Optimal Design of a Hybrid Electric Powertrain System*

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ABSTRACT

Optimal design of an electric hybrid powertrain system using a decomposition-based approach is presented. In this approach, a general system design problem is first formulated without specifying objectives. The mathematical model is analyzed using partitioning techniques, and an optimal design problem that can be readily decomposed and solved using an appropriate coordination strategy is derived. Basic concepts for hybrid powertrains in automotive applications and a mathematical design model are introduced. Different ways of synthesizing a hierarchically decomposed optimization problem statement are described, and one such problem is solved using a sensitivity-based coordination strategy.

I. INTRODUCTION

Using fossil fuels as an energy source for combustion in automotive engines results in the emission of several pollutants: carbon monoxide (CO), hydrocarbons (HCs), oxides of nitrogen (NOx), and particulate matter (PM) that is less than 10

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TABLE 1
Twenty years of federal emission standards [2] and emissions from an average passenger car [3]

<table>
<thead>
<tr>
<th>Years</th>
<th>HC (gm/mi)</th>
<th>CO (gm/mi)</th>
<th>NO$_x$ (gm/mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975–76</td>
<td>1.5</td>
<td>15.0</td>
<td>3.1</td>
</tr>
<tr>
<td>1985–86</td>
<td>0.41</td>
<td>3.4</td>
<td>1.0</td>
</tr>
<tr>
<td>1995–96</td>
<td>0.25</td>
<td>3.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Passenger Car Average</td>
<td>3.1</td>
<td>23.0</td>
<td>1.6</td>
</tr>
</tbody>
</table>

microns in size [1]. Efforts in the last 20 years to reduce pollution from automobiles leave substantial room for improvement (see Table 1) [2,3]. In an effort to reduce pollution, the EPA has classified future vehicles into the following categories: low emissions vehicles (LEV), ultra low emissions vehicles (ULEV), transitional low emissions vehicles (TLEV), and zero emissions vehicles (ZEV). Eventually, automobile manufacturers will be required to produce a specific percentage of their fleets in each category.

One way to reduce emissions is to use an electric energy source to power the vehicle instead of the conventional internal combustion (IC) engine. A combination of energy sources—i.e., hybrid powertrains—can also be used. Such systems require careful design trade-off analysis like that provided by mathematical optimization studies. This article focuses on the optimal design of a particular type of hybrid powertrain; however, section II first reviews different hybrid possibilities.

II. TYPES OF HYBRID POWERTRAINS

Automotive powertrains can be classified as IC engine, electric, and hybrid electric. Gas turbines used in military applications are not included here. Figures 1 and 2 show simple representations of a conventional IC engine-driven powertrain and an electric powertrain powered by a battery. In the conventional powertrain, a mechanical energy source such as an IC engine drives the wheels through a mechanical drive unit that usually comprises a clutch or a torque converter, a gear

![Fig. 1. An internal combustion engine (ICE) driven powertrain.](image-url)
box, a drive shaft, and a differential. In the electric powertrain, an electric energy source such as a battery powers the wheels through an electric drive and control unit (ECU). In its simplest form, the ECU consists of a motor, a controller, and sometimes a gear box coupled to the drive axle.

In a hybrid vehicle, propulsion energy is available from two or more kinds of energy stores, sources, or converters. In a hybrid electric vehicle, at least one of the energy stores, sources, or converters can deliver electric energy [4]. In a series configuration, all of the energy sources together drive a single drive unit that drives the wheels. In a parallel configuration, each energy store, source, or converter drives the wheels through a single predetermined drive unit. A mixed configuration has one or more energy stores, sources, or converters that can drive more than one drive unit; i.e., they can work in series and in parallel with other stores, sources, and converters. Figure 3 shows a general configuration that allows for all arrangements, as does Burke [5]. There are energy sources $E_1, \ldots, E_m$ and drive units $DU_1, \ldots, DU_m$, where $J_n$ is the junction that couples the drive units to the wheels. The inverter and control unit (ICU) is an inverter-controller combination that allows one form of energy to change to another and decides how the energy flow is directed. The ICU can send and receive signals from the energy.
sources and drive units. Energy flow into and out of the energy sources, as well as energy flow into the drive units, is controlled by the ICU.

In a series hybrid, a single drive unit drives the wheels. In a parallel configuration, each $E_i$ drives a $DU_i$ that is coupled to the wheels through junction $J_n$. A mixed configuration arises if $E_1$ can drive $DU_1$, as well as $DU_2$ (that is also driven by $E_2$). The definitions for series, parallel, and mixed hybrids are based on work by Willis and Radke [6].

Different modes of operation are possible for the above configurations: the power-assist, dual, and charging modes. In the power-assist mode, one or more energy sources provide the average power, and one or more energy sources provide assistance for performance needs such as acceleration and hill climbing. In the dual mode, the total range the vehicle is intended to travel is divided into two. One set of energy sources operates for a particular distance and a second set (with at least one energy source not included in the first set) operates to achieve the vehicle’s full range. In the charging mode, one or more energy sources are used exclusively for charging, while the remaining sources power the drive units directly through the ICU. The charging process results in energy loss similar to that of the energy conversion process, and it must be accounted for if it is considered significant. A design model of a hybrid electric powertrain operating in a power-assist mode is developed in the following section. This configuration will be used in the optimal design study.

III. MODELING OF A HYBRID POWERTRAIN

In this section, the system configuration for this study is described and model equations are derived from basic principles and empirical data.

A. System Configuration

The system representation is shown in Fig. 4. It is assumed that the battery provides the average power required to drive the vehicle and a flywheel surge

![Diagram](image)

Fig. 4. A series electric hybrid powertrain.
unit provides short-time power demands for acceleration and hill climbing. Efficiencies of different subsystems and the overall efficiency assumed in this study are presented in Table 2a, and the average pollution resulting from charging the batteries compared to California power plant emissions [2] for every KWh of electricity produced is presented in Table 2b. The infrastructure needed for charging is based on California data: power production capability of 44500 MW and energy consumption per year of $125750 \times 10^6$ KWh [7].

The specification of vehicle parameters assumed in this study is presented in Table 3. The 0–60 miles per hour (mph) acceleration time is usually the most stringent performance requirement, so it is the only vehicle acceleration specification considered here. The travel range is at least 80 miles of city driving, assumed to be the same as the Federal Urban Drive Schedule (FUDS).

Ttractive power as a function of vehicle mass and the energy per mile required to drive a vehicle of the specifications just described in the FUDS are given in Eqs. 1 and 2, respectively [8].

\[
TR \text{ (KW at wheels)} = 3.0 + 0.036 \text{ (Mass of Vehicle)} \quad (1)
\]

\[
\text{Wh/mile (at wheels)} = 0.0821 \text{ (Mass of Vehicle)} + 49.2 \quad (2)
\]

<table>
<thead>
<tr>
<th>TABLE 2</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Battery Powertrain Efficiency</th>
<th>(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>Efficiency</td>
</tr>
<tr>
<td>Battery (out/in)</td>
<td>0.8</td>
</tr>
<tr>
<td>Electric Drive + Control Unit</td>
<td>0.9(0.95) = 0.855</td>
</tr>
<tr>
<td>Overall</td>
<td>0.8(0.855) = 0.684</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Power Plant Pollution</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pollutant</td>
<td>gms/KWh</td>
</tr>
<tr>
<td>NO$_x$</td>
<td>0.6015</td>
</tr>
<tr>
<td>HC</td>
<td>0.0071</td>
</tr>
<tr>
<td>CO</td>
<td>0.0782</td>
</tr>
<tr>
<td>PM</td>
<td>0.0210</td>
</tr>
<tr>
<td>SO$_2$</td>
<td>0.4058</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 3</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>SPECIFICATION</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of vehicle without battery</td>
<td>1250 Kgs</td>
</tr>
<tr>
<td>Coefficient of rolling friction</td>
<td>0.0075</td>
</tr>
<tr>
<td>Frontal area</td>
<td>2 m$^2$</td>
</tr>
<tr>
<td>Drag coefficient ($C_D$)</td>
<td>0.25</td>
</tr>
<tr>
<td>Density of air ($\rho$)</td>
<td>1.20 Kg/m$^3$</td>
</tr>
</tbody>
</table>
Fig. 5. Relationship between energy density and power density of a NiMH battery.

The use of a NiMH battery is assumed in this study. The energy density of most batteries is a function of power density. In Fig. 5, which is plotted on a log-log scale, the energy and power delivered by a NiMH battery is shown as a function of its mass and volume [9]. In Fig. 6 the same information is shown for a flywheel used as a mechanical storage unit [10]. The cost of a battery is difficult to predict under mass production conditions. Based on several references [1, 8, 11], the average price of a NiMH battery is assumed to be $187.5/KWh using a price range of $150/KWh to $225/KWh. A battery is assumed to have lived its life when it can hold only 80 percent of the energy it could hold when it was new. Batteries live longer if discharged to a smaller depth of discharge (DOD) before recharging, where DOD is the percentage of energy capacity that has been

(a)

Fig. 6. Relationship between energy density and power density of a flywheel.
withdrawn from a battery. In addition, the cost of charging the batteries is assumed to be four cents per KWh [1] and the charging efficiency is assumed to be 90 percent. The maintenance cost of the hybrid electric vehicle is assumed to be 1.9 cents per mile, the same as for an electric vehicle [1].

B. Design Model Development

The battery curves relating energy density and power density can be described in equation form, using curve fitting techniques, as shown in Eqs. 3 and 4,

\[ E_{db} = f(P_{db}) = 80.747 - 0.18136(P_{db}) - 0.00036241(P_{db})^2 \]  \hspace{1cm} (3)
\[ E_{df} = f(P_{df}) = 85.033 - 0.053453(P_{df}) - 0.000046541(P_{df})^2 \]  \hspace{1cm} (4)

where \( P_{db} \) and \( P_{df} \) are the power densities of the battery and the flywheel, respectively, and \( E_{db} \) and \( E_{df} \) are the energy densities of the battery and the flywheel, respectively.

The energy consumed per mile, based on Eq. 2, and the distance the vehicle can travel are

\[ \frac{\text{Wh/mi}}{} = 0.0821(M_b + M_v + M_f) + 49.20 \]  \hspace{1cm} (5)
\[ R = E_{db}\eta_{db}M_b/(\text{Wh/mi}) \]  \hspace{1cm} (6)

where \( M_f \) is the mass of the flywheel unit, \( M_v \) is the mass of the vehicle excluding battery and flywheel, and \( \eta_{db} \) is the efficiency of the battery drivetrain. Based on Table 3, the variation of the power delivered by the battery or the flywheel as a function of power density is given in Eqs. 7 and 8. The expression of how the volume of the battery and the flywheel changes with the power density is given in Eqs. 9 and 10, where

\[ P_{lb} = f(P_{db}) = 3.1(P_{db}) \]  \hspace{1cm} (7)
\[ P_{lf} = f(P_{df}) = 0.85(P_{df}) \]  \hspace{1cm} (8)
\[ V_b = (1/P_{lb})(M_bP_{db}) \]  \hspace{1cm} (9)
\[ V_f = (1/P_{lf})(M_fP_{df}) \]  \hspace{1cm} (10)

The expressions for energy consumed in charging and the cost of charging assuming a cost of four cents per KWh are given in Eqs. 11 and 12, respectively. The battery cost and the cost per mile of driving are given in Eqs. 13 and 14, where

\[ E_{ch}(\text{KWh/mi}) = (\text{Wh/mi})/(\eta_{db}\eta_{ch}1000) \]  \hspace{1cm} (11)
\[ C_{ch}(\text{cents/mi}) = (4)E_{ch} \]  \hspace{1cm} (12)
\[ B_c = (100)187.5[(\text{Wh/mi})/(\eta_{db}1000)]/1000 \]  \hspace{1cm} (13)
\[ T_{ch}(\text{cents/mi}) = M_c + C_{ch} + B_c \]  \hspace{1cm} (14)
\( \eta_{db} \) is the efficiency of the battery drive, \( \eta_{ch} \) is the efficiency of charging, and \( M_c \) is the maintenance cost of the vehicle.

Battery emissions attributed to the power plants based on Table 3 are presented in Eqs. 15, 16, and 17, where

\[
\text{NO}_x = f(E_{ch}, \text{p.plant NO}_x) = (0.6015)E_{ch} \tag{15}
\]
\[
\text{CO} = f(E_{ch}, \text{p.plant CO}) = (0.0782)E_{ch} \tag{16}
\]
\[
\text{HC} = f(E_{ch}, \text{p.plant HC}) = (0.0071)E_{ch} \tag{17}
\]

The equations for the torque delivered by the battery and the flywheel at the wheels is shown in Eqs. 18 and 19. The variation of the mass of the flywheel assuming a linear approximation based on current data on flywheel mass with power \([10]\) is shown in Eq. 20. The torque required at wheels based on Eq. 1, and assuming a linear approximation, is shown in Eq. 21, where

\[
TR_b = (P_{db}M_b)\eta_{bd}/1000 \tag{18}
\]
\[
TR_f = (P_{df}M_f)\eta_{fd}/1000 \tag{19}
\]
\[
M_f = f(P_f) = 1.67(P_f) + 3.33 \tag{20}
\]
\[
TR = [3.0 + 0.036(M_b + M_v + M_f)](12.0/t_{0-60}) \tag{21}
\]

Design criteria and bounds in the model are summarized in Eqs. 22 through 52 below.

Minimum torque requirement at the wheels:

\[
TR \leq TR_b + TR_f \tag{22}
\]

Minimum energy to be stored in the flywheel:

\[
E_{df}M_f \geq E_{\text{min}} \tag{23}
\]

Limit on the total driving costs, assuming a limit of 10 cents per mile:

\[
T_{ch} \leq 10 \tag{24}
\]

Limits on emissions:

\[
\text{NO}_x \leq (\text{NO}_x)_{\text{limit}} \tag{25}
\]
\[
\text{CO} \leq (\text{CO})_{\text{limit}} \tag{26}
\]
\[
\text{HC} \leq (\text{HC})_{\text{limit}} \tag{27}
\]

Limits on 0–60 acceleration time:

\[
10 \leq t_{0-60} \text{ (secs)} \leq 14 \tag{28,29}
\]
HYBRID ELECTRIC POWERTRAIN

Limits on power density of the battery:

\[ 10 \leq P_{db} \ (\text{W/Kg}) \leq 200 \]  
(30,31)

Limits on power density of the flywheel:

\[ 10 \leq P_{df} \leq 900 \]  
(32,33)

Limits on energy density of the battery:

\[ 30 \leq E_{db} \leq 80 \]  
(34,35)

Limits on energy density of the flywheel:

\[ 1.1 \leq E_{df} \leq 85 \]  
(36,37)

Limits on the travel range of the vehicle:

\[ 80 \leq R \ (\text{miles}) \leq 140 \]  
(38,39)

Bounds on the mass of the flywheel:

\[ 80 \leq M_f \ (\text{Kgs}) \leq 250 \]  
(40,41)

Bounds on the mass of the battery:

\[ 10 \leq M_b \ (\text{Kgs}) \leq 1000 \]  
(42,43)

Bounds on the volume of the battery:

\[ 10 \leq V_b \ (\text{litres}) \leq 300 \]  
(44,45)

Bounds on the volume of the flywheel:

\[ 10 \leq V_f \ (\text{litres}) \leq 200 \]  
(46,47)

Bounds on the mass of the vehicle without battery and flywheel:

\[ 1100 \leq M_v \ (\text{Kgs}) \leq 1350 \]  
(48,49)

Minimum torque to be delivered by the battery:

\[ (TR_b)_{\text{limit}} \leq TR_b \]  
(50)

Limits on the flywheel surge power:

\[ 60 \leq P_f \leq 150 \]  
(51,52)

Values of parameters that are not given above but are used in the equations are listed in Table 4. The system design variables are summarized in Table 5.
TABLE 4
Additional Parameter Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{bd}$</td>
<td>0.684</td>
<td>(CO)$_{\text{limit}}$</td>
<td>1.7 gms/mile</td>
</tr>
<tr>
<td>$\eta_{fd}$</td>
<td>0.726</td>
<td>(HC)$_{\text{limit}}$</td>
<td>0.039 gms/mile</td>
</tr>
<tr>
<td>$\eta_{ch}$</td>
<td>0.90</td>
<td>$E_{\text{min}}$</td>
<td>800 Watts</td>
</tr>
<tr>
<td>(NO$<em>x$)$</em>{\text{limit}}$</td>
<td>0.2 gms/mile</td>
<td>($TR_b$)$_{\text{limit}}$</td>
<td>40 KW</td>
</tr>
</tbody>
</table>

TABLE 5
List of System Design Variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_b = x_1$</td>
<td>$M_v = x_2$</td>
<td>$M_f = x_3$</td>
<td>$t_{0-60} = x_4$</td>
</tr>
<tr>
<td>$P_{db} = x_7$</td>
<td>$TR_b = x_8$</td>
<td>$TR_f = x_9$</td>
<td>$P_{df} = x_{10}$</td>
</tr>
<tr>
<td>Wh/mi = $x_{13}$</td>
<td>$R = x_{14}$</td>
<td>$P_{lb} = x_{15}$</td>
<td>$P_{lf} = x_{16}$</td>
</tr>
<tr>
<td>$E_{ch} = x_{19}$</td>
<td>$C_{ch} = x_{20}$</td>
<td>$B_c = x_{21}$</td>
<td>$T_{ch} = x_{22}$</td>
</tr>
<tr>
<td>HC = $x_{25}$</td>
<td>$E_{db} = x_{11}$</td>
<td>$E_{df} = x_{12}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_b = x_{17}$</td>
<td>$V_f = x_{18}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$NO_x = x_{23}$</td>
<td>CO = $x_{24}$</td>
<td></td>
</tr>
</tbody>
</table>

IV. DECOMPOSITION SYNTHESIS

Decomposition synthesis is defined as the process of creating a decomposable optimal design problem (ODP) from a general design problem [12]. This allows the so-constructed ODP to be composed and solved by a desired coordination method, a process especially useful in the optimal design of large systems.

A general design problem (GDP) is modeled as follows:

$$\text{Find } \mathbf{x} \in \mathcal{F} \text{ subject to } \mathbf{h}(\mathbf{x}, \mathbf{p}) = 0$$
$$\mathbf{g}(\mathbf{x}, \mathbf{p}) \leq 0$$

(53)

where $\mathbf{h}$ and $\mathbf{g}$ are vectors of design requirements represented by equalities and inequalities that are functions of the design variables $\mathbf{x}$ and parameters $\mathbf{p}$, and $\mathcal{F}$ is the set constraint on the design variables. For this model, the relations in Eq. 53 are summarized in Table 6. The corresponding equation numbers in the previous section are given in parentheses. A GDP can be transformed to an ODP by selecting one or more design criteria from the vector $\mathbf{g}$ above and composing a scalar objective, namely,

$$\text{Minimize } f(\mathbf{x}, \mathbf{p}, \mathbf{w}) \text{ subject to } \mathbf{h}(\mathbf{x}, \mathbf{p}) = 0$$
$$\mathbf{g}(\mathbf{x}, \mathbf{p}) \leq 0$$

(54)

where $\mathbf{w}$ is a vector of parameters that includes any weights used in composing the scalar objective $f$. The development of an appropriate objective for this model
TABLE 6
Functional relations in the hybrid vehicle general design problem

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. 21:</td>
<td>$h_1(x_1, x_2, x_3, x_4, x_5)$</td>
</tr>
<tr>
<td>Eq. 18:</td>
<td>$h_3(x_1, x_7, x_8)$</td>
</tr>
<tr>
<td>Eq. 3:</td>
<td>$h_5(x_7, x_{11})$</td>
</tr>
<tr>
<td>Eq. 5:</td>
<td>$h_7(x_1, x_2, x_3, x_{13})$</td>
</tr>
<tr>
<td>Eq. 7:</td>
<td>$h_9(x_7, x_{15})$</td>
</tr>
<tr>
<td>Eq. 9:</td>
<td>$h_{11}(x_1, x_7, x_{15}, x_{17})$</td>
</tr>
<tr>
<td>Eq. 11:</td>
<td>$h_{13}(x_{13}, x_{19})$</td>
</tr>
<tr>
<td>Eq. 13:</td>
<td>$h_{15}(x_{13}, x_{21})$</td>
</tr>
<tr>
<td>Eq. 15:</td>
<td>$h_{17}(x_{19}, x_{23})$</td>
</tr>
<tr>
<td>Eq. 17:</td>
<td>$h_{29}(x_{19}, x_{25})$</td>
</tr>
<tr>
<td>Eq. 23:</td>
<td>$g_2(x_3, x_{12})$</td>
</tr>
<tr>
<td>Eq. 25:</td>
<td>$g_4(x_{23})$</td>
</tr>
<tr>
<td>Eq. 27:</td>
<td>$g_6(x_{25})$</td>
</tr>
<tr>
<td>Eq. 29:</td>
<td>$g_8(x_4)$</td>
</tr>
<tr>
<td>Eq. 31:</td>
<td>$g_{10}(x_7)$</td>
</tr>
<tr>
<td>Eq. 33:</td>
<td>$g_{12}(x_{10})$</td>
</tr>
<tr>
<td>Eq. 35:</td>
<td>$g_{14}(x_{11})$</td>
</tr>
<tr>
<td>Eq. 37:</td>
<td>$g_{16}(x_{12})$</td>
</tr>
<tr>
<td>Eq. 39:</td>
<td>$g_{18}(x_{14})$</td>
</tr>
<tr>
<td>Eq. 41:</td>
<td>$g_{20}(x_3)$</td>
</tr>
<tr>
<td>Eq. 43:</td>
<td>$g_{22}(x_1)$</td>
</tr>
<tr>
<td>Eq. 45:</td>
<td>$g_{24}(x_{17})$</td>
</tr>
<tr>
<td>Eq. 47:</td>
<td>$g_{26}(x_{18})$</td>
</tr>
<tr>
<td>Eq. 49:</td>
<td>$g_{28}(x_2)$</td>
</tr>
<tr>
<td>Eq. 51:</td>
<td>$g_{30}(x_6)$</td>
</tr>
<tr>
<td>Eq. 20:</td>
<td>$h_2(x_3, x_6)$</td>
</tr>
<tr>
<td>Eq. 19:</td>
<td>$h_4(x_3, x_9, x_{10})$</td>
</tr>
<tr>
<td>Eq. 4:</td>
<td>$h_{10}(x_{10}, x_{12})$</td>
</tr>
<tr>
<td>Eq. 6:</td>
<td>$h_8(x_1, x_{11}, x_{13}, x_{14})$</td>
</tr>
<tr>
<td>Eq. 8:</td>
<td>$h_{10}(x_{10}, x_{16})$</td>
</tr>
<tr>
<td>Eq. 10:</td>
<td>$h_{12}(x_3, x_{10}, x_{16}, x_{18})$</td>
</tr>
<tr>
<td>Eq. 12:</td>
<td>$h_{14}(x_{19}, x_{20})$</td>
</tr>
<tr>
<td>Eq. 14:</td>
<td>$h_{16}(x_{20}, x_{21}, x_{22})$</td>
</tr>
<tr>
<td>Eq. 16:</td>
<td>$h_{18}(x_{19}, x_{24})$</td>
</tr>
<tr>
<td>Eq. 22:</td>
<td>$g_4(x_5, x_9, x_9)$</td>
</tr>
<tr>
<td>Eq. 24:</td>
<td>$g_5(x_{22})$</td>
</tr>
<tr>
<td>Eq. 26:</td>
<td>$g_5(x_{24})$</td>
</tr>
<tr>
<td>Eq. 28:</td>
<td>$g_7(x_4)$</td>
</tr>
<tr>
<td>Eq. 30:</td>
<td>$g_9(x_7)$</td>
</tr>
<tr>
<td>Eq. 32:</td>
<td>$g_{11}(x_{10})$</td>
</tr>
<tr>
<td>Eq. 34:</td>
<td>$g_{13}(x_{11})$</td>
</tr>
<tr>
<td>Eq. 36:</td>
<td>$g_{15}(x_{12})$</td>
</tr>
<tr>
<td>Eq. 38:</td>
<td>$g_{17}(x_{14})$</td>
</tr>
<tr>
<td>Eq. 40:</td>
<td>$g_{19}(x_3)$</td>
</tr>
<tr>
<td>Eq. 42:</td>
<td>$g_{21}(x_1)$</td>
</tr>
<tr>
<td>Eq. 44:</td>
<td>$g_{23}(x_{17})$</td>
</tr>
<tr>
<td>Eq. 46:</td>
<td>$g_{25}(x_{18})$</td>
</tr>
<tr>
<td>Eq. 48:</td>
<td>$g_{27}(x_2)$</td>
</tr>
<tr>
<td>Eq. 50:</td>
<td>$g_{29}(x_5)$</td>
</tr>
<tr>
<td>Eq. 52:</td>
<td>$g_{31}(x_6)$</td>
</tr>
</tbody>
</table>

is discussed next. Further details on this type of ODP synthesis are given by Krishnamachari [13].

A hierarchical decomposition synthesis methodology focuses on synthesizing ODPs that can be solved by a primal hierarchical decomposition method [12–15]. A block-angular structure is first identified in the GDP. An ODP is then created that can be hierarchically decomposed based on this structure. Formally, a GDP is first cast into the form

\[
\begin{align*}
g_0(x_0) & \leq 0 \\
h_0(x_0) & = 0 \\
g_i(x_0, x_i) & \leq 0 & i &= 1, \ldots, K \\
h_i(x_0, x_i) & = 0 & i &= 1, \ldots, K
\end{align*}
\]

(55)

(56)

that has a master problem and $K$ subproblems with the block-angular structure of Fig. 7. A hierarchically decomposed ODP is synthesized by composing a weighted additive objective, selecting criteria from Eqs. 55 and 56, as shown in Eq. 57 below.
and illustrated in Figs. 8 and 9. In general, variables $x_0$ and $x_i$ ($i = 1, \ldots, K$) can correspond both to decision and to other intermediate variables. Elimination of intermediate variables in the GDP should be done judiciously because such elimination could restrict the number of possible partitions. Furthermore, it may not be possible to eliminate some intermediate variables.
HYBRID ELECTRIC POWERTRAIN

\[
\begin{align*}
\text{minimize } f &= f_0(x_0, w_0) + \sum_{i=1}^{K} f_i(x_0, x_i, w_i) \\
\text{subject to: } &\quad g_0(x_0) \leq 0 \\
&\quad h_0(x_0) = 0 \\
&\quad g_i(x_0, x_i) \leq 0 \quad i = 1, \ldots, K \\
&\quad h_i(x_0, x_i) = 0 \quad i = 1, \ldots, K 
\end{align*}
\]  

(57)

There are several ways of identifying the best block angular structure from a GDP. The process of finding the “most suitable” structure is called optimal decomposition synthesis [13,14]. The formulation essentially consists of partitioning the graph of the GDP into a master cluster and subclusters that are composed of functions and variables of the original GDP. An integer linear programming (ILP) formulation is used in this study [13,15]. The general ILP partitioning formulation is presented in the appendix. The GDP consists of 50 design criteria and 25 variables. Figure 10 shows the result of optimally partitioning the GDP into two subclusters with a partitioning size ratio upper bound of \(K_x = 1.25\) (largest to smallest partition size, as shown in the appendix). The linking variables \(\{x_1, x_8, x_{13}\}\) divide the GDP into two subclusters. Subcluster 1 (SC1) contains design criteria that determine 0–60 time and flywheel packaging, and subcluster 2 (SC2) contains range, emissions, cost-per-mile, and battery packaging criteria.

The second step in the synthesis process is the composition of the overall system objective. With the two subclusters described above, a weighted objective can be composed that maximizes range and 0–60 time and minimizes battery mass. The Euler form of the multi-objective formulation [16] is shown in Eq. 58.

\[
\begin{align*}
\text{minimize } w_1 f_1(\text{Range}) + w_2 f_2(\text{Battery Mass}) + w_3 f_3(t_{0-60}) \\
\text{where } f_1 &= \left[ (\text{Range} - \text{Maximum Range})/\text{Maximum Range} \right]^2 \\
f_2 &= \left[ (\text{Battery Mass} - \text{Minimum Battery Mass})/\text{Maximum Battery Mass} \right]^2 \\
f_3 &= \left[ (0-60 \text{ Time} - \text{Lowest 0-60 Time})/\text{Maximum 0-60 Time} \right]^2
\end{align*}
\]  

(58)

Partitioning the GDP into three and four subclusters is shown in Figs. 8 and 9, respectively. As expected, the number of linking variables increases with the number of subclusters. For three subclusters, the linking variables are \(\{x_1, x_3, x_5, x_9, x_{13}\}\); SC1, SC2, and SC3 contain the criteria that determine the 0–60 time, range, and emissions/cost-per-mile, respectively. Note that division into three subclusters with a size requirement causes a separate partition that deals with emissions. Similarly, further division into four subclusters defines another way of partitioning the GDP with linking variables \(\{x_1, x_3, x_8, x_9, x_{13}, x_{20}\}\); SC1, SC2, SC3, and
SC4 are composed of criteria that determine the range, 0–60 time/cost-per-mile, flywheel packaging, and emissions, respectively.

Decisions about the number of subclusters and the value of $K_s$ are made by the system designer. They must include a meaningful engineering interpretation of the subclusters, and that may not always be possible. The ILP formulation is flexible and computationally inexpensive, so that different scenarios may be easily studied and evaluated from both a mathematical or computational and an engineering viewpoint.

V. DESIGN OPTIMIZATION

Once a decomposable ODP has been synthesized, it can be solved with a proper coordination method. A sensitivity-based coordination method similar to one proposed by Sobieski [17] is used here. The method has the following steps:
HYBRID ELECTRIC POWERTRAIN

Step 1: Solve the subproblems in the local variables \( (x_i, i = 1, \ldots, p) \), after fixing linking variables \( (x_o) \).

Step 2: Using sensitivity analysis, find how the optimal solution of the subproblems changes as a function of the linking variables. The change in the optimal solution \( x^*_o(x_o) \) is obtained as a function of the linking variables \( x_o \) using linear sensitivity analysis.

Step 3: Solve for the overall system objective in the master problem as a function of the linking variables. This is achieved by substituting \( x_i \) in the overall objective function (see Eq. 57) with \( x^*_i(x_o) \).

Step 4: Repeat steps 1–3 until termination criteria are met. These criteria are stationarity of the overall system objective and maximum number of iterations.

In implementation of the procedure described above, an SQP algorithm is used to solve each separate optimization problem [18]. Results obtained by solving both decomposed and undecomposed versions of the problem for different weights are shown in Table 7. The solution to the undecomposed and decomposed versions converges only when the move limits in the linking variable are small. This is expected, since the linear sensitivity approximation is valid only in a small region. Also, the number of master-subproblem iterations decreased when the constraints in the subproblems were projected as a function of the linking variables in the master problem. This projection allows larger steps to be taken in the linking variable space, but requires more computation. Decreasing the weight \( w_1 \) on \( f_1 \) and increasing the weight \( w_2 \) on \( f_2 \) decreases the range and reduces the mass of the battery as expected. The weight on the 0–60 time objective is maintained to validate results based on just two criteria.

VI. CONCLUDING REMARKS

The hybrid electric vehicle design model developed is very simple and contains numerous assumptions. The current system model does not include an internal

<table>
<thead>
<tr>
<th>Weights</th>
<th>Undecomposed Problem</th>
<th>Decomposed Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>( W_2 )</td>
<td>( W_3 )</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>0.45</td>
<td>0.45</td>
<td>0.1</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>
combustion engine. An engine model can be included, but development of good emissions models for transient behavior, while highly desirable, is not an easy task.

The partitioning of the GDP obtained using the ILP formulation is meaningful from an engineering viewpoint. The number of linking variables increases with the number of partitions, while the ILP objective function value decreases (see Figs. 10–12). The optimal solution obtained by synthesizing an ODP from the partitioned GDP in Fig. 10 is also meaningful.

The objective function defined in Eq. 53 is one form of expressing a scalar objective formulation of a multi-criteria optimization problem. Eschenauer et al. [16] and Athan [17] describe other methods of formulating such an objective, including nonlinear scalar objectives that may be useful when the attainable set is nonconvex. The weights shown in Table 7 are only examples. A complete Pareto trade-off study can be conducted by varying all the weights.

![Diagram](image)

linking variables = 5
objective value = 31

Fig. 11. Optimal partitioning of the hybrid powertrain GDP (three subclusters, $K_s = 1.25$).
The sensitivity-based coordination method is limited by the need to carefully choose move limits in the linking variable space for the subproblems to have a feasible solution at a particular $x_o$. Also, it may be necessary to project constraints in the master problem space, if maintaining feasibility of the subproblems becomes an issue. This increases the number of functions in the master problem substantially. Higher-order sensitivity information could allow for larger steps in the linking variable space, but would increase the computational cost. In any case, although a rigorous convergence proof for such a coordination strategy is not available, the method may be attractive from an engineering viewpoint.

VII. ACKNOWLEDGMENT

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APPENDIX: ILP FORMULATION FOR PARTITIONING THE GDP

In this formulation, the GDP is partitioned into a master and a given number of subclusters. Each cluster is designated by its variables and functions. All variables and functions in the GDP must be assigned. The master cluster contains all design criteria that are functions exclusively of the linking variables. A local variable belongs to a subcluster if the function that depends on that variable is in the subcluster. Each function can belong to only one subcluster. Two different functions belonging to two different subclusters cannot have any common variables other than the linking ones. An integer linear programming (ILP) model with zero-one variables that indicate which cluster the design variables and functions are assigned to is created.

To proceed with the ILP model, define the following:

\[ i \quad \text{cluster index, } i = 0, \ldots, K \text{ (zero corresponding to the master cluster)} \]
\[ j \quad \text{function (criterion) index in the GDP, } j = 1, \ldots, T \]
\[ v \quad \text{variable index in the GDP, } v = 1, \ldots, N \]
\[ d_j \quad \text{number of variables in function } j \]
\[ a_{jv} = \begin{cases} 1 & \text{if function } j \text{ contains variable } v \\ 0 & \text{otherwise} \end{cases} \]
\[ e_{iv} = \begin{cases} 1 & \text{if cluster } i \text{ contains variable } v \\ 0 & \text{otherwise} \end{cases} \]
\[ s_{ij} = \begin{cases} 1 & \text{if cluster } i \text{ contains function } j \\ 0 & \text{otherwise} \end{cases} \]
\[ K_s \quad \text{cluster relative size constant, usually } 3 \geq K_s \geq 1 \]
\[ S_i \quad \text{size of cluster } i \]

The mathematical model for the optimal synthesis problem is now stated as follows:

\[
\begin{align*}
\text{minimize} & \quad \sum_{v=1}^{N} e_{iv} s_{ij} + \sum_{j=1}^{T} s_{0j} + (1/K) \sum_{i=1}^{K} S_i \\
\text{subject to:} & \\
& \quad h_1: S_i = \sum_{j=1}^{T} s_{ij} + \sum_{v=1}^{N} e_{iv} \quad i = 1, \ldots, K
\end{align*}
\] (A-1)
\[ h_2: \sum_{i=0}^{K} s_{ij} = 1 \quad j = 1, \ldots, T \]

\[ h_3: \sum_{i=0}^{K} e_{iv} = 1 \quad v = 1, \ldots, N \]

\[ g_1: K_s S_i \geq S_q, q \neq i, \quad i, q = 1, 2, \ldots, K \]

\[ g_2: \sum_{v=1}^{N} a_{jv} e_{0v} \geq d_j s_{0j} \quad j = 1, \ldots, T \]

\[ g_3: \sum_{v=1}^{N} a_{jv} e_{0v} \leq d_j - \sum_{i=1}^{K} s_{ij} \quad j = 1, \ldots, T \]

\[ g_4: e_{iv} \geq \left( \frac{\sum_{j} a_{jv} s_{ij}}{\sum_{j} a_{jv}} \right) - e_{0v} \quad v = 1, \ldots, N \]

\[ g_5: e_{iv} \leq \sum_{j} a_{jv} s_{ij} \quad v = 1, \ldots, N \]

\[ g_6: \sum_{v=1}^{N} e_{iv} \geq 1 \quad i = 1, \ldots, K \]

Constraint \( h_1 \) defines the size of a subcluster, while constraints \( h_2 \) and \( h_3 \) enforce the requirement that each function and variable belong to only one cluster, respectively. Constraint \( g_1 \) restricts the relative sizes of subclusters, constraint \( g_2 \) states that a function belonging to the master cluster must only depend on the linking variables, and constraint \( g_3 \) precludes such functions from being in any subcluster. Constraint \( g_4 \) says that if a function \( j \) depending on a variable \( v \) is in subcluster \( i \), then variable \( v \) is also in \( i \), unless \( v \) is a linking variable. Constraint \( g_5 \) says that if the functions depending on variable \( v \) are not in a subcluster \( i \), then \( v \) does not belong to subcluster \( i \). Finally, constraint \( g_6 \) says that each subcluster must have at least one design variable that is a local variable.

REFERENCES


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