MULTICRITERIA OPTIMIZATION
OF ABS CONTROL ALGORITHMS
USING A QUASI-MONTE CARLO METHOD

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Abstract. The Quasi-Random Weighted Criteria method is proposed for multicriteria design optimization. This quasi-Monte Carlo method features increased computational efficiency and is particularly suitable for exploring alternative design configurations. A quasi-random sequence generates a set of candidate solutions representative of the range of available solutions for each design alternative. The method can be used recursively to produce more detailed Pareto surface descriptions near selected points.

In this paper the method is used to select between vehicle anti-lock brake system (ABS) control algorithm approaches and to optimize the parameters within each. An ABS system is highly nonlinear and therefore the control algorithms draw upon the methods of nonlinear control theory. Stochastic optimization was incorporated to ensure that the ABS system will perform well despite the uncertainties in the vehicle and in the environment. A variety of ABS design studies are presented.

KEY WORDS: ABS control, optimal design, multicriteria, optimization

1 INTRODUCTION
Computer assisted vehicle brake systems are commonly referred to as anti-lock brake systems (ABS). An anti-lock brake system makes adjustments to the fluid pressures that govern the four brakes on a vehicle. These adjustments modulate the outputs from the brakes to increase utilization of the road-to-tire adhesion.

The design of ABS control algorithms is a difficult task for the traditional approaches of intuitive design, test evaluation and analytical derivation. Many competing criteria must be considered. An intuitive understanding is difficult to develop for such a complex, nonlinear system that must operate with a wide range of stochastic conditions.

A multicriteria optimization method is proposed which is particularly suited to design problems with many competing criteria, with preferences between criteria difficult to decide a priori, and with alternative design strategies to compare, as is the case with the many approaches to non-linear control algorithm design.
Section 2 discusses the method, while Section 3 describes ABS control algorithm design studies.

2 THE QUASI-RANDOM WEIGHTED CRITERIA METHOD
This section begins with a brief review of three similar methods. The proposed method is then described.

2.1 Similar Methods
Methods for problems with large numbers of criteria can carry large computational costs. In those described here the decision maker's iterative exploration of preference structure is no longer the focus of the solution effort. Rather, these methods attempt to provide many candidate solutions from which the decision maker can pick the most preferred.

2.1.1 Meisel's Method
Meisel proposed the use of a random input to direct the selection of weights for a weighted criteria formulation in repeated scalar substitute optimizations [11]. Meisel's method can generate large numbers of points distributed over the attainable set. Because the weights are randomly distributed the method is not computationally efficient. Also, Meisel's method does not capture Pareto optimal points corresponding to an even distribution across the ranges of possible criteria weight ratios, because the weight ratios are not evenly distributed.

2.1.2 Osyczka's Method
Osyczka [13] suggests the use of a Monte Carlo method to pick design variables over their estimated ranges. Criteria values are calculated for each set of design variables. This process generates a set of feasible points. In general many points will not be Pareto optimal and therefore the Pareto set must be extracted. There is no certainty that the set adequately represents the attainable Pareto set.

2.1.3 Statnikov's Method
Rather than utilizing randomness to pick design variables, R.B. Statnikov suggested a search strategy to generate a representative set of feasible points (22, 21, 19, 10). A uniformly distributed scattering of feasible trial points is generated using quasi-random sequences (described below). Statnikov’s method uses these sequences to deliver a uniform and efficient distribution for the design variables.

Because Statnikov's method is similar to Osyczka's in that it is based upon the generation of a large set of feasible points from which Pareto optimal points must be extracted, it too will require many calculations. Stadler and Dauer note that there is presently no assurance that this approximation converges in some fashion to the set of Pareto optimal solutions for a given multicriteria optimization problem [20].

2.2 Proposal of a New Multicriteria Optimization Method
The Quasi-Random Weighted Criteria (QRWC) Method [2] solves a series of weighted criteria scalar substitute problems with systematically adjusted scalar criteria weights. The method utilizes quasi-random sequences to generate the
weights. Quasi-random sequences cover a hypervolume evenly and efficiently, while maintaining random-like relations between the components.

A distribution over the ranges of all criteria weight ratios is produced with the quasi-random sequences. The distribution is chosen to provide optimal solutions that can be considered representative of the potential combinations of criteria weights, making the method useful in quantitative comparisons between competing design strategies. Furthermore, points are spaced in the most efficient manner in that the weights for each additional scalar optimization are selected to provide a maximum amount of new information.

The method can be used recursively. After a set of candidate solutions is generated, a subset is selected by the decision maker and a new set of candidate solutions is generated within the neighborhood of the selection. The process can be repeated with increasingly smaller neighborhoods until the decision maker selects a final solution.

2.2.1 Generation of the Quasi-Random Sequence
In the QRWC Method, I - 1 components of the weight vector are calculated as independent quasi-random numbers from the Hammersley point set. This set is a finite sequence of points created to cover a hypercube uniformly and with low discrepancy. Discrepancy is a measure of the efficiency of a series in covering a volume. A low discrepancy sequence is one that is distributed such that each point is near a large amount of hypervolume that is not near other points. Niederreiter in his book on quasi-Monte Carlo methods [12] states that it is widely believed that the Hammersley point set with pairwise relatively prime bases attains the discrepancy of smallest possible magnitude.

Because the Hammersley point set was devised to cover a hypercube, not a simplex, a projection into a simplex must be made. It can be shown that the discrepancy of the projected series is lower than the discrepancy of the original series in the hypercube [2].

2.2.2 The Hammersley and the Shifted Hammersley Point Sets
There are a number of quasi-random series and point sets. The point set used in this work is termed the shifted Hammersley sequence, originally suggested by van der Corput [27] and developed by Halton [7] and Hammersley [8].

Shifting the first component is advantageous. Wozniakowski [28] shows that shifting the first component to $(l+t)/L$, with $0 \leq l + t \leq L$ reduces the average case error in numerical integration for one class of functions. Wozniakowski did not specify a value for $t$. This parameter was set to -0.5 in the studies reported in this paper. That value makes the first component of the series symmetric over the segment $(0,1)$. The Hammersley sequences approach this symmetry asymptotically as $L$ increases, but because the QRWC method uses small point sets the establishment of symmetry was judged desirable.

3 ANTI-LOCK BRAKE SYSTEM ALGORITHM DESIGN STUDY
In the present study the criterion vector $c$ has six components and the parameter vector $b$ has four components representing stochastic influences. The dimension of the design variable vector $x$ varies with the algorithms, from two to five.
Variables related to ABS hardware are also included in some of the design studies. All design variables are unconstrained except for simple upper and lower bounds.

3.1 Criteria Functions
(1) Minimize ABS Inefficiency $c_1$ to reduce stopping distance.
(2) Minimize Average Tire Slip $c_2$ to improve steerability.
(3) Minimize Deceleration Variability $c_3$ to improve passenger comfort.
(4) Minimize Chatter (actuator reversals) $c_4$ to increase hardware life.
(5) Minimize Sensitivity $c_5$ of the ABS efficiency criterion to errors in the wheel deceleration measurement.
(6) Minimize Transition Response Inefficiency $c_6$ to improve response to a sudden change in tire-pavement adhesion.

3.2 Stochastic Parameters
The simulation includes four stochastic influences upon ABS performance: vehicle loading $b_1$, tire-pavement adhesion $b_2$, initial vehicle speed $b_3$, and brake output hysteresis $b_4$.

The relative importance of the stochastic influences in the ABS design studies presented in the next section was determined by accident data [27,26]. If accidents occur five times more often on dry pavement than on wet pavement, an improvement in dry pavement performance is given five times the importance of an improvement in wet pavement performance. Stochastic conditions are integrated into a single quantity for the objective function by expressing their relative importance in terms of accident probability:

$$c(x) = \int_{b_{low}}^{b_{high}} \text{(Probability that } b \text{ occurs in an accident) } c(x,b) \, db. \tag{1}$$

Here $c(x)$ is evaluated over the range of stochastic parameters and is then used in an optimization conducted over all feasible $x$. In numerical evaluation the integral is replaced by a summation of a discrete set of stochastic conditions.

3.3 ABS Control Algorithms
Three ABS control algorithms are compared in this study: the common phase plane and sliding mode methods, and a recent modification of sliding mode control. In phase plane control look-up tables determine a controller output for any combination of state variable values [4]. The sliding mode algorithm was based upon a description by Tan [23] that does not explicitly consider implementation with discrete controllers. Therefore, Tan's algorithm was adapted by defining four bands parallel to the sliding mode surface, as suggested by Gibson's "Dual Mode Switching" [6]. In the modified sliding mode scheme proposed by Tomizuka and Tan [24] the bounded or slow-varying uncertainties of the brake system are grouped together and treated as disturbances, estimated by a derivative feedback. The resultant system is locally linearized by a nonlinear prefilter and linear digital control theory is used to determine the controller design variables.
These three algorithms were adapted to a simulation controller capable of five outputs: fast and slow pressure release, fast and slow pressure apply, and hold pressure. The second and third algorithms utilize a surface identification estimation algorithm to update estimates of the tire slip level that corresponds to peak adhesion. A least squares estimation with a forgetting factor was used.

Theoretical control schemes must be modified when implemented on a vehicle system to accommodate system limitations. Modifications increase the difficulty for intuitive grasp or theoretical analysis of the control problem, and a systematic design optimization process becomes more advantageous.

3.4 THE SIMULATION
A quantity termed wheel slip is important in the analysis of braking. It is a measure of the difference between vehicle velocity and the velocity of the tire at its point of contact with the road:

\[
\text{% Wheel slip} = \frac{V - w r}{V} \times 100\% \tag{2}
\]

where \( V \) is vehicle velocity, \( w \) is wheel angular velocity and \( r \) is wheel radius. The ABS controller is designed to maintain wheel slip near the value corresponding to peak adhesion. The basic elements of an ABS system are a wheel sensor, an anti-lock pressure modulating valve, and an anti-lock controller. A four-wheel ABS system would consist of four sets of these components. This study uses a simulation of one set, termed a one-quarter vehicle model [9]. The state equations are

\[
s = v, \quad \dot{v} = g \mu + 0.5 C_d v^2, \quad \dot{\omega} = (\mu F_z R - B)/I \tag{3}
\]

where \( s \) is distance covered by the vehicle, \( v \) is vehicle velocity, \( \omega \) is wheel rotational velocity, \( g \) is the gravitational constant, \( \mu \) is coefficient of tire-to-road adhesion, \( C_d \) is coefficient of aerodynamic drag, \( F_z \) is normal force between tire and road, \( R \) is tire radius, \( B \) is brake torque to retard the wheel, and \( I \) is wheel rotational inertia.

The simulation begins with a simulated driver input consisting of a fixed ramp pressure apply rate of 1900 N/m\(^2\)s lasting one second. Line pressure is then held constant until the ABS system is triggered. For simplicity, driver input is not considered after the ABS system has been triggered.

The following assumptions were made for the simulation:
1. The ABS system is a sampled data system receiving updated inputs every 8 milliseconds and responding at that same time, maintaining the response until the next update.
2. There is no delay in the reception of ABS input information.
3. There is no delay nor hysteresis in ABS actuator response.
4. ABS actuator response levels are exactly delivered.
5. Wheel speeds are accurately known.
6. Vehicle deceleration input contains noise. In hardware implementations vehicle deceleration is estimated with imprecision. In the simulation a normal distribution is used assuming that noise is generated by independent, additive factors. If independent, multiplicative factors were assumed a log-normal distribution would be more appropriate [1] The standard deviation of 10 m/s² was selected by engineering judgment because no published data were available. The same random sequence is generated for each simulation (a common random series) to provide fair comparison among simulations.

7. Wheel inertia is accurately known and is set at 1.1 kg m².

8. Aerodynamic drag is included, considered proportional to the square of vehicle velocity, \( D = 0.5 C_d V^2 \), where \( C_d = 0.3 \) is the coefficient of aerodynamic drag and \( V \) is vehicle velocity.

9. The ABS actuator controlling brake input pressure has two rates of change, 8,500 and 17,000 N/m²/s in both apply and release, and a pressure hold. Due to cost considerations, most ABS systems use discrete type actuators producing only a limited number of rates of pressure change.

10. Powertrain braking is negligible.

11. The brake input-to-output function is exactly known. In actual operation the controller modulates brake line pressure but actual brake torque output is unknown.

12. The brake input-to-output function is linear.

13. Wheel radius is accurately known and is set at 0.307 m.

3.5 RESULTS
The ABS models described in the previous sections generally result in criteria functions that have many local minima and that may be non-smooth. The controller works on a sampled system so controller input and output occur at intervals. Furthermore, a discrete output ABS actuator must call for one rate from a very limited set rather than provide a continuous adjustment of pressure levels.

Local gradient-based optimization algorithms will fail under such conditions. One alternative is to use a global optimization method. Another is to average the functions over runs repeated with stochastic variable variations. The averaging results in smoothing of the response surface because in effect it performs an integration of the function over the ranges of stochastic variables [3].

In the present study scalar optimizations were conducted using both a local optimizer, the sequential quadratic programming method NLPQL [17,16] and a global optimizer, the simulated annealing method Hide and Seek [14]. The local optimizer results were sensitive to starting point and consistently had a larger objective value than did the global optimizer solutions, even with stochastic variable integration. All subsequent optimizations therefore relied upon the global optimizer.

A number of ABS design studies are summarized in the following subsections. For details please refer to [2].

3.5.1 Comparison Between Algorithms
Solution sets of six points each were found for each of the three algorithms given above. Overall, sliding mode control did not match the capabilities of the other
two control algorithms, see Fig 1. Note that for each algorithm there is little variation among the six solutions. The objective function criteria weights were varied between the optimizations to represent the ranges of possible relative weights, and so the limited variation indicates that these algorithms are not capable of much range in their performance trade-offs.

3.5. Inclusion of Control Hardware in MSM Controller Optimization

Other quantities can be designated as design variables in the optimizations. For example, the controller rates, previously fixed at 17000 N-m/sec and 8500 N-m/sec, can be allowed to vary. This was done with the MSM algorithm. The average improvement in the weighted criteria objective function was 7.9%. The results suggest that a slightly lower slow rate and a slightly higher fast rate would improve performance. Chatter was reduced 19%.

3.5.3 Inclusion of ABS Activation in MSM Controller Optimization

In both sliding mode and MSM algorithms the ABS system is activated when tire slip rises above the value estimated to correspond to peak adhesion. This activation was set intuitively. A pattern of optimization runs was conducted to optimize ABS activation in the MSM controller. An offset from the previous trigger value was added as design variable. Including the trigger offset brought a decrease in the average objective function value across six optimizations of 6.11%.

3.5.4 Impact of Sampling Rate

Optimizations were conducted with the MSM algorithm to examine the impact of slower (10 ms) and faster (6 ms) sampling rates. Faster sampling brings a 12.5% decrease in the 6 objectives, while slower sampling brings a 5.83% increase. Comparing the 10 ms results with the 6 ms results, the faster sampling reduced average slip from 0.233 to 0.166, but actually increased average chatter and sensitivity.

4 CONCLUDING REMARKS

The simulation used in this article was adequate to demonstrate method capabilities, yet could be enlarged to better represent the complete ABS design problem. The studies are representative examples from many that could be conducted. In actual application further analysis would be undertaken to generate information pertinent to the selection of a final solution. The QRWC method can be focused upon a region of particular interest, and the focus can be narrowed with each set of evaluations until a solution is selected.

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Figure 3.1 Comparison between the Three Algorithms

REFERENCES


