that coordinate subsystem responses and subsystem design linking variables.

$$\begin{aligned}
\mathbf{P}_{s,j}:\ &\text{Minimize}\ \ \|\mathbf{R}_{s,j}-\mathbf{R}_{s,j}^U\|+\|\mathbf{y}_{s,j}-\mathbf{y}_{s,j}^U\|+\varepsilon_{\mathbf{R}}+\varepsilon_{\mathbf{y}}\\
&\text{with respect to}\ \ \tilde{\mathbf{x}}_{s,j},\mathbf{y}_{s,j},\mathbf{y}_{ss},\mathbf{R}_{ss},\varepsilon_{\mathbf{R}},\varepsilon_{\mathbf{y}}\\
&\text{where}\ \ \mathbf{R}_{s,j}=r_{s,j}(\mathbf{R}_{ss},\tilde{\mathbf{x}}_{s,j},\mathbf{y}_{s,j})\\
&\text{subject to}\\
&\sum_{k\in C_{s,j}}\|\mathbf{R}_{ss,k}-\mathbf{R}_{ss,k}^L\|\leqslant\varepsilon_{\mathbf{R}}\\
&\sum_{k\in C_{s,j}}\|\mathbf{y}_{ss,k}-\mathbf{y}_{ss,k}^L\|\leqslant\varepsilon_{\mathbf{y}}\\
&\mathbf{g}_{s,j}(\mathbf{R}_{s,j},\tilde{\mathbf{x}}_{s,j},\mathbf{y}_{s,j})\leqslant\mathbf{0},\quad \mathbf{h}_{s,j}(\mathbf{R}_{s,j},\tilde{\mathbf{x}}_{s,j},\mathbf{y}_{s,j})=\mathbf{0}\\
&\tilde{\mathbf{x}}_{s,j}^{\min}\leqslant\tilde{\mathbf{x}}_{s,j}\leqslant\tilde{\mathbf{x}}_{s,j}^{\max},\quad \mathbf{y}_{s,j}^{\min}\leqslant\mathbf{y}_{s,j}\leqslant\mathbf{y}_{s,j}^{\max}
\end{aligned}\tag{3}$$

where $C_{s,j}=\{k_1,\ldots,k_{c_{s,j}}\}$, $c_{s,j}$ being the number of child element of system-level problem, and $\mathbf{R}_{ss}=(\mathbf{R}_{ss,1},\ldots,\mathbf{R}_{ss,c_{s,j}})$. The objective function minimizes the discrepancy between system level responses $\mathbf{R}_{s,j}$ and the targets set at the upper (supersystem) level $\mathbf{R}_{s,j}^U$, as well as between system linking variables $\mathbf{y}_{s,j}$ and the targets set at the supersystem level $\mathbf{y}_{s,j}^U$. Therefore, $\mathbf{R}_{s,j}^U$ and $\mathbf{y}_{s,j}^U$ are determined by solving Eq. (2). Target deviation tolerances are minimized to achieve a consistent design with minimum discrepancies between the subsystem level responses $\mathbf{R}_{ss}$ and the target responses $\mathbf{R}_{ss}^L$ from the subsystem design problem, as well as between the subsystem level linking variables $\mathbf{y}_{ss}$ and the target values $\mathbf{y}_{ss}^L$ from the subsystem design problem. Since the system level is located in the middle of the overall hierarchy, this formulation is the most comprehensive, capturing all interactions, through linking variables, target responses from the lower level (superscript $L$), and target responses from the upper level (superscript $U$).

**Target Cascading at the Bottom Level (Subsystem Level).** The $j$th subsystem level problem is stated in Eq. (4): minimize the deviations for subsystem responses and subsystem-level linking variables subject to subsystem design constraints. Formally,

$$\begin{aligned}
\mathbf{P}_{ss,j}:\ &\text{Minimize}\ \ \|\mathbf{R}_{ss,j}-\mathbf{R}_{ss,j}^U\|+\|\mathbf{y}_{ss,j}-\mathbf{y}_{ss,j}^U\|\\
&\text{with respect to}\ \ \tilde{\mathbf{x}}_{ss,j},\mathbf{y}_{ss,j}\\
&\text{where}\ \ \mathbf{R}_{ss,j}=r_{ss,j}(\tilde{\mathbf{x}}_{ss,j},\mathbf{y}_{ss,j})\\
&\text{subject to}\\
&\mathbf{g}_{ss,j}(\mathbf{R}_{ss,j},\tilde{\mathbf{x}}_{ss,j},\mathbf{y}_{ss,j})\leqslant\mathbf{0},\quad \mathbf{h}_{ss,j}(\mathbf{R}_{ss,j},\tilde{\mathbf{x}}_{ss,j},\mathbf{y}_{ss,j})=\mathbf{0}\\
&\mathbf{x}_{ss,j}^{\min}\leqslant\mathbf{x}_{ss,j}\leqslant\mathbf{x}_{ss,j}^{\max},\mathbf{y}_{ss,j}^{\min}\leqslant\mathbf{y}_{ss,j}\leqslant\mathbf{y}_{ss,j}^{\max}
\end{aligned}\tag{4}$$

At the bottom of the model hierarchy, subsystem design variables are input to the analysis models $r_{ss,j}$ returning responses to the subsystem level as output. In Eq. (4), the objective is to minimize the deviations between the subsystem responses $\mathbf{R}_{ss,j}$ and the targets set at the system level $\mathbf{R}_{ss,j}^U$, as well as between the subsystem linking variables $\mathbf{y}_{ss,j}$ and the targets from the system level $\mathbf{y}_{ss,j}^U$. Target deviation tolerance constraints are not introduced in Eq. (4) because there are no lower-level design models that need to be coordinated.

**General Formulation of Target Cascading Problem.** The modeling structure presented in Fig. 1 can be extended to a general multilevel hierarchical structure. Problem $P_{ij}$ for the $j$ partition element at the $i$ level is defined as follows: Minimize the deviations between current level responses $\mathbf{R}_{ij}$ and linking variables $\mathbf{y}_{ij}$ and the targets $\mathbf{R}_{ij}^U$ and $\mathbf{y}_{ij}^U$, subject to design constraints and tolerance constraints that coordinate responses and linking variables from one level below. Formally,

$$\begin{aligned}
\mathbf{P}_{ij}:\ &\text{Minimize}\ \ \|\mathbf{R}_{ij}-\mathbf{R}_{ij}^U\|+\|\mathbf{y}_{ij}-\mathbf{y}_{ij}^U\|+\boldsymbol{\varepsilon}_{\mathbf{R}}+\boldsymbol{\varepsilon}_{\mathbf{y}}\\
&\text{with respect to}\ \ \tilde{\mathbf{x}}_{ij},\mathbf{y}_{ij},\mathbf{y}_{(i+1)j},\mathbf{R}_{(i+1)j},\boldsymbol{\varepsilon}_{\mathbf{R}},\boldsymbol{\varepsilon}_{\mathbf{y}}\\
&\text{where}\ \ \mathbf{R}_{ij}=r_{ij}(\mathbf{R}_{(i+1)j},\tilde{\mathbf{x}}_{ij},\mathbf{y}_{ij})\\
&\text{subject to}\\
&\sum_{k\in C_{ij}}\|\mathbf{R}_{(i+1)k}-\mathbf{R}_{(i+1)k}^L\|\leqslant\boldsymbol{\varepsilon}_{\mathbf{R}}\\
&\sum_{k\in C_{ij}}\|\mathbf{y}_{(i+1)k}-\mathbf{y}_{(i+1)k}^L\|\leqslant\boldsymbol{\varepsilon}_{\mathbf{y}}\\
&\mathbf{g}_{ij}(\mathbf{R}_{ij},\tilde{\mathbf{x}}_{ij},\mathbf{y}_{ij})\leqslant\mathbf{0},\mathbf{h}_{ij}(\mathbf{R}_{ij},\tilde{\mathbf{x}}_{ij},\mathbf{y}_{ij})=\mathbf{0}\\
&\tilde{\mathbf{x}}_{ij}^{\min}\leqslant\tilde{\mathbf{x}}_{ij}\leqslant\tilde{\mathbf{x}}_{ij}^{\max},\mathbf{y}_{ij}^{\min}\leqslant\mathbf{y}_{ij}\leqslant\mathbf{y}_{ij}^{\max}.
\end{aligned}\tag{5}$$

Here $c_{ij}$ is the number of child element of $i$th level problem, $C_{ij}=\{k_1,\ldots,k_{c_{ij}}\}$, and $\mathbf{R}_{(i+1)}=(\mathbf{R}_{(i+1),1},\ldots,\mathbf{R}_{(i+1),c_{ij}})$. This concludes our discussion of the formal statement of the problem. The next section illustrates these ideas with two examples.

## Illustrative Examples

The first example is a geometric programming problem, which is general enough to illustrate the key ideas of the target cascading formulation. The second example is an automotive chassis system design problem involving two simple quarter-car models.

**A Geometric Programming Problem.** Geometric programming problems with posynomials are known to have a unique globally optimal solution [17]. The example here has a quadratic objective function, 14 design variables, 4 equality and 6 inequality constraints, and nonnegativity constraints for all design variables, as shown in Eq. (6). Equality constraints $h_1,\ldots,h_4$ can be directed as active inequality constraints in negative unity form [13].

$$\begin{aligned}
&\text{Minimize}\quad f=x_1^2+x_2^2\\
&x_3,x_4,\ldots,x_{14}\\
&\text{subject to}\\
&g_1:\frac{x_3^{-2}+x_4^2}{x_5^2}\leqslant1\quad g_2:\frac{x_5^2+x_6^{-2}}{x_7^2}\leqslant1\quad g_3:\frac{x_8^2+x_9^2}{x_{11}^2}\leqslant1\\
&g_4:\frac{x_8^{-2}+x_{10}^2}{x_{11}^2}\leqslant1\quad g_5:\frac{x_{11}^2+x_{12}^{-2}}{x_{13}^2}\leqslant1\quad g_6:\frac{x_{11}^2+x_{12}^2}{x_{14}^2}\leqslant1
\end{aligned}\tag{6}$$

**Table 1 Optimal designs from All-At-Once (AAO) and Target Cascading (TC) formulations**

| | AAO | TC |
|---|---|---|
| $x_1$ | 2.84 | 2.80 |
| $x_2$ | 3.09 | 3.03 |
| $x_3$ | 2.36 | 2.35 |
| $x_4$ | 0.76 | 0.76 |
| $x_5$ | 0.87 | 0.87 |
| $x_6$ | 2.81 | 2.79 |
| $x_7$ | 0.94 | 0.95 |
| $x_8$ | 0.97 | 0.97 |
| $x_9$ | 0.87 | 0.87 |
| $x_{10}$ | 0.80 | 0.80 |
| $x_{11}$ | 1.30 | 1.30 |
| $x_{12}$ | 0.84 | 0.84 |
| $x_{13}$ | 1.76 | 1.75 |
| $x_{14}$ | 1.55 | 1.54 |
| $f$ | 17.61 | 17.02 |
| $\varepsilon_1$ | N/A | 0.0001 |
| $\varepsilon_2$ | N/A | 0.0017 |
| $\varepsilon_3$ | N/A | 0.0029 |

$$h_1: x_1^2 = x_3^2 + x_4^{-2} + x_5^2 \quad h_2: x_2^2 = x_5^2 + x_6^2 + x_7^2$$
$$h_3: x_3^2 = x_8^2 + x_9^{-2} + x_{10}^{-2} + x_{11}^2 \quad h_4: x_6^2 = x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2$$
$$x_3, x_4, \ldots, x_{14} \geqslant 0$$

If we assume that $x_1$, $x_2$, $x_3$, $x_6$ are responses from analysis models $r_1$, $r_2$, $r_3$, $r_4$, the equality constraints can be regarded as analysis models and the overall problem can be stated as in Eq. (7). This problem can be partitioned as shown in Fig. 3. Supersystem analysis models take supersystem design variables $x_4$, $x_5$, $x_7$ and system responses $x_3$ and $x_6$ and return supersystem responses $x_1$, $x_2$. System analysis models take system design variables $x_8$, $x_9$, $x_{10}$, $x_{11}$, $x_{12}$, $x_{13}$, $x_{14}$ and return system responses $x_3$, $x_6$. The linking variable between system level optimal design problems is $x_{11}$. Only the values of $x_3$, $x_6$ and $x_{11}$ are passed up and down between the supersystem and system optimal design problems. Top level target values are set to zero. Table 1 shows optimization results obtained using a single All-At-Once (AAO) formulation, and a target cascading (TC) formulation. Objective, variables and tolerance values at the optima are given in the table. The solution using the AAO formulation is the unique global optimum, whereas the TC solution is the same within a tolerance. Note that the value of the objective in TC is smaller than that of the objective in AAO. This is because of the deviation tolerances $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ allowed after matching the lower-level responses $x_3^L$ and $x_6^L$.

$$\text{Minimize} \quad f = x_1^2 + x_2^2$$
$$x_3, x_4, \ldots, x_{14}$$
$$\text{where}$$
$$R_1 = x_1 = r_1(x_3, x_4, x_5) = (x_3^2 + x_4^{-2} + x_5^2)^{1/2}$$
$$R_2 = x_2 = r_2(x_5, x_6, x_7) = (x_5^2 + x_6^2 + x_7^2)^{1/2}$$
$$R_3 = x_3 = r_3(x_8, x_9, x_{10}, x_{11}) = (x_8^2 + x_9^{-2} + x_{10}^{-2} + x_{11}^2)^{1/2}$$
$$R_4 = x_6 = r_4(x_{11}, x_{12}, x_{13}, x_{14}) = (x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2)^{1/2} \quad (7)$$
$$\text{subject to}$$
$$g_1: \frac{x_3^{-2} + x_4^2}{x_5^2} \leqslant 1 \quad g_2: \frac{x_5^2 + x_6^{-2}}{x_7^2} \leqslant 1 \quad g_3: \frac{x_8^2 + x_9^2}{x_{11}^2} \leqslant 1$$
$$g_4: \frac{x_8^{-+2} + x_{10}^2}{x_{11}^2} \leqslant 1 \quad g_5: \frac{x_{11}^2 + x_{12}^{-2}}{x_{13}^2} \leqslant 1 \quad g_6: \frac{x_{11}^2 + x_{12}^2}{x_{14}^2} \leqslant 1$$
$$x_3, x_4, \ldots, x_{14} \geqslant 0$$

**Chassis Design Problem.** Figure 4 shows a schematic diagram of a half-vehicle analysis model at the vehicle and system levels. The vehicle model is composed of two symmetric quarter-car models, which takes system level responses and variables as inputs and returns outputs (such as acceleration of body mass and relative displacement between sprung and unsprung masses) as vehicle level responses. Sprung mass is divided into two parts, body-in-white $m_b$ and the rest of the system $m_s$. At the vehicle level, the body-in-white (BIW) is represented by a single rigid body mass with stiffness $k_b$ and damping coefficient $c_b$ mounted on a spring with stiffness $k_s$ and a damper with coefficient $c_s$ at the front and rear of the vehicle. At the system level, the mass representation is expanded into a finite element beam model that takes eight different section thicknesses as input design variables and returns the weight as the system level response. For simplicity, suspension springs in the vehicle model are modeled as two parallel springs at the system level; the model takes two spring constants as input variables and calculates the suspension spring constant as the system level response. As we move down from the top vehicle level, complexity and completeness of models are increased.

The vehicle-level optimal design problem $\mathbf{P}_v$ is stated in Eq. (8). Design targets are set for NVH and packaging that correspond to acceleration of the body $z_b''$ and the relative displacement of sprung and unsprung masses, $(z_s - z_{us})$, respectively. The road profile input is $z_0$. Targets $T_1$ and $T_2$ for NVH and packaging, respectively, are set as in Table 2, and values for upper and lower bounds for design variables are given in Table 3. The objective is to minimize the sum of deviations of the two responses from the targets, with target deviation tolerances $\varepsilon_1$ and $\varepsilon_2$.

$$\mathbf{P}_v: \quad \text{Minimize} \|z_b'' - T_1\| + \|(z_s - z_{us}) - T_2\| + \varepsilon_1 + \varepsilon_2$$
$$\text{with respect to} \quad k_s, m_b, c_b, k_b, c_b, m_s, \varepsilon_1, \varepsilon_2$$
$$\text{where}$$
$$m_{s2} z_{s2}'' + c_{s2}(-z_{s1}' + z_{s2}') + k_{s2}(-z_{s1} + z_{s2}) = 0$$
$$m_{s1} z_{s1}'' + c_{s2}(z_{s1}' - z_{s2}') + k_{s2}(z_{s1} - z_{s2}) + c_{s1}(z_{s1}' - z_{us}') + k_{s1}(z_{s1} - z_{us}) = 0$$
$$m_{us} z_{us}'' + c_{s1}(z_{us}' - z_{s1}') + k_{s1}(z_{us} - z_{s1}) + c_{us}(z_{us}' - z_0') + k_{us}(z_{us} - z_0) = 0 \quad (8)$$
$$\text{subject to}$$
$$\|m_b - m_b^L\| \leqslant \varepsilon_1, \|k_s - k_s^L\| \leqslant \varepsilon_2, c_b^{\min} \leqslant c_b \leqslant c_b^{\max}$$
$$k_b^{\min} \leqslant k_b \leqslant k_b^{\max}, m_b^{\min} \leqslant m_b \leqslant m_b^{\max}, c_s^{\min} \leqslant c_s \leqslant c_s^{\max}$$
$$k_s^{\min} \leqslant k_s \leqslant k_s^{\max}, m_s^{\min} \leqslant m_s \leqslant m_s^{\max}$$

The system-level design problem for the BIW is given in Eq. (9) below. The objective is to minimize the deviation of BIW weight from the target calculated at the vehicle level. A finite element analysis code takes section thicknesses of all the beams comprising the BIW structure and returns weight and strain energy as outputs. Constraints on the strain energy per unit volume $J_i$ for each beam are specified.

$$\mathbf{P}_{s1}: \text{Minimize} \|m_b - m_b^U\|$$
$$\text{with respect to} \quad (t_1, \ldots, t_g)$$
$$\text{where} \quad (\mathbf{J}, m_b) = r(t_1, \ldots, t_8)$$
$$\text{subject to} \quad (9)$$
$$\mathbf{J} \leqslant \mathbf{J}^{\max}$$

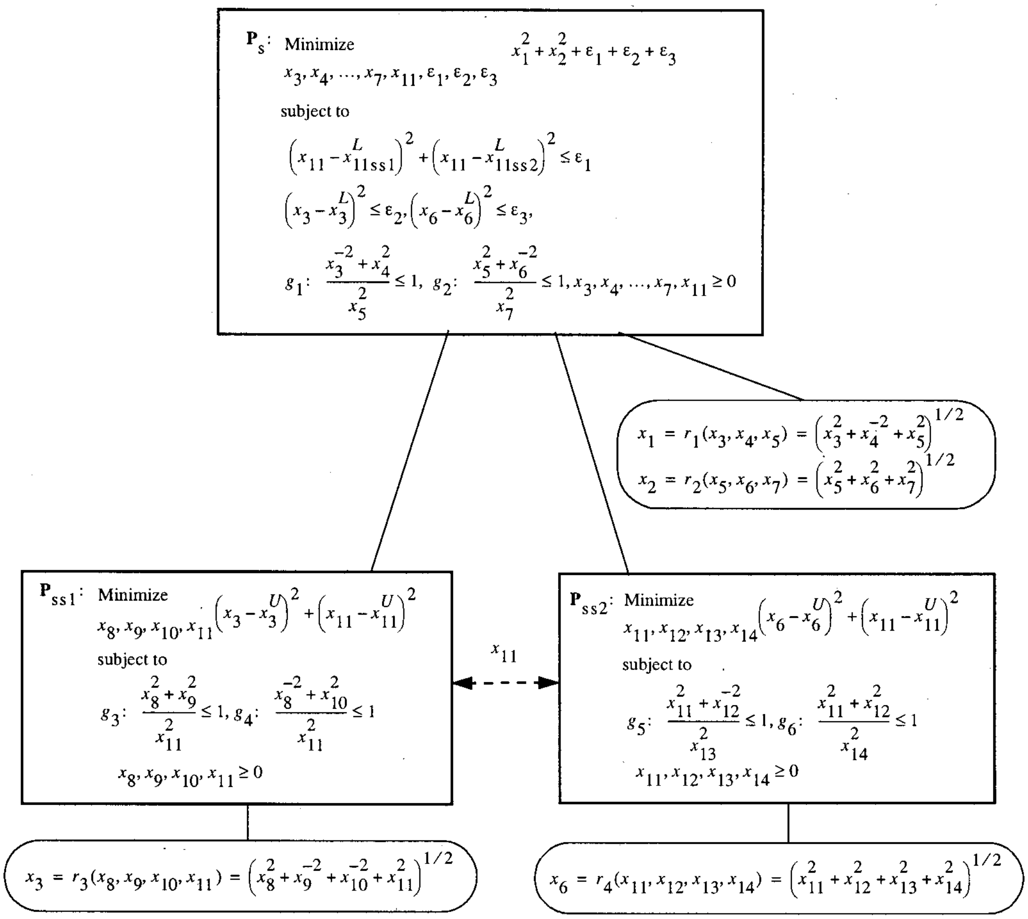


**Fig. 3 Partitioning of geometric programming example problem**

$$m_b^{\min} \leq m_b \leq m_b^{\max}$$

The system-level design problem for the suspension spring is stated in Eq. (10). The objective is to minimize the deviation of suspension stiffness $k_s$ from the target $k_s^U$ calculated at the vehicle level. The simple analysis model identifies the relationship between the overall suspension stiffness and the two component spring constants $k_{s1}$ and $k_{s2}$ as follows:

$$\mathbf{P}_{s2}: \text{Minimize} \|k_s - k_s^U\|$$

$$\text{with subject to } k_{s1}, k_{s2}$$

$$\text{where } k_s = k_{s1} + k_{s2}$$

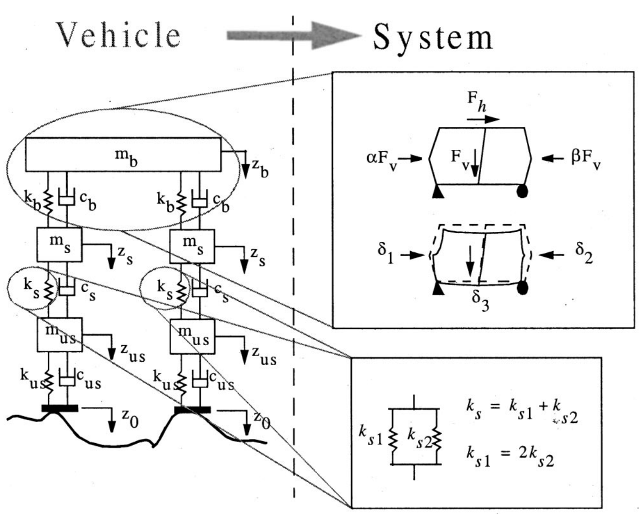


**Fig. 4 Schematic of half-vehicle analysis models**

**Table 2 Chassis problem targets**

| Target Variables | Target Values | AAO | TC |
|---|---|---|---|
| NVH (m/s$^2$) | 0.01 | 0.023 | 0.01 |
| Packaging (m) | 0.002 | 0.0016 | 0.0016 |

**Table 3 Chassis problem design variables**

| Variable | Lower Bound | Upper Bound | AAO | TC | Units |
|---|---|---|---|---|---|
| $c_s$ | 125 | 2500 | 1530 | 349 | kg/s |
| $k_s$ | 3000 | 12000 | 12000 | 3161 | kg/s$^2$ |
| $c_b$ | 25 | 500 | 500 | 500 | kg/s |
| $k_b$ | 18000 | 36000 | 18000 | 18000 | kg/s$^2$ |
| $m_s$ | 400 | 500 | 500 | 500 | kg |
| $m_b$ | 100 | 150 | 143 | 143 | kg |
| $k_{s1}$ | 1000 | 12000 | 8000 | 2107 | kg/s$^2$ |
| $k_{s2}$ | 1000 | 12000 | 4000 | 1054 | kg/s$^2$ |

$$\text{subject to} \tag{10}$$

$$k_{s1} = 2k_{s2}$$

$$k_{s1}^{\min} \leq k_{s1} \leq k_{s1}^{\max}, k_{s2}^{\min} \leq k_{s2} \leq k_{s2}^{\max}$$

Tables 2 and 3 show the AAO and TC results for targets and design variables, respectively. As shown in Figure 4, three analysis models are stand-alone codes integrated into a single model for the AAO solution strategy. Both approaches achieve the target values for the packaging target. For the NVH targets, the AAO formulation does not achieve the target while the TC formulation does. Suspension designs are different, while BIW structure designs are the same. If one suspects that the problem has more than one local minima, further validation of the numerical results using global search methods will be necessary.

## Conclusion

Target cascading is a generic formulation for large-scale, multidisciplinary system design problems with a multilevel structure. Responses, linking variables, and local variables capture interactions between design problems and analysis models. Proper hierarchical coordination ensures convergence to the optimal design of the original system target problem. The examples illustrated the process and indicated that convergence to the global optimum is an issue to be kept in mind when evaluating the results. While it is worth while to compare convergence characteristics between target cascading and other MDO approaches, the main focus of this paper is on illustrating the target cascading process and its ability to converge to an optimal solution. The convergence issue in target cascading is further discussed in Michelena et al. [11].

From a design viewpoint, the main benefits of target cascading are reduction in system design cycle time, avoidance of design iterations late in the development process, and increased likelihood that physical prototypes will be closer to production quality. Design iterations are reduced by integrating the target propagation and target matching processes into a single procedure. Using partitioning by systems, subsystems, and components reduces the complexity of the overall design problem and allows more systematic concurrent design of the system's elements. Thus target cascading is more beneficial when applied to large-scale multidisciplinary design problems, where the modeling hierarchy is composed of subproblems of smaller size compared to the original problem.

## Acknowledgments

This research was partially supported by the U.S. Army Automotive Research Center for Modeling and Simulation of Ground Vehicles at the University of Michigan, and by a grant from Ford Motor Company. This support is gratefully acknowledged. The views presented here do not necessarily reflect those of the sponsors.

## Nomenclature

J = strain energy per unit volume
$\mathbf{P}_o$ = original design problem
$\mathbf{P}_s$ = system-level target cascading optimization problem
$\mathbf{P}_{\text{sup}}$ = supersystem-level target cascading optimization problem
$\mathbf{P}_{ss}$ = subsystem-level target cascading optimization problem
$\mathbf{R}^L$ = target values of $\mathbf{R}$ from a lower level
$\mathbf{R}^U$ = target values of $\mathbf{R}$ from an upper level
$\mathbf{R}$ = responses computed by analysis models
$\mathbf{T}$ = design targets
$f$ = objective for the design problem
$\mathbf{g}$ = inequality constraints for the design problem
$\mathbf{h}$ = equality constraints for the design problem
$r$ = response function
$\mathbf{x}$ = vector of all design variables $(\tilde{\mathbf{x}}, \mathbf{y})$
$\tilde{\mathbf{x}}$ = local design variables
$\mathbf{x}^{\min}$ = lower bound of $\mathbf{x}$
$\mathbf{x}^{\max}$ = upper bound of $\mathbf{x}$
$\mathbf{y}$ = linking design variables
$\mathbf{y}^L$ = target values of $\mathbf{y}$ from a lower level
$\mathbf{y}^U$ = target values of $\mathbf{y}$ from an upper level
$\varepsilon_{\mathbf{R}}$ = target deviation tolerance for responses
$\varepsilon_{\mathbf{y}}$ = target deviation tolerance for linking variables
$m_i$ = number of inequality constraints
$m_e$ = number of equality constraints
$n$ = number of design variables
$s$ = system level
sup = supersystem level
$ss$ = subsystem level

## References

[1] Kodiyalam, S., and Sobiesczanski-Sobieski, J., 2000, ''Bilevel Integrated System Synthesis with Response Surfaces,'' AIAA J., **38**(8), pp. 1479–1485.
[2] Sobieszczanski-Sobieski, J., James, B., and Riley, M., 1987, ''Structural Sizing by Generalized, Multilevel Optimization,'' AIAA J., **25**(1), pp. 139–145.
[3] Cramer, E., Dennis, J., Frank, P., Lewis, R., and Shubin, G., 1994, ''Problem Formulation for Multidisciplinary Optimization,'' SIAM J. Control Optim., **4**(4), pp. 754–776.
[4] Braun, R., 1996, *Collaborative Optimization: An Architecture For Large-Scale Distributed Design*, Doctoral Dissertation, Stanford University.
[5] Braun, R., Moore, A., and Kroo, I., 1996, ''Use of the Collaborative Optimization Architecture for Launch Vehicle Design,'' *Proceedings of the 6th AIAA/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, AIAA-96-4018, Bellevue, WA.
[6] Tappeta, R., and Renaud, J., 1997, ''Multiobjective Collaborative Optimization,'' ASME J. Mech. Des., **119**(3), pp. 403–411.
[7] Alexandrov, N. M., and Lewis, R. M., 2002, ''Analytical and Computational Aspects of Collaborative Optimization for Multidisciplinary Design,'' AIAA J., **40**(2), pp. 301–309.
[8] Bazaraa, M. S., Sherali, H. D., and Shetty, C. M., 1993, *Nonlinear Programming*, 2nd edition, John Wiley & Sons.
[9] Kim, H. M., 2001, ''Target Cascading in Optimal System Design,'' Doctoral Dissertation, University of Michigan, Ann Arbor, MI.
[10] Michelena, N. F., Papalambros, P. Y., Park, H. A., and Kulkarni, D., 1999, ''Hierarchical Overlapping Coordination for Large-Scale Optimization by Decomposition,'' AIAA J., **37**(7), pp. 890–896.
[11] Michelena, N. F., Park, H. A. and Papalambros, P. Y., 2002, ''Convergence Properties of Analytical Target Cascading,'' *Proceedings of the 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, AIAA-2002-5506, Atlanta, GA.
[12] Park, H. A., Michelena, N. F., Kulkarni, D., and Papalambros, P. Y., 2000, ''Convergence Criteria for Overlapping Coordination Under Linear Constraints,'' Journal of Computational Optimization and Applications, **18**(3), pp. 273–293.
[13] Papalambros, P. Y., and Wilde, D., 2000, *Principles of Optimal Design: Modeling and Computation* (2nd Ed.), Cambridge University Press, New York.
[14] Wagner, T. C., 1993, ''A General Decomposition Methodology for Optimal System Design,'' Doctoral Dissertation, University of Michigan, Ann Arbor, MI.
[15] Michelena, N. F., and Papalambros, P. Y., 1995, ''Optimal Model-Based Decomposition of Powertrain System Design,'' ASME J. Mech. Des., **117**(4), pp. 499–505.
[16] Krishnamachari, R. S., and Papalambros, P., 1997, ''Optimal Hierarchical Decomposition Synthesis Using Integer Programming,'' ASME J. Mech. Des., **119**(4), pp. 440–447.
[17] Beightler, C. S., and Phillips, D. T., 1976, *Applied Geometric Programming*, John Wiley and Sons, Inc., New York.