

Antenna Optimization Using Sequential Quadratic Programming (SQP) Algorithms

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1. Introduction

Antenna design is a topic of great importance to electromagnetics and involves the selection of antenna physical parameters to achieve optimal gain, pattern performance, VSWR, bandwidth and so on, subject to specified constraints. Over the past 10 years, a variety of sophisticated computer codes have been developed for antenna analysis based on a variety of popular methods. The utility of these codes is, of course, greatly enhanced if they provide the user with a design capability. However, to date, the codes have not been extended to include design capabilities primarily because of their complexity and non-linearity with respect to the physical properties of the antenna (material constants, dimensions, feed location and type, etc.). Some design algorithms have been proposed but these are only applicable to specialized antenna shapes and do not address the general antenna optimization.

Recently, genetic algorithms (GAs) have been examined for array design and absorber optimization [1-2]. However, in both cases the response function was available in analytic form and required minimal computation time. This is consistent with the GA algorithm which, although robust, requires a large number of function evaluations to complete the optimization study. In contrast, antenna simulations rely on complex and time consuming integral equations or finite element codes. It may therefore be impractical to generate a sufficiently large sample space for carrying out an optimization using GAs which are best suitable for discrete objective functions.

An alternative optimization algorithm is the Sequential Quadratic Programming (SQP) [3] suitable for continuous non-linear objective functions such as the input impedance, gain, pattern shape and so on, with both equality and inequality constraints. The basic principle of sequential approximations is to replace the given nonlinear problem by a sequence of subproblems that are easier to solve. Convergence is typically achieved in a few iterations, and therefore their interface with rigorous but expensive numerical antenna analysis codes is much more practical. In this paper, we examine with an example the performance of a general SQP code for designing patch antennas in conjunction with a finite

element-boundary integral code.

The SQP algorithms consider the equality constrained problem:

$$\begin{aligned} & \min f(\mathbf{x}) \\ & \text{subject to } \mathbf{h}(\mathbf{x}) = \mathbf{0} \end{aligned}$$

where \mathbf{x} represents the design variable set, and $f(\mathbf{x})$ is the objective function. The other functions $\mathbf{h}(\mathbf{x})$ are equality constraints.

Using the Lagrange-Newton method described in most optimization books, we have:

$$\begin{bmatrix} \mathbf{W}_k & \mathbf{A}_k^T \\ \mathbf{A}_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{s}_k \\ \lambda_{k+1} \end{bmatrix} = \begin{bmatrix} -\nabla f_k^T \\ -\mathbf{h}_k \end{bmatrix} \quad (1)$$

where $\mathbf{W} = \nabla^2 f + \lambda^T \nabla^2 \mathbf{h}$, $\mathbf{A} = \nabla \mathbf{h}$.

Solving the above equation iteratively, we can obtain the iterates $\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{s}_k$ and λ_{k+1} which should eventually approach \mathbf{x} and λ , the optimal values.

Alternatively, we may observe that the above equation can be viewed as the KKT conditions for the quadratic model:

$$\begin{aligned} & \min_{\mathbf{s}_k} q(\mathbf{s}_k) = f_k + \nabla_{\mathbf{x}} L_k \mathbf{s}_k + \frac{1}{2} \mathbf{s}_k^T \mathbf{W}_k \mathbf{s}_k \\ & \text{subject to } \mathbf{A}_k \mathbf{s}_k + \mathbf{h}_k = \mathbf{0} \end{aligned} \quad (2)$$

where $\nabla_{\mathbf{x}} L_k = \nabla f_k + \lambda_k^T \nabla \mathbf{h}_k$

Solving the quadratic programming subproblem (2) gives the same \mathbf{s}_k and λ_{k+1} as solving equation (1) and thus the two formulations are equivalent. In the second formulation, the values of \mathbf{x} and λ will be obtained from solving a sequence of quadratic programming (QP) subproblems, hence comes the name of SQP methods for the relevant algorithms. The QP subproblem can be solved efficiently by some general NLP algorithms such as reduced space or projection methods or augmented Lagrangian methods. Using active set strategy, we can also handle problems with both equality and inequality constraints.

2. Antenna Optimization Using SQP

In this section we give an example of microstrip antenna optimization using SQP algorithms. The flowchart of the whole process is shown in Figure 1. For the calculation of the objective function, a hybrid finite element algorithm is used to compute electromagnetic scattering and radiation by an open three-dimensional rectangular cavity recessed in an infinite ground plane [4] (see Figure 2). The cavity may support microstrip patch or slot antennas and may be filled with layered dielectric material. The problem is formulated using the finite element method and boundary integral equation and the resulting system of equations is solved via the biconjugate gradient method. Besides different layers of dielectrics, the cavity may also have lumped loads, probe feeds, and short circuit pins.

In accordance with the flowchart, the FEM code first computes the values of initial

points which are used by the optimizer to determine the new search direction and step size. The process is repeated till convergence within the given tolerance is reached.

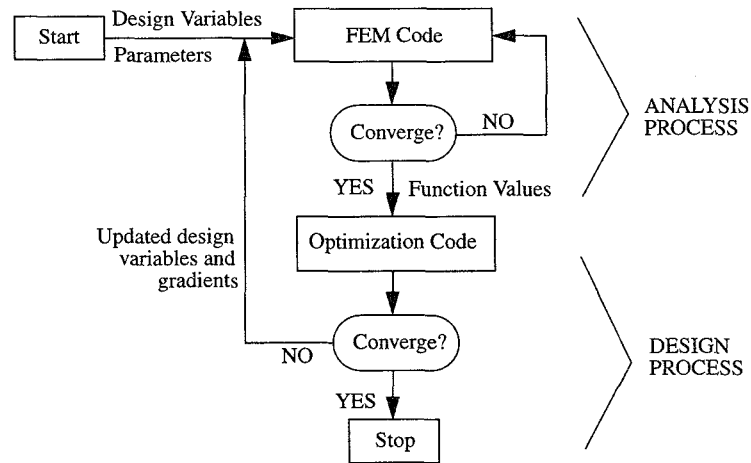


Figure 1: Flowchart of SQP combined with FEM.

Figure 2 illustrates the configuration of the stacked antenna under investigation. The top and lower substrates have a dielectric constant of $\epsilon_r = 2.2$ and thickness $h=1.59\text{mm}$, and the driven patch is designed to operate at 1.53 GHz. The middle substrate has a relative permittivity of $\epsilon_r = 1.1$. We wish to find the optimum length, width and separation of the patches to achieve a 15% bandwidth. As a starting point in the optimizer, we will use the values from the cavity model, i.e., $L = 5.73\text{ cm}$ and $W = 6.60\text{ cm}$, respectively. The bottom patch is probe-fed and the top patch is parasitically coupled to the driven patch. The top patch has the same dimensions.

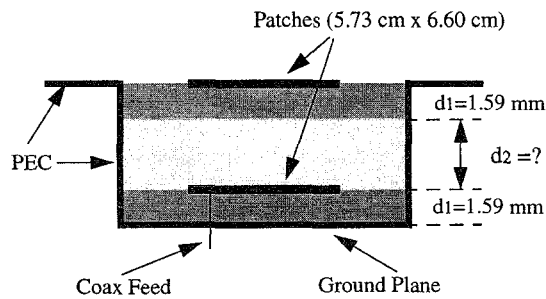


Figure 2: Geometry of the dual patch antenna.

Keeping the patch lengths and widths at $L = 5.73$ cm and $W = 6.60$ cm, respectively, the problem statement is: minimize $(R_{in}(f))/R_0(f_0)$. In our case R_{in} is the input resistance with f_0 denoting the center frequency within the band. After 9 iterations of the optimizer, a value of $d_2 = 14.25$ mm was determined which delivers a bandwidth of 15%. The VSWR is less than 2.09, within the entire bandwidth and this is a great improvement from the original single patch. The center frequency is 1.53 GHz if the patch size is 5.73 cm \times 6.6 cm. Figure 3 gives the iteration history for the dual patch optimization.

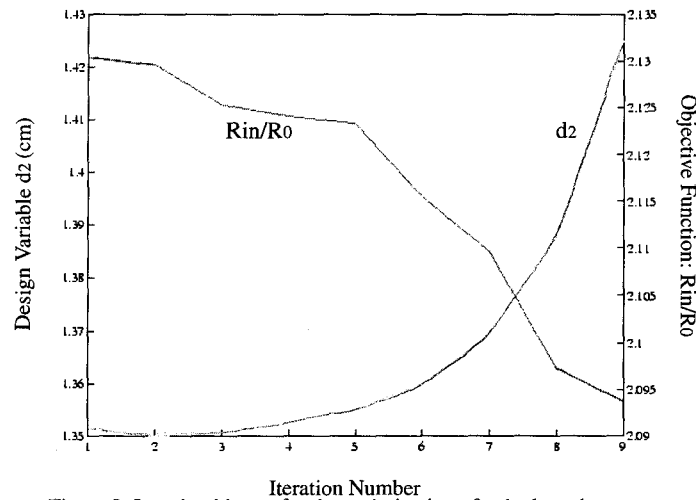


Figure 3: Iteration history for the optimization of a dual patch antenna.

3. References

- [1] Haupt, R.L. "Thinned arrays using genetic algorithms", *IEEE Transactions on Antennas and Propagation*, vol. 42, no. 7, pp. 993-999, 1994.
- [2] Weile, D.S., Michielssen, E. and Goldberg, D.E. "Genetic algorithm design of Pareto optimal broadband microwave absorbers", *IEEE Transactions on Electromagnetic Compatibility*, vol. 38, pp. 518-525, 1996.
- [3] Papalambros, P.Y. and Wilde, D. Chap. 7, *Principles of Optimal Design*, Cambridge University Press, 1988.
- [4] Volakis, J.L., Gong, J. and Alexanian, A. "A finite element-boundary integral method for antenna RCS analysis", *Electromagnetics*, vol. 14, no. 1, pp. 63-85, 1994.