

# **Coupling and Controllability in Optimal Design and Control**

by

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A dissertation submitted in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy  
(Mechanical Engineering)  
in The University of Michigan  
2010

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Dedicated to my late, beloved American Eskimo Dog, Igloo; he started with me on this journey, but couldn't be here to celebrate the completion.

## ACKNOWLEDGEMENTS

When I first told my family and friends that I was going to quit my job and go back to school for a Ph.D., I wasn't sure what kind of reaction I would get. I was prepared for everything from wholehearted support to questions about my sanity. What I received was universal support, approbation, and enthusiasm (even from those who did, in fact, question my sanity). My family and friends stood behind me every step of the way, even those who weren't quite sure what I was doing or why I wanted to do it. My thanks go out to my parents, Thomas and Margaret Peters, who put up with my books and laptop coming to every holiday gathering for the past four years and listening to me talk about what I'm doing, and to my siblings, Catherine, Patrick, Michael, and Jennifer, for their patience. Special thanks to my best friend, Anne Lucietto, her husband Richard Scheffrahn, and their daughter Elisabeth. Their encouragement has been a lifeline to me when things were tough, and they have shared in my joy when things went well. Anne has heard more than she probably ever wanted to know about my research, and yet has been able to share my enthusiasm for neat technical stuff. Jo Lynn Sedgewick has been a good sounding board for mathematical issues who was willing to discuss Taylor series, matrix algebra, or differential geometry, as well as serving as my 'Maple Guru'. Other friends from my pre-University of Michigan life who have provided moral support are Donn Dengel, Mary Mueller, Deanna Heffron, Wendy Landwehr, Sue Biancheri, Tony Puntuzs, and Karen Preston Wozniak.

As I planned to go back to school and worked to choose a university, there were a number of people who provided advice, help, and support of various types. I'd like to thank Professors Steven Skaar and Michael Staniscic at the University of Notre Dame, who gave

me their time and the benefit of their experience, and wrote letters of recommendation for schools and fellowships. Thanks also to Mike Dunn, a former boss of mine from Western Printing Machinery (WPM), who wrote recommendation letters, and to Kent Troxel, my last boss at WPM, who signed off on all those vacation requests as I visited various schools. I'm also grateful to the company as a whole, for throwing me a terrific send-off party with good food, gifts, and good wishes as I went on to a new phase of life. I also had the support of my colleagues at Oakton Community College, where I taught as adjunct faculty. In particular, my thanks to Joe Cirone, who wrote recommendation letters for me.

As I joined the University of Michigan community, I gained colleagues who have at various times served as friends, mentors, sounding boards, a workout partner, a co-author, and a carpool. In particular, I'd like to mention Tahira Reid, Bart Frischnecht, Steven Hoffenson, Michael Alexander, Kwang Jae Lee, Abigail Mechtenberg, Sun Yi, Rachel Bis, Shifang Li, Yongseob Lin, Shanna Daly, Kukhyun Ahn, John and Katie Whitefoot, Yi Ren, Jarod Kelley, Erin MacDonald, Andreas Malikopoulos, James Allison, and everyone connected with the Optimal Design Laboratory. Within the larger Ann Arbor community, I've been privileged to meet some really terrific people, who have served at times as a cheering section and at other times as a source of balance and perspective. I'd particularly like to acknowledge my fellow volunteers at the Humane Society of Huron Valley (HSHV), who have often asked me how my work was coming along as we walked dogs. The dogs at HSHV have always been good listeners, as have my own dogs - my late American Eskimo, Igloo, and Kari and Sydney, who have heard countless presentations before I practiced them on humans.

As I approached the end of my time at the University of Michigan, I spent two months in the summer of 2009 interning at the Tank Army Research and Development Center (TARDEC). This was an interesting experience due to the efforts of several people. I'd like to thank Dave Gorsich, Geri Neal, Greg Hudas, and Mark Brudnak in particular for their time and efforts while I was there.

While the encouragement of the people in my life has been vital, financial support is also a necessity, and I've been fortunate to benefit from several sources of funding. My thanks go out to the Rackham Graduate School, for the Rackham Merit Fellowship; the National Science Foundation, for funding my advisors' grant; the Automotive Research Center; and the Teresan Scholarship Fund, which paid for my first year's textbooks.

And, finally, these acknowledgements wouldn't be complete without mentioning my advisors and committee. My advisors, Dr. Papalambros and Dr. Ulsoy, have taught me a tremendous amount about my research and about academic life. My thanks also to Dr. Kurabayashi and Dr. Sun, who have served on my committee. Without their guidance, this dissertation would never have been started, much less completed.

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# CHAPTER I

## Introduction

### 1.1 Summary

Many of the new products and systems being designed today require the design of both a physical system, or artifact, and a controller. A significant number of such systems exhibit coupling between the artifact and its controller, i.e., the performance of the artifact itself may depend upon the controller, and the performance of the controller may depend on the physical configuration of the device. In designing the complete system via optimization, this coupling between the product and its controller can be critical for achieving the best system performance. Previous research has shown that, when coupling is present, optimal system design presents special challenges. In particular, failure to address coupling appropriately results in sub-optimal systems.

This chapter introduces the motivation for the dissertation research work in Section 1.2. The concept of coupling is defined, and existing measures used to quantify coupling are explained in Section 1.3. Examples of coupled systems are then given in Section 1.4, and optimization methods in the existing literature are discussed in Section 1.5. The use of controllability in system design is summarized in Section 1.6. Section 1.7 lists the dissertation's original contributions to the literature. The chapter concludes in Section 1.8 with an outline of the contents of the dissertation.

## 1.2 Motivation

New technologies and ‘smart’ products have the potential to improve life dramatically and to transform our understanding of the world. New technologies at the smallest scale promise to radically change people’s lives. Nanotechnology and biological microelectrical mechanical systems (bioMEMS) carry the potential to allow people with a wide variety of medical conditions, such as epilepsy, diabetes, and high cholesterol, to monitor and control their health with minimal intrusion into their ability to live a normal life. Hearing aids, pacemakers, and many other devices can be drastically improved, allowing our aging population to remain active and productive. Scientific instruments utilizing these technologies in the hands of talented researchers will facilitate new discoveries.

On a larger scale, smart systems address challenges in our society’s needs for energy and transportation. Smart electrical grids, intelligent hybrid cars, and smart appliances can improve the reliability of our infrastructures, reduce wasted energy, and limit our impact on the environment. Achieving these revolutions, however, requires a change in engineering design practices. All of these applications require the design of both an artifact and a controller, and it can be reasonably asserted that optimal designs of artifact and controller are required in order to realize the full benefit of these technologies. The problem of designing both the artifact and its controller for such smart products will be referred to here as the *co-design problem*. Coupling between the artifact and controller has been demonstrated to be critical in the proper co-design of many systems. The existence of such coupling seems to indicate that a simultaneous problem formulation is preferable to a sequential one in order to achieve a system-optimal design. However, the simultaneous formulation presents challenges. Computationally, it is a larger problem and is more difficult to solve. Even if the problem is tractable, though, formulating it requires multiple areas of expertise. It is unlikely that a single person, or even a single group within an organization, would possess all of the necessary expertise. Multiple groups need to be involved in the design of a typical artifact. Thus, separation of the two problems, design and control, and solution