Designing Broad-Band Patch Antennas Using the Sequential Quadratic Programming Method

Zhifang Li, Panos Y. Papalambros, and John L. Volakis, *Fellow, IEEE*

Abstract—**The utility of numerical codes is greatly enhanced if they can be used in design, a situation that typically involves iterative optimization algorithms. An attractive way is to use gradient-based algorithms developed for solving nonlinear programming (NLP) problems. In this letter, we examine the performance of a general sequential quadratic programming (SQP) optimization algorithm for designing patch antennas in conjunction with a finite-element boundary-integral code.**

***Index Terms*—Microstrip antennas.**

I. Introduction

ANTENNA design involves the selection of the physical antenna parameters to achieve optimal gain, pattern performance, VSWR, bandwidth, and so on, subject to specified constraints. Over the past ten years, sophisticated computer codes have been developed for antenna analysis [1]–[3] based on a variety of popular methods. By and large, these codes have not been extended to include design capabilities primarily because of their complexity and nonlinearity with respect to the physical properties of the antenna (material constants, dimensions, feed location, and type, etc.). Some design algorithms have been proposed but these are applicable to specialized antenna shapes and do not address the general antenna optimization problem [4].

Recently, genetic algorithms (GA's) have been examined for array design and absorber optimization [5]–[7]. However, GA's, although robust, require large number of function evaluations to complete the optimization study. Also, GA's are more suitable for discrete variable problems. In contrast, antenna simulations rely on complex computationally intensive codes, which generate continuous functions. It may, therefore, be impractical to generate a sufficiently large sample space for carrying out an optimization study using GA's.

An alternative optimization algorithm is the sequential quadratic programming (SQP) method, suitable for continuous nonlinear objective functions such as the input impedance, gain, pattern shape, etc. with both equality and inequality constraints. Convergence is typically achieved in a few iterations and, therefore, their interface with rigorous (but expensive) numerical antenna analysis codes is much more practical. SQP and other similar algorithms are routinely used for large structural design problems involving finite-element analysis [8] and, thus, we can benefit from the extensive experience available in other disciplines.

In this letter, we examine the performance of a general SQP code [9] for designing patch antennas in conjunction with a finite-element boundary-integral code [10]. Both are rigorous general-purpose codes. The main point of the paper is to examine the suitability of SQP for antenna parameter optimization to achieve the design objectives subject to constraints. We will illustrate the performance of the optimizer using a few illustrative examples from simple to more complex.

Manuscript received April 18, 1997; revised July 10, 1997.

Z. Li and J. L. Volakis are with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109 USA.

P. Y. Papalambros is with the Department of Mechanical Engineering and Applied Mechanics, University of Michigan, Ann Arbor, MI 48109 USA.

Publisher Item Identifier S 0018-926X(97)07992-1.

II. Sequential Quadratic Programming Algorithm Description

SQP is a gradient-based class of methods that became prominent in the late 1970's [11]. They are considered the most efficient general-purpose nonlinear programming algorithms today. The basic principle of sequential approximations is to replace the given nolinear problem by a sequence of quadratic subproblems that are easier to solve.

Consider the equality constrained problem

$$\begin{aligned}&\min\ f(\boldsymbol{x})\\&\text{subject to } \boldsymbol{h}(\boldsymbol{x})=\boldsymbol{0}\end{aligned} \tag{1}$$

where $\boldsymbol{x}$ is the design variable vector, $f(\boldsymbol{x})$ is the objective function, and $\boldsymbol{h}(\boldsymbol{x})$ is the vector of equality constraints. Using a Lagrange–Newton method (see, for example, [11]), at the kth iteration, we have

$$\begin{bmatrix}\boldsymbol{W}_k & \boldsymbol{A}_k^T\\ \boldsymbol{A}_k & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{s}_k\\ \lambda_{k+1}\end{bmatrix}=\begin{bmatrix}-\nabla f_k^T\\ -\boldsymbol{h}_k\end{bmatrix} \tag{2}$$

where $\boldsymbol{W}=\nabla^2 f+\lambda^T\nabla^2\boldsymbol{h}, \boldsymbol{A}=\nabla\boldsymbol{h}$, and λ is the vector of Lagrange multipliers. Solving the above equations iteratively, we obtain the iterates $\boldsymbol{x}_{k+1}=\boldsymbol{x}_k+\boldsymbol{s}_k$ and λ_{k+1} which should eventually approach $\underset{\sim}{\boldsymbol{x}}$ and $\underset{\sim}{\lambda}$, the optimal values.

We observe that the above equation can be viewed as the first-order optimality (Karush–Kuhn–Tucker) conditions for the quadratic model

$$\begin{aligned}&\min\ q(\boldsymbol{s}_k)=f_k+\nabla_{\boldsymbol{x}}L_k\boldsymbol{s}_k+\tfrac{1}{2}\boldsymbol{s}_k^T\boldsymbol{W}_k\boldsymbol{s}_k\\&\text{subject to } \boldsymbol{A}_k\boldsymbol{s}_k+\boldsymbol{h}_k=\boldsymbol{0}\end{aligned} \tag{3}$$

where $\nabla_{\boldsymbol{x}}L_k=\nabla f_k+\lambda_k^T\nabla\boldsymbol{h}_k$. Solving the quadratic programming subproblem (3) gives the same $\boldsymbol{s}_k$ and λ_{k+1} as solving (2) and thus the two formulations are equivalent. The values of $\underset{\sim}{\boldsymbol{x}}$ and $\underset{\sim}{\lambda}$ can be obtained from solving a sequence

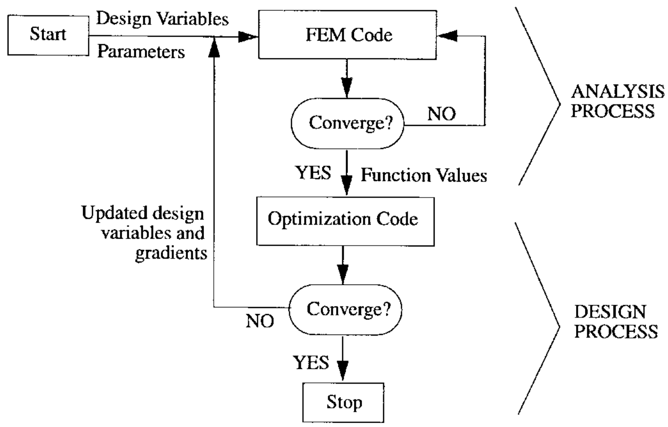

Fig. 1. Flow chart of SQP combined with FEM.

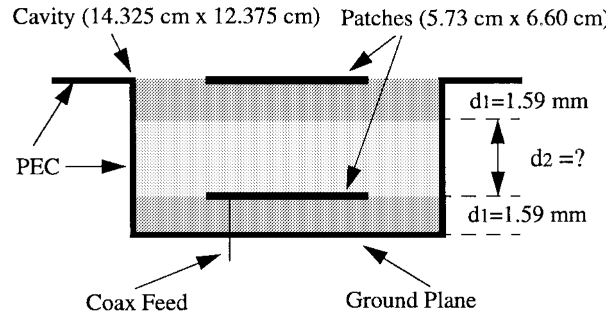

Fig. 2. Geometry of the dual-patch antenna.

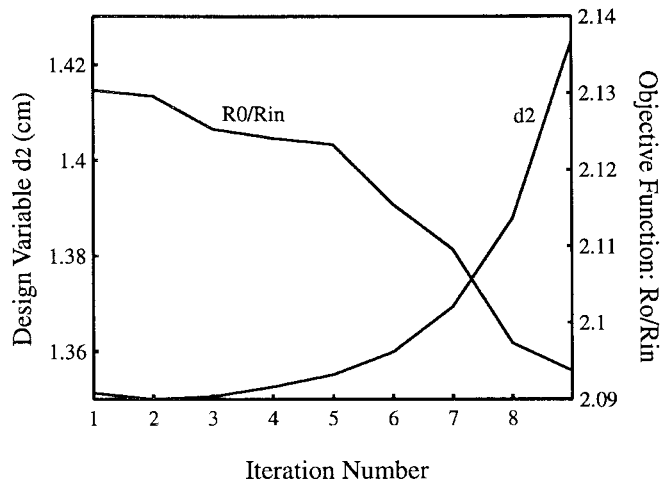

Fig. 3. Iteration history for the optimization of the dual-patch antenna.

of quadratic programming (QP) subproblems, hence the name SQP methods for the relevant algorithms.

Proper convergence proeprties are achieved with some modifications on this basic SQP algorithm. We may view $\boldsymbol{s}_k$ as a search direction and define the iterate as $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha_k \boldsymbol{s}_k$ where the step size α_k is introduced and computed by minimiznig an appropriate merit function along the search direction. The QP subproblem can be solved efficiently by well-developed QP solvers, based, for example, on projection or augmented Lagrangian methods. Using an active set strategy, problems with both equality and inequality constraints can be solved.

III. Combining SQP with the Finite-Element Method

In the next section, we give two examples of microstrip antenna optimization using the SQP algorithms. For the calculation of the objective function, a hybrid finite-element algorithm is used to compute electromagnetic scattering and radiation by an open three-dimensional rectangular cavity recessed in an infinite ground plane [10]. The cavity may support microstrip patch or slot antennas and may be filled with layered dielectric material. The problem is formulated using the finite-element boundary-integral method and the resultinng system of equations is solved via the biconjugate gradient method. Besides different layers of dielectrics, the cavity may also have lumped loads, probe feeds, and short circuit pins. The flow chart of the whole process is shown below in Fig. 1.

Accordingly, the finite-element method (FEM) code first computes the objective function using an intial set of antenna parameters which is used by the optimizer to determine the new search direction and step size. The process is repeated until convergence within the given tolerance is achieved.

IV. Example Applications

A. Probe-Fed Dual Patch

A number of techniques have been suggested and implemented to improve the bandwidth of the microstrip patch antenna. One of them is stacking patches horizontally or vertically [12]. Several parameters (size of patches, substrate thicknesses, feed locations, etc.) can be used to maintain a low voltage standing wave ratio (VSWR) over a given frequency range. For our purposes, we choose the VSWR to be around two, which corresponds to a return loss of about 3 dB. Thus, we can write the problem statement as $R_0/R_{\text{in}} < 2$ where R_0 refers to the input resistance at resonance and R_{in} is the corresponding resistance at nearby frequencies.

Fig. 2 illustrates the configuration of the stacked antenna under investigation. The top and lower substrates have a dielectric constant of $\varepsilon_r = 2.2$ and thickness $h = 1.59$ mm and the driven patch is designed to operate at 1.53 GHz. The middle substrate has a relative permittivity of $\varepsilon_r = 1.1$. We wish to find the optimum length, width, and separation of the patches to achieve a 15% bandwidth. As a starting point in the optimizer, we use the values from the cavity model [13], i.e., $L = 5.73$ cm and $W = 6.60$ cm, respectively. The top patch was chosen to have the same dimensions.

After nine iterations of the optimizer, a value of $d_2 = 14.25$ mm was determined which delivers a bandwidth of 15%. The VSWR is less than 2.09 within the entire bandwidth. The center frequency is 1.53 GHz, if the patch size is 5.73 cm $\times$ 6.6 cm and Fig. 3 gives the iteration history for the dual-patch optimization.

B. Slot-Fed Dual Patch

There are many ways to improve impedance bandwidth such as impedance matching and multiple resonances [14].

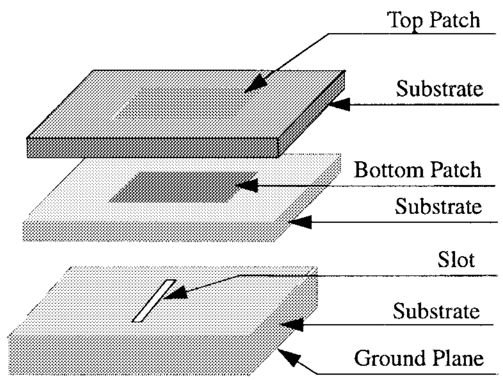

Fig. 4. Three-dimensional view of the stacked patches with aperture.

The latter (borrowed from tuned electronic amplifier design) is a popular approach and introduces additional resonant patches to provide two or more closely spaced resonances. The design principle of the previous example falls into this category. However, that example dealt with only a single-variable optimization. To improve the bandwidth for a dual-patch configuration, we now consider a multivariable optimization.

The structure of the stacked-patch antenna is shown in Fig. 4. The size of the top patch is slightly larger than that of the bottom patch to get a better bandwidth and a slot feed is put below the bottom patch. Among all geometric parameters which are sensitive to the design, we choose three for optimization. Specifically, the slot length, and the two substrate thicknesses were chosen for optimization with the goal of achieving a bandwidth more than 15% with VSWR < 2. The dimensions of the other parameters are given below:

TOP PATCH: length = 3.89 mm/ width = 5 mm
thickness to be decided; $\varepsilon_r = 2.33$;
BOTTOM PATCH: length = 3.5 mm
width = 4.5 mm
thickness to be decided; $\varepsilon_r = 2.2$;
SLOT FEED: slot-width = 0.5 mm
slot length to be decided;
GROUND SUBSTRATE: thickness = 0.508 mm;
$\varepsilon_r = 2.2$.

The stacked patches are residing in a cavity 1.4 cm × 1.8 cm in size.

The problem statement is the same as in the last example, i.e. min R_0/R_{in} plus some size constraints, and the antenna analysis simulation is also based on the same hybrid finite-element code. After six iterations, the SQP optimizer found the following optimal values for the three unknown parameters: thickness of the substrate supporting top patch = 0.85 mm; thickness of the substrate supporting the bottom patch = 0.55 mm; and slot length = 4 mm. The performance of this antenna is shown in Fig. 5, with a VSWR equal to only 1.414 over an 18% bandwidth.

V. CONCLUSION

The advantages of the SQP algorithm are fast convergence and reduced number of function evaluations. This is attractive when expensive computations are needed within the optimization loop. SQP achieves its speed by restricting its search in a more narrow range (local optimization) than genetic algorithms and by making use of gradient information. This is suitable for a large class of antennas where a limited range of parameter values is sufficient for good performance.

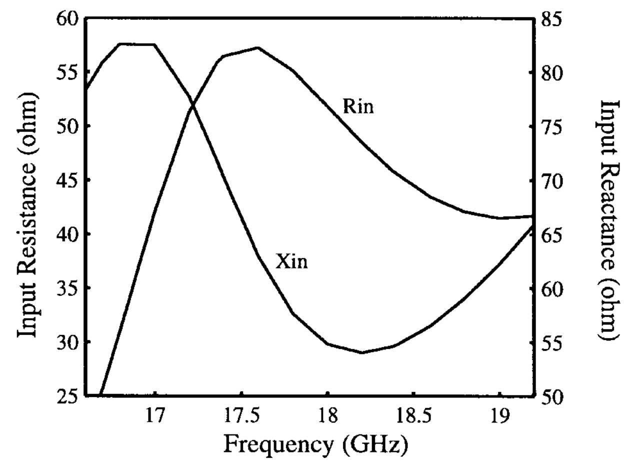

Fig. 5. Performance of the optimized dual patch-slot antenna.

REFERENCES

[1] D. M. Pozar and D. H. Schaubert, *Microstrip Antennas*. New York: IEEE Press, 1995 (reprints volume).
[2] J. L. Volakis, J. Gong, and T. Ozdemir, "Application of the FEM to conformed antennas," in *Finite Element Software for Microwave Engineering*, T. Itoh, G. Pelosi, and P. Silvester, Ed. New York: Wiley, 1996, pp. 313–345.
[3] C. Penny and R. J. Luebbers, "Radiation and scatteirng of a square archimedean spiral antenna using FDTD," *Electromagn.*, vol. 14, no. 1, pp. 87–98, 1994.
[4] R. L. Haupt, "Introduction to genetic algorithms for electromagnetics," *IEEE Antennas Propagat. Mag.*, vol. 37, pp. 7–15, Apr. 1995.
[5] R. L. Haupt, "Thinned arrays using genetic algorithms," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 993–999, July 1994.
[6] D. S. Weile, E. Michielssen, and D. E. Goldberg, "Genetic algorithm design of Pareto optimal broadband microwave absorbers," *IEEE Trans. Electromagn. Compat.*, vol. 38, pp. 518–525, Aug. 1996.
[7] E. Michielssen, M. J. Sajer, S. Ranjithant, and R. Mittra, "Design of lightweight, broad-band microwavew absorbers using genetic algorithms," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1024–1030, June/July 1993.
[8] N. Olhoff and G. I. N. Rozvany, Eds., *Structural and Multidisciplinary Optimization.* Oxford, U.K.: Pergamon-Elsevier, 1995.
[9] R. L. Crane, K. E. Hillstrom, and M. Minkoff, "Solution of the general nonlinear programming problems with subroutine VMCON," Argonne National Labs., 1990.
[10] J. L. Volakis, J. Gong, and A. Alexanian, "A finite elemen t-boundary integral method for antenna RCS analysis," *Electromagn.*, vol. 14, no. 1, pp. 63–85, 1994.
[11] P. Y. Papalambros and D. J. Wilde, *Principles of Optimal Design.* New York: Cambridge Univ. Press, 1988.
[12] J. S. Dahele, S. H. Tung, and K. F. Lee, "Normal and inverted configurations of the broadband electromagnetically coupled microstrip antennas," in *IEEE Antennas Propagat. Soc. Int. Symp. Dig.*, Philadelphia, PA, June 1986, pp. 841–844.
[13] J. R. James and P. S. Hall, "Handbook of microstrip antenna," *Inst. Elect. Eng. Electromagn. Waves*. London, U.K.: Peter Peregrunus, 1989, ser. 28.
[14] D. M. Pozar, "A review of bandwidth enhancement techniques for microstrip antennas," *Microstrip Antennas: The Analysis and Design of Microstrip Antennas*, D. M. Pozar and D. H. Schaubert, Eds. New York: IEEE Press, 1995, pp. 157–166.
[15] F. Croq and D. M. Pozar, "Millimeter-wave design of wide-band aperture-coupled stacked microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 1770–1776, Dec. 1991.

Zhifang Li was born in China in June 1972. She received the B.S. degree in mathematics from Peking University, China, in 1995, and the M.S. degree in electrical engineering from the University of Michigan, Ann Arbor, in 1996. She is currently working toward the Ph.D. degree in electrical engineering and computer science at the University of Michigan.

Her research interests include antenna optimization and numerical methods.

Panos Y. Papalambros received the Diploma degree in mechanical and electrical engineering from the National Technical University of Athens, Greece, in 1974, and the M.S. and Ph.D. degrees from the Design Division of Mechanical Engineering, Stanford University, Stanford, CA, in 1976 and 1979, respectively.

He joined the University of Michigan faculty in 1979 where he currently serves as the Chair of the Department. He has published over 100 articles in the field of design automation and has co-authored (with D. J. Wilde) the textbook, *Principles of Optimal Design: Modeling and Computation*. He serves on the editorial boards for the following journals: *Artificial Intelligence in Engineering Design and Manufacturing*, *Engineering Design*, *Engineering Optimization*, *Integrated Computer-Aided Engineering*, and *Structural Optimization*. He is also past editor of the *Japan Society of Mechanical Engineers International Journal*, *Global Optimization* and the *ASME Journal of Mechanical Design*.

Dr. Papalambros is the Founding Director of the multi-university Automotive Research Center in Modeling and Simulation of Ground Vehicles, funded by the United States Army. He is a member of ASME, INFORS, MPS, SIAM, SME, SAE, and ASEE and a Fellow of ASME.

John L. Volakis (S'77–A'79–S'80–M'82–SM'88–F'96), for photograph and biography, see p. 507 of the March 1997 issue of this TRANSACTIONS.