

**A QUANTIFICATION OF PROPORTIONALITY AESTHETICS
IN MORPHOLOGICAL DESIGN**

by

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“because we look not at what can be seen but at what cannot be seen; for what can be seen is temporary, but what cannot be seen is eternal.” (II Corinthian 4:18)

To my parent, my children, and my beloved wife Sang-Hee for their faith in the LORD

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ABSTRACT

A QUANTIFICATION OF PROPORTIONALITY AESTHETICS IN MORPHOLOGICAL DESIGN

by

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The study of beauty in morphological design is conducted using a computer-based system for quantification of aesthetics in the presence of proportionality. The system is formulated, implemented, and tested in the analysis and synthesis of various design artifacts.

With an assumption that universal aesthetic principles exist and are quantifiable, proportionality (known as the theory of means) is proposed as the apparatus for this research. When the computation of proportionality is encoded as quantifiable criteria into a design machine, in turn, a quantification of proportionality aesthetics is defined as the "mechanization" of the human process in selecting an "optimum" design. In this quantification, a description of a given design artifact consists of the dimensions of the

artifact. The dimensions are used to compute proportionality values. These values are assigned to the given design artifact as its reference characteristic. When an artifact has the best reference characteristic, it is selected as the optimum among the alternatives generated from the given artifact using a design optimization methodology.

This quantification process is implemented computationally in an analysis and synthesis system called Hermes. Hermes performs optimization using genetic algorithms. The analysis component of Hermes is designed as a plug-in application for AutoCAD and is tested with a group of 38 buildings designed by the Italian architect Palladio to discover the extent of proportionality rules used in his work. Factor analysis is employed for the classification of the results. The synthesis component is written in MATLAB and in AutoLISP. This synthetic function is tested with Palladio's Villa Rotonda and Mondrian's "Composition with Red, Yellow, and Blue." The program provides a better understanding of the master's usage of proportionality in the design. In return, it allows exploration of how these designs may change by varying the presence of proportionality.

The theoretical and computational results obtained using Hermes suggest that quantification of proportionality aesthetics is feasible and useful as an aesthetic measure for analyzing existing and creating new works of art. Providing an opportunity to begin to explore aesthetic pleasure, this approach may address a variety of issues about aesthetics in design, and about design in general.