SIMULTANEOUS TOPOLOGY AND MATERIAL MICROSTRUCTURE DESIGN

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Abstract

We present an overview of Project MAXWELL, a rigorous methodology for the integrated design and fabrication of efficient mechanical components. A key component of this project is the homogenization method, a technique for performing optimal structural layout design. This method can perform topology optimization with only the specification of a designable area, appropriate loads and boundary conditions. Both the optimum global shape of the structure and optimal material properties for each discrete element in the design domain can be found using this method. This methodology has been applied to an extensive and growing body of examples, which range from the design of simple plane strain/plane stress structures to the design of automotive body panels, suspension components, and complete vehicle structural layouts. Several examples will be presented, illustrating the applications of these tools to component design; multibody layout design, and design for natural frequency tailoring. Additionally, a description of material microstructures designed using homogenization is included.

1. Introduction

Structural optimization has been an active area of research since the early 1970s; see for example Botkin [5]; Fleury [8]; Bennet and Botkin [4]; Kikuchi [11]; Esping [7]. The two basic optimization problems typically addressed in structural optimization have been sizing and shape optimization. In sizing optimization, variables define local geometric characteristics such as height, width, thickness, and radius of specific portions of the structure. A typical design task is to find the minimum weight shell structure to withstand applied thermo-mechanical loads. In shape optimization, the optimum shape of a structure is sought by varying the boundary shape defined by an appropriate spline function, with the design variables defined in a function form.

In most design problems, the topology of a structure is not known a priori. Topology is related to the number of holes in a structure. If the topology is fixed, the configuration is defined easily by spline functions. Significant difficulties are encountered when the topology of a structure must be designed, since its representation with spline-type functions is unwieldy. As a result, design problems involving both shape and topology have not been solved satisfactorily. Several approaches to the topology optimization problem have been proposed: see, for example, Rozvany [15], and the proceedings of a recent NATO Advanced Study Institute [3]. The homogenization method, described by Bendsoe and Kikuchi [2], and used in Project Maxwell is unique in that it yields the optimal shape and topology at a macro- and micro-level of description.

2. Homogenization Design Methodology

We can formulate a generalized topology optimization problem by introducing microstructural perforations into the structure, and then minimizing the mean compliance subject to a constraint on the total volume of material used. Formally,

$$\text{Minimize} \sum_{i} \int_{\Omega} f_{i} \mu_{i} d\Omega + \sum_{\Gamma_{i}} \int_{\Gamma_{i}} t_{i} \mu_{i} d\Gamma,$$

subject to equilibrium equations, and

$$\int_{\Omega} (1-\alpha b) d\Omega \leq \Omega,$$

Here, $u$ is the vector of virtual displacements, $f$ is the applied body force, $t$ is the applied traction on the boundary $\Gamma_{i}$, $i$ is the number of finite elements used to discretize the structure, and $\Omega$ is the total volume of solid material forming the porous structure. The microstructural model used in this method is shown in Figure 1. There are three design variables per element: void dimensions $a$ and $b$, and void orientation angle $\theta$. Variables $a$ and $b$ are restricted to values $[0,1]$, i.e., 0 to 1 inclusive.

![Figure 1: Element Microstructural Model](image)
The equilibrium equation and its associated loading and support conditions, i.e., the structural analysis problem, are solved using the finite element method. The domain for stress analysis is the initial design domain. This domain is discretized into finite elements, with the design variables $a$, $b$, and $\theta$ for each element evaluated at the centroid. The number of design variables for the present problem is quite large, e.g., five to thirty thousand variables, and it is difficult to apply standard mathematical programming techniques. The optimization method used for this problem is a simple resizing scheme based on the optimality conditions. For details, refer to Bendsoe and Kikuchi [2].

In the remainder of this paper we present several examples illustrating a range of applications of the homogenization design technique.

3. Examples

3.1 Lower Control Arm Design

This component is typical of vehicle chassis structural components; their design is usually driven by stiffness requirements. Figure 2 shows the description of the initial design domain, boundary conditions, and loads. In this example, packaging requirements for other suspension and wheel component severely limit the allowable design space. In addition, appropriate attachment material for the strut must be provided in the final design. Translational displacements are constrained at the two pivots (to the right in Figure 2), and loads are applied at the strut and at the ball joint. Three separate loading conditions are considered, with the primary load case being vertical (z-direction) loading at the strut attachment point. More details about the design and fabrication of this component can be found in Johanson [10].

![Figure 2: Design Domain for Lower Control Arm](image2)

![Figure 3: Cross-Section of Lower Control Arm at 30% Section](image3)

![Figure 4: Cross-Section of Lower Control Arm at 45% Section](image4)

The homogenization process produces a topology which possesses the maximum stiffness for a (user-supplied) constraint on the percentage of the design domain to be filled with material. Figures 3 and 4 show the result of the homogenization process, showing two sectional views of the resultant structure.

![Dark areas on these figures indicate areas of solid material, i.e., areas where the size of the microscale voids has been reduced to zero. White areas indicate regions where material is not required in the optimal structure. Areas of intermediate gray-scale shading indicate regions of composite material where the structure possesses microstructure. In many cases, these areas can be considered to be regions which possess different directional stiffness characteristics. This topology possesses the minimum compliance for a given volume constraint; in this case, forty percent of the initial design domain is filled with material. The design is independent of the numerical value of the load P, as stress constraints are not considered at this stage of the analysis.](image5)
Note that the optimal topology suggested using the homogenization procedure would be difficult to manufacture using traditional techniques. In Project Maxwell, we consider two approaches to this problem. One approach is to interpret the resultant structural layouts in terms of available manufacturing technologies. The other approach is to consider novel manufacturing techniques, such as layered manufacturing, for fabrication of these designs. Depending on the problem application and the manufacturing technology available, regions of intermediate density may either be interpreted as representing isotropic material or as representing composite structure. This layout design is then ready to be subjected to detail sizing and shape optimization, where additional constraints on local measures, such as stresses, are considered.

3.2 Multbody Structures

To extend the homogenization design technique to the design of multbody structures requires only slight modifications to the earlier formulation. In particular, we consider the minimization of the mean compliance of the total structural system:

\[
\min_{\mu} \left[ \min_{\alpha, \beta} \sum_b \left( \sum_{i \in \Omega} \int_{\Omega_i} f_i \mu_i d\Omega_i + \sum_{i \in \Gamma_b} \int_{\Gamma_b} f_i \mu_i d\Gamma_i \right) \right]
\]

subject to equilibrium equations, and

\[
\sum_{b \in \Omega} \int (1 - ab) d\Omega \leq \Omega,
\]

where \( B \) is the number of bodies, \( j \) is the number of joints between structures, \( j_k \) is a value describing the location of joint \( j \), and all other variables are as described above.

Joints between bodies in the system may possess a variety of different compliance characteristics. Joints in the examples described below are modeled as connections between nodes that are rigid in translation but allow relative rotational deformation between bodies. The rotational stiffness provided by the joints is supplied by the modeler and is not changed in the topology optimization procedure. Currently, this formulation is limited to solving two-dimensional problems.

The optimization procedure for the multbody formulation requires some modification. For any particular selection of joint locations \( j_k \), it is possible to determine an optimal structural layout. Rather than treating joint location as a variable and solving the resultant optimization problem, the joint location is considered to be a parameter, and the minimum compliance topology is determined for each selection of joint location.

In this example, we compare the optimal layout of two flanged structures, one with an integral flange and another with a flange welded to a base structure. Figure 4 shows the initial design domain. The structure is constrained at the four points on the flange shown in the figure, and is subjected to a vertical load at the top left corner of the structure. Note that these structures are two-dimensional and not cross-sections of shell structures.

Figure 4 -- Design Domain for Flanged Structure

Figure 5 shows the results of the homogenization procedure for the structure with integral flange. In these figures, light areas indicate areas where the void size has gone to zero, i.e., areas where material should be distributed. Dark areas on these figures indicate areas where material is not required in the optimal structure.

Figure 5 -- Optimal topology for structure with integral flange.
Figures 6 and 7 show two structural layouts for two different sets of joint locations. Figure 6 shows the layout for an arbitrary set of joint locations, which was one of the intermediate iterations in the optimization procedure. Figure 7 shows layout of the structure with the optimal joint location. In these figures, a joint is represented by two closely spaced parallel lines, connecting one element on each of the two bodies. In both Figure 6 and Figure 7, two joints connect the two bodies. Mathematically, the joint is treated as an element that is rigid in translation, but allows relative rotations between the two bodies. It is possible to modify the rotational stiffness of the joint element to simulate different joint rates.

The non-dimensional compliance for the structure with integral flange was 9.28. The compliance for the structure shown in Figure 6 was 8.82, while the compliance for the optimal structure shown in Figure 7 was 5.3312. Clearly, the joint location shown in Figure 7, at the top and bottom of the attached flange, produces a much stiffer structure. This result is as expected; however, it was obtained without using any a priori knowledge about the structure. It is also interesting to compare the layouts suggested in Figures 5 and 7. Both structures possess similar layouts in areas away from the flange. Near the flange, both layouts distribute material near the edges of the design domain. For this set of loading and boundary conditions, tall flanges attached at the top and bottom to the base, will give the stiffest structures.

3.3 Dynamic Structural Design

All of the previous examples illustrated the application of the homogenization method to solve layout optimization problems where the objective was to minimize the mean compliance of the structure. In this example, we modify the method to allow the design of structures with prescribed natural frequency properties. Details of the formulation can be found in the works by Ma [1] and Cheng [9].

Consider the design of the cabin structure shown in Figure 8. This structure is formed of isotropic plates with a uniform thickness of 2.0 mm. We consider the reinforcement of the cabin structure with a given mass, such that the eigenvalues of the final design match the desired values. The maximum reinforcement thickness in this case is 6.0 mm. We consider three cases. In case a, we prescribe the first three desired eigenfrequencies. In cases b and c, we specify desired values for the first four eigenfrequencies.
The results are summarized in Table 1.

<table>
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<tr>
<th>Case</th>
<th>Desired Eigenfrequency</th>
<th>Obtained Eigenfrequency</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1st Mode</td>
<td>2nd Mode</td>
</tr>
<tr>
<td>a</td>
<td>6.5</td>
<td>8.0</td>
</tr>
<tr>
<td>b</td>
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<td>8.28</td>
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<tr>
<td></td>
<td>8.0</td>
<td>10.00</td>
</tr>
<tr>
<td>c</td>
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<td></td>
<td>9.23</td>
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</table>

Table 1. Desired and Obtained Natural Frequencies of Cabin Box

In this case, the stiffened structure very nearly possesses the desired natural frequency characteristics. The resultant structural layouts are shown in Figures 9, 10, and 11. In these figures, the dark areas represent areas where the plate should be thick, and the white areas represent areas where no stiffener is required. Areas of intermediate grayscale shading represent areas where the optimal layout suggests a plate thickness of intermediate thickness.

4. Material Microstructure Design

Just as it is possible to perform structural layout optimization using a homogenization technique, it is also possible to design materials which possess desirable or optimal properties. In fact, these two procedures can be performed simultaneously. In this section, we describe some recent work concerning the design of optimal material microstructures.

Some recent works by Bendsøe and collaborators describe the construction of optimal structures using materials of prescribed constitutive behavior. In [17], Bendsøe et. al. show that the lower bound on compliance optimization for a single load case is obtained using an orthotropic material with the material directions aligned with the principal stresses and without shear rigidity. The optimum structure thus obtained is stable only under the loads that the structure was designed for; any other loading condition will cause its collapse. When there are multiple load cases, reference [18] obtains an optimal design using anisotropic materials, with the elastic properties varying along the continuum.

Recently, Sigmund [16] has demonstrated the feasibility of designing materials with prescribed elastic behavior. By using a properly formulated topology optimization model, it is possible to create materials which possess such behavior. Most of Sigmund's materials were realized by using a truss topology optimization model on a material microstructural level. As suggested by Sigmund, it is also possible to develop these materials by using a continuum optimization approach, similar to that used for the above examples. Essentially, the optimization problem becomes an inverse homogenization problem: given a particular elasticity tensor $E_{ijkl}$, determine a material microstructure that possesses such a constitutive matrix. Details of the technical approach can be found in Sigmund [16]. We present two simple examples of the application of such a technique to the design of two-dimensional materials.

4.1: Material with Poisson's Ratio = 1.0

In this example, we seek to design a material which possesses equal strength in each of the principal directions, and thus possesses a Poisson's ratio of approximately 1.0. Note that this material has no shear stiffness. The microstructure of such a material is shown in Figure 12.
A lattice made of this material resembles an octagonal honeycomb structure. The response of this structure is similar to that of a more typical hexagonal honeycomb structure.

4.2: Orthotropic Material

In this example, we designed an orthotropic material which possesses stiffnesses in the two principal directions in the ratio of 2 to 1. Figure 13 illustrates the microstructural layout of this material. The stiffness in the vertical direction is one-half of the stiffness in the horizontal direction.

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References


