Sigurd A. Nelson II
Research Fellow

Matthew B. Parkinson
Doctoral Student

Panos Y. Papalambros*
Professor
pyp@umich.edu

Optimal Design Laboratory,
Department of Mechanical Engineering,
University of Michigan,
Ann Arbor, MI 48109

Multicriteria Optimization in Product Platform Design

A product platform is a set of common components, modules or parts from which a stream of derivative products can be created. Product platform design requires selection of the shared parts and assessment of the potential sacrifices in individual product performance that result from parts sharing. A multicriteria optimization problem can be formulated to study such decisions in a quantitative manner at the product performance level. Studying the Pareto sets that correspond to various derivative products leads to a systematic methodology for design decision making. Design of a nail gun platform is used to illustrate the concepts presented. [DOI: 10.1115/1.1355775]

1 Introduction

Traditional design processes typically address design of a single product. However, manufacturing firms increasingly make several variations of a product, each directed to a different market niche. There is even some evidence to suggest a firm must have an appreciable product line to attain market share, stay competitive, and remain profitable (see, for example, [1–3]).

In a product line, products are likely to be related in some way. Exactly how the similarities among products exhibit themselves can either simplify or complicate a number of business issues. For example, if it is possible to manufacture two different products using some of the same parts, a company often sees benefits through reduction of inventory [4,5], reduction in the proliferation of different parts [6], reduction in the design lead-in time for products [7], ease of designing for new market niches [3], and reduction in the number and type of factory machinery and processes [8]. Penalties may be incurred by the sharing of components, however. Components used in multiple products must be designed to the criteria of the most demanding product—which may result in a component that far exceeds the requirements of the other models in which it is used. This sharing may also result in a lack of variation across the product family [9].

This is the idea behind a *product platform*, namely, a set of common components, modules, or parts from which a stream of derivative products can be efficiently created and launched. Several researchers have addressed interesting aspects of product platforms. For example, Gonzalez-Zugasti et al. [10] consider the market niche as well as the platform cost. Ishii et al. [11] look to minimize life-cycle cost while still providing a broad product line. In the automotive field MacDuffie et al. [12] examined overall factory performance, while Siddique et al. [8] have studied the effects of using common assembly methods and factory lines as well as common parts.

In this article the focus is on using nonlinear programming methods for product platform design. In particular, optimization is used as a method for designing several individual products for market niches. The argument is made that if two or more products are to share the same part, the *performance* of each product within its own niche will likely change in comparison to its performance if it were designed to not share any parts. Optimization formulations can quantify this change in performance, which is one of the criteria used to justify or deny the use of the common part. Furthermore, if the common part is to be used in two or more products, the design objectives of the two products will often be in competition—leading to the use of multicriteria optimization.

*Corresponding author.

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A design example is introduced early in the body of the paper (rather than as an example at the end). This motivates the development of the methodology and illustrates its application. The product is a nail gun (Fig. 1), a device used in construction or woodworking projects to drive nails into wood. A trigger activates the hammer mechanism which drives the nails with an appropriate force. A model for the performance of a nail gun is given in Section 2. In Section 3 the model is used to formulate a multicriteria optimization problem representing a possible platform design. The general approach for quantifying the change in performance due to the introduction of common parts is presented in Sections 4 and 5. Some conclusions are offered in Section 6.

2 Modeling a Nail Gun

In the present design example we focus on the spring-hammer mechanism using the representation in Fig. 2 to derive the mathematical model. A potential vector of design variables is $\mathbf{x} = [l_h, l_n, l_p, N]$ where l_h is the starting height of the hammer, l_n is the length of the nail, l_p is the preloaded length of the spring, and N is the number of active spring coils. The remaining dimensions are considered as parameters, namely, the wire diameter d, spring diameter D, unstretched length l_u, and hammer mass m_h. The spring constant of the spring within the hammer mechanism is given by

$$k_s = \frac{d^4 G}{8D^3 N}, \quad (1)$$

where G is the shear modulus of the wire. The maximum shear stress in the spring is given by

$$\tau = \frac{8FD}{\pi d^3}, \quad (2)$$

where F is the maximum force in the spring

$$F = k_s(l_u - (l_p - l_h)). \quad (3)$$

The energy used to deliver the nail is the energy released from the spring during its travel:

$$E = \frac{1}{2} k_s[(l_u - (l_p - l_h))^2 - (l_u - l_p)^2]. \quad (4)$$

Additionally, the solid height of the spring cannot be less than the height of the spring at its maximum compression:

$$Nd - (l_p - l_h) \leq 0. \quad (5)$$

The positions of the nail (x_n) and nail gun (x_t) are modeled by three sets of differential equations. The first set governs the acceleration of the hammer until it reaches the end of its travel

$$\ddot{x}_n = -F_s/(m_h + m_n)$$

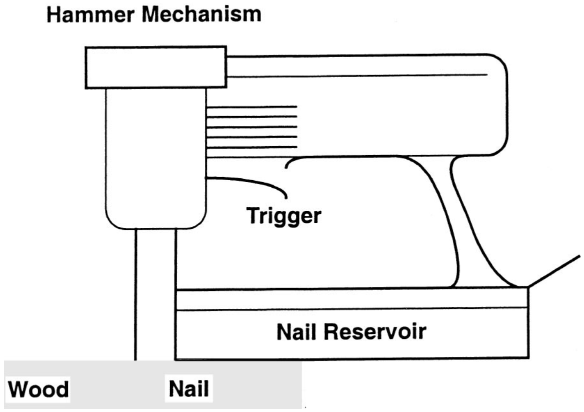

Fig. 1 A typical nail gun

$$\ddot{x}_t=(F_s-F_a)/m_l \tag{6}$$

where

$$F_a=F_p+k_a x_t \tag{7}$$

$$F_s=k_s(l_u-l_p+x_n-x_t). \tag{8}$$

The initial values for x_n and x_t and l_h and 0, respectively.

Once the hammer comes into contact with the nail (but is still being forced down by the spring), the equation governing the motion of the nail becomes

$$\ddot{x}_n=(F_n-F_s)/(m_h+m_n) \tag{9}$$

where

$$F_n=\begin{cases} 0 & \text{if } x_n>l_n \\ x_n\beta & \text{if } x_n\leq l_n \end{cases}. \tag{10}$$

At some point the resistive force of the wood becomes too great and the acceleration of the nail, $\ddot{x}_n$, becomes zero.

After the hammer has reached the end of its travel the motion is governed by the equations

$$\ddot{x}_n=F_n/m_n$$

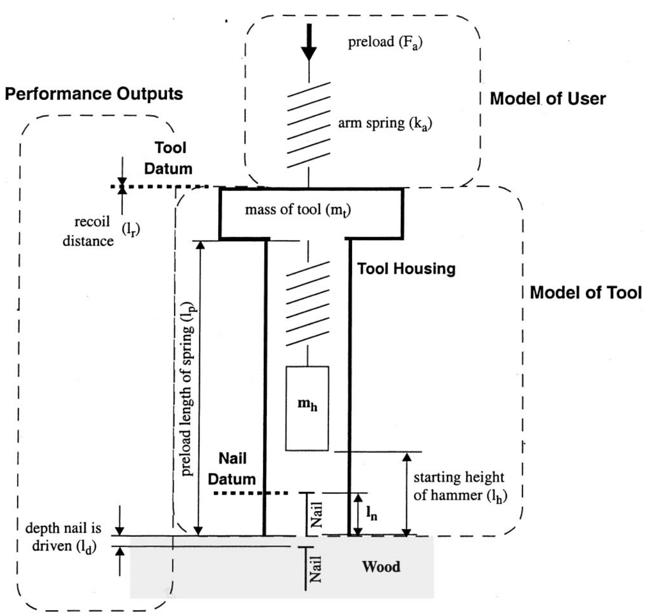

Fig. 2 Lumped-mass model of a nail gun

$$\ddot{x}_t=-F_a/(m_h+m_l). \tag{11}$$

The recoil experienced by the user is the maximum height reached by the tool. The depth to which the nail is driven is the minimum value of the height reached by the nail head:

$$l_r=\max(x_t) \tag{12}$$

$$l_d=\min(x_n) \tag{13}$$

3 Formulating an Optimization Problem

When targeting a particular market niche, the performance criteria modeled in Section 2 are used to formulate a design optimization problem. However, if two products are meant to target two separate niches, the respective optimization problems may have different objective functions and constraints, along with different limits and parameters assigned to the corresponding constraints.

For example, suppose that a nail gun manufacturer wishes to create a product family consisting of two separate nail guns. Nail gun A will be an industrial-quality gun for use by professional carpenters, and nail gun B will be a less expensive entry-level model for use by the typical weekend enthusiast. Since model A is the flagship model there is a premium placed on performance, so the objective for model A is to maximize the size of the nail that can be driven into the wood. Model B, intended for the casual user, prescribes a smaller nail and puts a premium on user comfort. Its objective function is to minimize the recoil the user experiences. Two independent optimal design problems can be stated. The model for Product A is

$$\begin{aligned} \underset{x_A}{\text{minimize}} \quad & -m_{n,A} \\ \text{subject to} \quad & l_{h,A}-l_{n,A}\geq 0 \\ & l_{d,A}\geq l_{d\,\min,A} \\ & \tau_A\leq S_u \\ & N_A d_A-(l_{p,A}-l_{h,A})\leq 0. \end{aligned} \tag{14}$$

The maximum nail size ($m_{n,A}$) is multiplied by -1 in the objective to keep a common formalism of minimizing the objective. The model for Product B is

$$\begin{aligned} \underset{x_B}{\text{minimize}} \quad & l_{r,B} \\ \text{subject to} \quad & l_{h,B}-l_{n,B}\geq 0 \\ & l_{d,B}\geq l_{d\,\min,B} \\ & \tau_B\leq S_u \\ & N_B d_B-(l_{p,B}-l_{h,B})\leq 0. \end{aligned} \tag{15}$$

The different parameter values in Eqs (14) and (15) are given in Table 1. Both models (14) and (15) are specific cases of the general optimal design problem

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & \mathbf{g}(\mathbf{x})\leq \mathbf{0} \\ & \mathbf{h}(\mathbf{x})=\mathbf{0}. \end{aligned} \tag{16}$$

When making comparisons between two platforms, the best designs possible in one platform must be compared to the best designs possible in another platform. For instance, suppose that the internal spring mechanism housing is a candidate common part for both nail guns, requiring a consistent starting height l_h. This prompts the following question:

> Does the use of the same l_h in both designs significantly degrade the performance of the individual products?

In order to answer this question, one must compare the best possible designs when sharing parts to the best possible designs when the products are not sharing parts. In the present discussion a product platform is a specific set of shared parts among the

Table 1 Parameter values for the optimal design models in Eqs. (14) and (15)

Symbol	Description	Units	Prod. A	Prod. B
S_u	maximum shear stress	Pa	6.4E8	
G	shear modulus of wire	Pa	1.235E11	
d	wire diameter	m	0.002	
D	coil diameter	m	0.02	
l_p	preloaded length	m	0.12	
l_u	unstretched length	m	0.139	0.141
N	number of coils		25	31
k_a	spring constant of user's arm	N/m	25	10
F_p	force in user's arm	N	20	10
m_t	mass of other components of gun	kg	2.5	2.0
m_n	mass of the longest nail to be driven	kg	3.95E-3	4.29E-4
β	parameter determining nail-wood interaction		1590	324

different products. To model a product platform, the different optimization problems are combined into a single multicriteria optimization problem:

$$\begin{aligned} \underset{\mathbf{x}=[\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_p]}{\text{minimize}} \quad & f_i(x_i) && i=1\ldots p\\ \text{subject to} \quad & g_i(x_i)\leqslant 0 && i=1\ldots p\\ & h_i(x_i)\leqslant 0 && i=1\ldots p\\ & x_{i,k_1}=x_{j,k_2} && (k_1,k_2)\in P_{ij}\\ & && i,j=1\ldots p\\ & && i<j. \end{aligned} \tag{17}$$

The set P_{ij} is a set of index pairs used to represent the equality constraints associated with parts sharing. If products *i* and *j* share some of the same parts, then P_{ij} contains the index pairs of the design variables describing the common parts, effectively forcing the part to be the same for both products *i* and *j*. Thus a platform configuration is defined by a distinct set of index pairs. To compare two different product platforms, the solutions to model (17) are compared with different sets of index pairs, say $\{P_{ij}\}$ and $\{Q_{ij}\}$.

To illustrate this idea consider again two nail guns, *A* and *B*, designed such that they use the same spring-resetting mechanism, allowing a common housing. The variable describing the starting height of the hammer (I_h) is forced to be the same through the use of equality constraints. The two separate optimal design problems modeled in (14) and (15) are combined to form the multicriteria optimization problem

$$\begin{aligned} \underset{[\mathbf{x}_A,\mathbf{x}_B]}{\text{minimize}} \quad & f_A(x_A)=-m_{n,A}\\ & f_B(x_B)=l_{r,B}\\ \text{subject to} \quad & l_{h,A}-l_{n,A}\geqslant 0\\ & l_{d,A}\geqslant l_{d\,\min,A}\\ & \tau_A\leqslant S_u\\ & N_A d_A-(l_{p,A}-l_{h,A})\leqslant 0\\ & l_{h,B}-l_{n,B}\geqslant 0\\ & l_{d,B}\geqslant l_{d\,\min,B}\\ & \tau_B\leqslant S_u\\ & N_B d_B-(l_{p,B}-l_{h,B})\leqslant 0\\ & l_{h,A}=l_{h,B}. \end{aligned}$$

The equality constraint $l_{h,A}=l_{h,B}$ is represented by a set of index pairs containing one element, $P_{AB}=\{(1,1)\}$, because l_h is the first variable in $\boldsymbol{x}_A$ and $\boldsymbol{x}_B$. The platform consisting of $\{P_{AB}\}$ can be compared with *the null platform* where no parts are shared. The null platform is denoted by $\{O_{ij}\}$ where $O_{ij}=\{\}$ for every *i* and *j*.

For a specific P_{ij} set the solutions of model (17) and, therefore, model (18) form a *Pareto set*, defined such that for each point in the Pareto set it is not possible to improve the objective function of one product without making the other objective worse. Pareto optimality has been extensively studied, so no background will be presented here. For extensive reviews of its use in design the reader is referred to the books by Eschenauer et al. [13], Osyczka [14], and Statnikov and Matusov [15], or to articles by Freiheit and Rao [16], and Koski [17]. Because Pareto optimality gives a set of solutions rather than a unique solution, the next section discusses the limits of the change in performance by defining individual minima and bounds for the Pareto set.

4 Bounding the Pareto Optimal Solutions

Before determining the Pareto set for a particular platform, bounds can be placed on the performance of the products within the platform.

As a convention, different superscripts will represent optimal values from different platform configurations. For the nailer example a superscript circle represents optimal quantities for the null platform (f_A° and f_B°). A superscript bullet ($f_A^\bullet$ and $f_B^\bullet$) represents optimal quantities for the platform with the common parts. The individual minima $f_i^\bullet$ of Eq. (17) are defined as the extreme values of the Pareto set. These are the solutions to Eq. (17) with only one of the scalar functions (f_A or f_B) used as an objective.

There are three possible designs, (f_A°, f_B°), $(f_A^\bullet, f_B)$, and $(f_A, f_B^\bullet)$. An additional fictitious design $(f_A^\bullet, f_B^\bullet)$ called the *utopia point*, is also considered. Plotted together in Fig. 3, the four designs bound the Pareto set and provide a means of visualizing the cost of commonality. In most instances, the utopia point (an idealized best design under the commonality constraint) is outperformed by the null platform (which has no such constraint). In other words, forcing two products to share parts changes both designs, and no solution to model (18) can be better than the solutions to the separate optimal design problems. This leads to a general statement about product platforms:

If there are two sets of index pairs $\{R_{ij}\}$ and $\{S_{ij}\}$ such that $R_{ij}\subseteq S_{ij}$ for each *i* and *j*, then the feasible space for platform *S* cannot be larger than the feasible space for platform *R*. Therefore the performance for platform *R* will be at least that of platform *S* as measured by the objective function values.

In fact, the solution to the separate optimal design problems (the null platform) will typically be *better* than the utopia point of Eq. (18), and the designer of a product platform should *expect* to give up some acceptable amount of performance. The magnitude of this sacrifice is an indication of the cost of commonality.

This is important for three reasons. First, by defining the individual minima, the limits of the Pareto set and therefore the changes in performance are known. Second, simply quantifying the change in performance is useful to justify further investigation. If there is too much degradation in performance (i.e.,

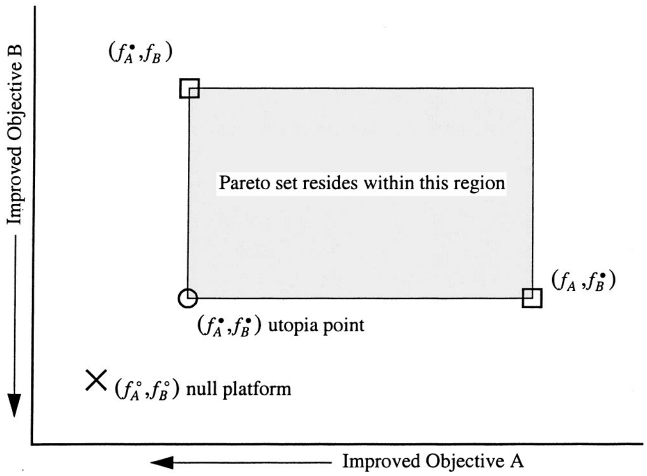

Fig. 3 The bounds on the Pareto set. The distance from the null platform to the utopia point is an indication of the cost of commonality

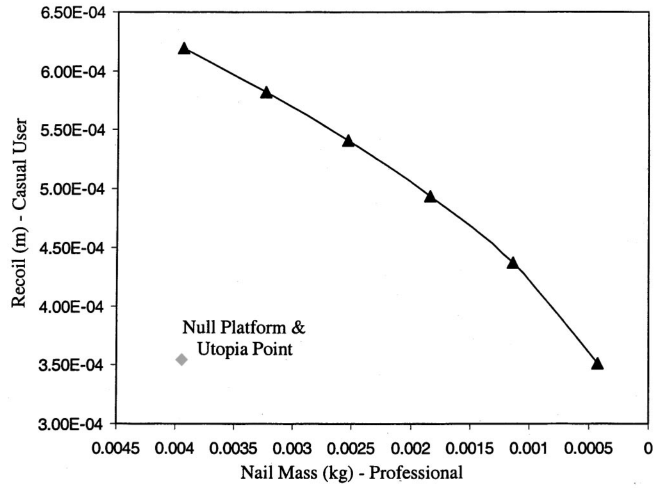

Fig. 5 The Pareto set of the multi-criteria optimal design problem for the nail gun

$f_A^\circ \ll f_A^\bullet$ or $f_B^\circ \ll f_B^\bullet$) the platform can be disregarded and there is no need to find other points within the Pareto set. Finally, most methods of determining the Pareto set use the individual minima.

For the nailer example, the optimum for the null-platform is located at $(f_A^\circ, f_B^\circ) = (3.95\text{E-}3\text{ kg}, 3.55\text{E-}4\text{ m})$, while the individual minima for the common spring-resetting platform are located at $(f_A^\bullet, f_B) = (3.95\text{E-}3\text{ kg}, 6.19\text{E-}4\text{ m})$, and $(f_A, f_B^\bullet) = (4.29\text{E-}4\text{ kg}, 3.51\text{E-}4\text{ m})$. Figure 4 is a plot of the performance of the three possible designs and the utopia point. Note that the entire Pareto set for the configuration lies in the rectangle with corners at $(f_A^\bullet, f_B)$, and $(f_A, f_B^\bullet)$. Moreover, the utopia point $(f_A^\bullet, f_B^\bullet)$, is as good as the null platform point representing the separate designs (f_A°, f_B°). As described earlier, this is an atypical situation, and is an indication that holding the spring-resetting mechanisms in common may be a good choice for a product platform and the full Pareto set should be generated.

5 Using the Pareto Set

There are two general approaches used to make single design decisions in multicriteria optimization. In the first approach, single points on the Pareto set are found based on a decision-maker's a priori preferences and trade-off decisions. In the second approach,

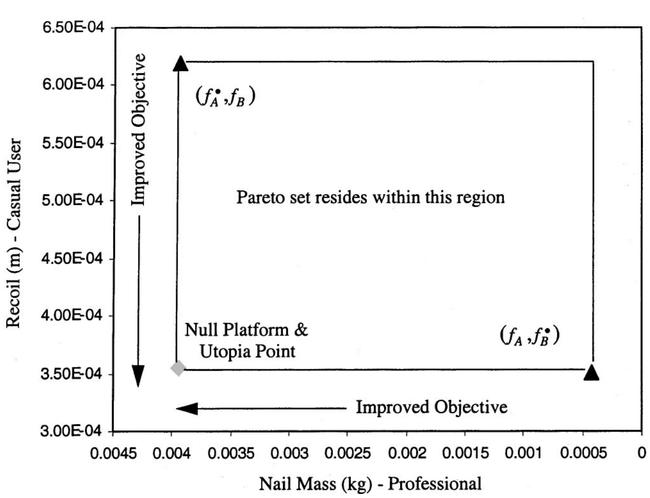

Fig. 4 Limiting cases of the optimal design of two separate nail guns using the same spring-resetting mechanism

the entire set is computed first and the decision-maker uses it to establish a posteriori preferences. In order to explore the trade-offs presented by different potential platform designs, automated methods that capture the complete Pareto set are more appropriate here. The reader is referred to the theses by Athan [18] or Das [19] for synopses of methods.

The Pareto set for the nailer example is given in Fig. 5. Notice that the utopia point $(f_A^\bullet, f_B^\bullet)$ is *not* part of the Pareto set. Only when there is no trade-off between designs does the utopia point belong to the Pareto set. Therefore, a final decision of whether or not to use the common part should not be made until the Pareto set (or at least part of the Pareto set) has been explored.

Some of the benefits of using a platform have not been explicitly taken into account in the present formulation. It is inherently difficult to quantify considerations such as the competitive advantage via faster product development or the reduced inventory, tooling, production costs and factory floor-space via smaller variety of parts. The problem is made more complex by the fact that sharing some parts may be more beneficial than sharing others, and the benefit is usually measured on the company level instead of the product performance level. The reader is referred to Meyer and Lehnerd [3] who make arguments for competitive advantages as well as Fujita et al. [20] who use simple models for quantifying these advantages monetarily.

Regardless, the discussion and example presented here is suggestive of a general method consisting of the following six steps:

1. Identify a set of parts that could be shared between two or more products. For each possible pair of products, assign a set of index pairs, i.e., $\{O_{ij}\},\{P_{ij}\},\{Q_{ij}\}, \ldots ,\{Y_{ij}\},\{Z_{ij}\}$.
2. Formulate the multicriteria design problem as modeled in Eq. (17).
3. Determine the individual optima (i.e., extreme points) of the multicriteria design problem for each possible configuration.
4. Use the individual optima for each configuration to decide if investigating the Pareto set of that configuration is worthwhile.
5. For each combination of common parts that has an acceptable degradation in performance calculate the Pareto set (or an appropriate approximation of it).
6. From the candidate Pareto sets choose the design that offers the best value for all appropriate products while still allowing for the benefits of having a flexible product platform.

The primary focus here is on the investigation and comparison of a number of possible combinations of parts. Each possible configuration has an associated set of index pairs. For example, many

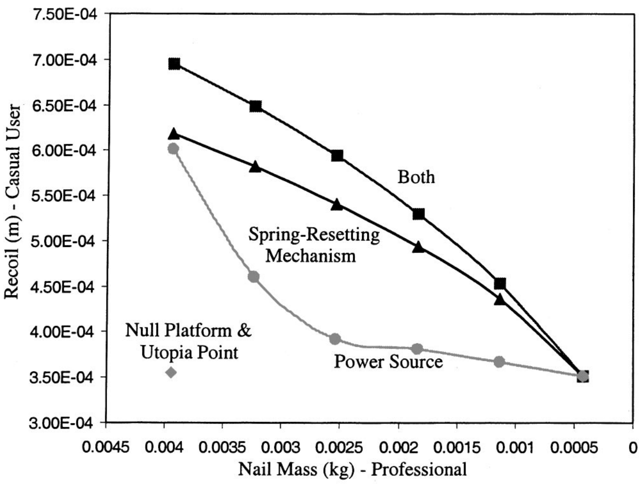

Fig. 6 Pareto sets for three possible platforms

product platforms are possible from the nail gun model, such as a common spring-housing, common spring-wire, common power source, or any combination of these. Using the methodology presented above two more Pareto curves can be generated, as shown in Fig. 6. One curve investigates the effect of having a common power source across the product platform. The nature of this curve is quite different from that obtained in the previous example. Note that when the spring-resetting mechanism is held in common, as in the previous example, the performance trade-off between the two modes (or objective functions) is nearly linear—there is no point on the curve that is significantly nearer the null platform than any other. That is not the case when the power source is the common part. Although the individual optima $(f^{\bullet}_A, f_B)$ and $(f_A, f^{\bullet}_B)$ are in nearly the same location as the previous example, the curve dips in towards the null platform/utopia point. The area nearest the null platform/utopia point, the ''knee,'' represents the portion of the design space where the least amount of performance is sacrificed to commonality (see Das [21] for an in-depth discussion of this region).

A third Pareto curve is created to represent the performance when both the spring-resetting mechanism and the power source are common across the platform. Note that the knee that existed in the power-source curve no longer exists. Indeed, the constraint inducing most of the performance reduction, namely that of the common spring-resetting mechanism, dominates the response. The relationship between this last curve and the first two is similar to that between the utopia point and the null platform: when a new commonality constraint is introduced, the performance of the product cannot improve—at best it remains the same.

The optimal platform design should lie in one of these Pareto sets, but exactly which Pareto set is ''the best'' is a question of performance as well as of other business issues. Defining the product platform configuration is a combinatorial problem because it requires searching through several possible configurations and making decisions based on the Pareto sets of each configuration.

6 Conclusion

A product platform, being a set of common components, modules or parts from which a stream of derivative products can be efficiently created and launched, is becoming a popular business practice today. The main premise in this paper was that a product platform can be formulated as a multicriteria optimization problem. In so doing, it has been shown that the performance of the products within the platform will degrade, and that the amount given up in performance can be quantified. The designer of a product platform should therefore *expect* to make compromises that can be rigorously studied through a multicriteria optimization process.

Acknowledgments

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Nomenclature

A = Subscript denoting product A
B = Subscript denoting product B
f = Objective function
f° = The optimal value of the objective functions for the null platform
$f^{\bullet}$ = The optimal value of the objective functions for a given platform
g = Vector of inequality constraints
h = Vector of equality constraints
i = Subscript index denoting a particular product
j = Subscript index denoting a particular product
k = Design variable index used to define a platform
p = Number of products in a particular platform
x = Vector of design variables
$\{O_{ij}\}$ = Empty index set (null platform)
$\{P_{ij}\},\{Q_{ij}\},\ldots$ = Index set denoting a specific product platform

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