Computational Tools For Optimal Structural Layout Design

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Abstract

We present an overview of the design capabilities in Project MAXWELL, a rigorous methodology for the integrated design and fabrication of efficient mechanical components. A key component of this project is the homogenization method, a technique for performing optimal structural layout design. This method can perform topology optimization with only the specification of a designable area, appropriate loads and boundary conditions. Both the optimum global shape of the structure and optimal material properties for each design element in the design domain can be found using this method. This methodology has been applied to an extensive and growing body of examples, which range from the design of simple plane stress structures to the design of automotive body panels, suspension components, and complete vehicle structural layouts. Several examples will be presented, illustrating the applications of these tools to component design, multibody layout design, multimaterial layout design, and design for natural frequency tailoring.

Introduction

Structural optimization has been an active area of research since the early 1970s, see for example Botkin [5]; Fleury [8]; Bennet and Botkin [4]; Kikuchi [11]; Esping [7]. The two basic optimization problems typically addressed in structural optimization have been sizing and shape optimization. In sizing optimization, variables define local geometric characteristics such as height, width, thickness, and radius of specific portions of the structure. A typical design task is to find the minimum weight shell structure to withstand applied thermomechanical loads. In shape optimization, the optimum shape of a structure is sought by varying the boundary shape defined by an appropriate spline function, with the design variables defined in a function form.

In most design problems, the topology of a structure is not known a priori. Topology is related to the number of holes in a structure. If the topology is fixed, the configuration is defined easily by spline functions. Significant difficulties are encountered when the topology of a structure must be designed, since its representation with spline-type functions is unwieldy. As a result, design problems involving both shape and topology have not been solved satisfactorily. Several approaches to the topology optimization problem have been proposed: see, for example, Rozvany [15], and the proceedings of a recent NATO Advanced Study Institute (Bendsoe, 1992) [3]. The homogenization method, described by Bendsoe and Kikuchi (Bendsoe, 1988) [2], and used in Project Maxwell is unique in that it yields the optimal shape and topology at a macro- and micro-level of description.

Homogenization Design Methodology

We can formulate a generalized topology optimization problem by introducing microstructural perforations into the structure, and then minimizing the mean compliance subject to a constraint on the total volume of material used. Formally,

\[
\text{Minimize } \sum_{i} \int_{\Omega} f_i u_i d\Omega + \sum_{i} \int_{\Gamma_f} t_i u_i d\Gamma_f
\]

subject to equilibrium equations, and

\[
\int_{\Omega} (1 - ab) d\Omega \leq \Omega_c
\]

Here, \( u \) is the vector of virtual displacements, \( f \) is the applied body force, \( t \) is the applied traction on the boundary \( \Gamma_f \), \( i \) is the number of finite elements used to discretize the structure, and \( \Omega_c \) is the total volume of solid material forming the porous structure. The microstructural model used in this method is shown in Figure 1. There are three design variables per element: void dimensions \( a \) and \( b \), and void orientation angle \( \theta \). Variables \( a \)

![Fig. 1: Element microstructural model](image-url)
and b are restricted to values (0,1), i.e., 0 to 1 inclusive.

The equilibrium equation and its associated loading and support conditions, i.e., the structural analysis problem, are solved using the finite element method. The domain for stress analysis is the initial design domain. This domain is discretized into finite elements, with the design variables a, b, and θ for each element evaluated at the centroid. The number of design variables for the present problem is quite large, e.g., five to thirty thousand variables, and it is difficult to apply standard mathematical programming techniques. The optimization method used for this problem is a simple resizing scheme based on the optimality conditions. In the remainder of this paper we present several examples illustrating a range of applications of the homogenization design technique.

**Examples**

**Lower Control Arm Design**

This component is typical of vehicle chassis structural components; their design is usually driven by stiffness requirements. Figure 2 shows the description of the initial design domain, boundary conditions, and loads. In this example, packaging requirements for other suspension and wheel component severely limit the allowable design space. In addition, appropriate attachment material for the strut must be provided in the final design. Translational displacements are constrained at the two pivots (to the right in Figure 2), and loads are applied at the strut and at the ball joint. Three separate loading conditions are considered, with the primary load case being vertical (z-direction) loading at the strut attachment point. More details about the design and fabrication of this component can be found in Johanson [10].

![Fig. 2: Design domain for lower control arm](image)

The homogenization process produces a topology which possesses the maximum stiffness for a (user-supplied) constraint on the percentage of the design domain to be filled with material. Figures 3 and 4 show the result of the homogenization process, showing two sectional views of the resultant structure.

![Fig. 3: Cross-section of lower control arm at 30% section](image)

![Fig. 4: Cross-section of lower control arm at 45% section](image)
Dark areas on these figures indicate areas of solid material, i.e., areas where the size of the microscale voids has been reduced to zero. White areas indicate regions where material is not required in the optimal structure. Areas of intermediate gray-scale shading indicate regions of composite material where the structure possesses microstructure. In many cases, these areas can be considered to be regions which possess different directional stiffness characteristics. This topology possesses the minimum compliance for a given volume constraint; in this case, forty percent of the initial design domain is filled with material. The design is independent of the numerical value of the load P, as stress constraints are not considered at this stage of the analysis.

Note that the optimal topology suggested using the homogenization procedure would be difficult to manufacture using traditional techniques. In Project Maxwell, we consider two approaches to this problem. One approach is to interpret the resultant structural layouts in terms of available manufacturing technologies. The other approach is to consider novel manufacturing techniques, such as layered manufacturing, for fabrication of these designs. Depending on the problem application and the manufacturing technology available, regions of intermediate density may either be interpreted as representing isotropic material or as representing composite structure. This layout design is then ready to be subjected to detail sizing and shape optimization, where additional constraints on local measures, such as stresses, are considered.

**Multibody Structures**

To extend the homogenization design technique to the design of multibody structures requires only slight modifications to the earlier formulation. In particular, we consider the minimization of the mean compliance of the total structural system:

\[
\text{Min} \sum_{k} \text{Min} \sum_{a,b} \sum_{\theta} \left( \sum_{i} f_{i} u_{i} d \Omega + \sum_{j} t_{i} u_{i} d \Gamma_{i} \right)
\]

subject to equilibrium equations, and

\[
\sum_{B} \int_{\Omega} (1 - ab) d \Omega \leq \Omega_{s}
\]

where B is the number of bodies, j is the number of joints between structures, j_k is a value describing the location of joint j, and all other variables are as described above.

Joints between bodies in the system may possess a variety of different compliance characteristics. Joints in the examples described below are modeled as connections between nodes that are rigid in translation but allow relative rotational deformation between bodies. The rotational stiffness provided by the joints is supplied by the modeler and is not changed in the topology optimization procedure. Currently, this formulation is limited to solving two-dimensional problems.

The optimization procedure for the multibody formulation requires some modification. For any particular selection of joint locations j_k, it is possible to determine an optimal structural lay-

![Fig. 5: Design domain for flanged structure](image)

out. Rather than treating joint location as a variable and solving the resultant optimization problem, the joint location is considered to be a parameter, and the minimum compliance topology is determined for each selection of joint location.

In this example, we compare the optimal layout of two flanged structures with an integral flange and another with a flange welded to a base structure. Figure 5 shows the initial design domain. The structure is constrained at the four points on the flange shown in the Figure, and is subjected to a vertical load at the top left corner of the structure. Note that these structures are two-dimensional and not cross-sections of shell structures.

Figure 6 shows the results of the homogenization procedure for the structure with integral flange. In these figures, light areas indicate areas where the void size has gone to zero, i.e., areas where material should be distributed. Dark areas on these figures indicate areas where material is not required in the optimal structure.

Figures 7 and 8 show two structural layouts for two different sets of joint locations. Figure 7 shows the layout for an arbitrary set of joint locations, which was one of the intermediate iterations in the optimization procedure. Figure 8 shows layout of the structure with the optimal joint location. In these figures, a joint is represented by two closely spaced parallel lines, connecting one element on each of the two bodies. In both Figure 7 and Figure 8, two joints connect the two bodies. Mathematically, the joint is treated as an element that is rigid in translation, but allows relative rotations between the two bodies. It is possible to modify the rotational stiffness of the joint element to simulate different joint rates.

The non-dimensional compliance for the structure with integral flange was 9.28. The compliance for the structure shown in Figure 7 was 8.82, while the compliance for the optimal struc-
Multimaterial Structural Design

A different variation of the initial topology optimization methodology is to allow the topology design of continua composed of regions of different material properties. Tailored blanks for stamping operations are examples of such structures. Figure 9 shown an initial design domain for a tailored blank inner door stiffener. In this case, the initial tailored blank is composed of two different regions of material, and the stiffness of the upper region is three times greater than the stiffness of the lower region.

Fig. 6: Optimal topology for structure with integral flange

Fig. 7: Topology for structure with attached flange - arbitrary joint location

Fig. 8: Optimal topology for structure with attached flange

Fig. 9: Multimaterial design domain
The result of the optimal layout design procedure is shown in Figure 10. On this figure, light areas indicate areas of material distribution, and dark area indicate areas where material is not required.

Note that the optimal structural layout uses more material in the region of the design domain which is composed of less stiff material, and less material in the region of high stiffness. This design is somewhat different than the design that would be suggested by performing topology optimization on a design domain composed of one isotropic material. This procedure can be extended to structures of arbitrarily many regions of different material stiffness.

**Dynamic Structural Design**

All of the previous examples illustrated the application of the homogenization method to solve layout optimization problems where the objective was to minimize the mean compliance of the structure. In this example, we modify the method to allow the design of structures with prescribed natural frequency properties. Details of the formulation can be found in the works by Ma [1] and Cheng [9].

Consider the design of the cabin structure shown in Figure 11. This structure is formed of isotropic plates with a uniform thickness of 2.0 mm. We consider the reinforcement of the cabin structure with a given mass, such that the eigenvalues of the final design match the desired values. The maximum reinforcement thickness in this case is 6.0 mm. We consider three cases. In case a, we prescribe the first three desired eigenfrequencies. In cases b and c, we specify desired values for the first four eigenfrequencies.

**Table 1. Desired and obtained natural frequencies of cabin box**

In this case, the stiffened structure very nearly possesses the desired natural frequency characteristics. The resultant structural layouts are shown in Figures 12, 13, and 14. In these figures, the dark areas represent areas where the plate should be thick, and the white areas represent areas where no stiffener is required. Areas of intermediate grayscale shading represent areas where the optimal layout suggests a plate thickness of intermediate thickness.
Fig. 12: Optimal layout - Case a

Fig. 13: Optimal layout - Case b

Fig. 14: Optimal layout - Case c
Conclusions

Effective computational tools exist for the structural layout design of typical automotive body components. In this paper, we have provided an overview of the design capabilities of Project Maxwell. In particular, we have demonstrated this capability for the design of structural components, multibody welded structures, and multimaterial structures for tailored blanking applications. Additionally, we have shown that these tools can also address problems in the layout and design of structures with specified dynamic characteristics. Ongoing work in Project Maxwell involves extending the methodology to the design of new classes of structures, and integrating these layout designs with more traditional sizing tools to create an integrated environment for the realization of superior components.

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References


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