

# MONOTONICITY, ACTIVITY AND SEQUENTIAL LINEARIZATION IN PROBABILISTIC DESIGN OPTIMIZATION

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## Abstract

Design optimization under uncertainty is considered in the context of problems with probabilistic constraints. Probabilistic optimization has been studied for several years; in this dissertation, some theoretical developments in certain classes of deterministic optimization problems are extended to probabilistic ones. Specifically, the optimality conditions of probabilistic optimization and properties such as monotonicity, activity, criticality, and dominance of constraints are investigated and their applicability to probabilistic optimizations is demonstrated.

Solution methods for probabilistic optimization problems to date have focused on improving efficiency rather than accuracy and global convergence behavior of solution algorithms. A new strategy utilizing an adaptive sequential linear programming (SLP) algorithm is proposed to balance accuracy, efficiency, and convergence. The method is enabled by a proposed reliability contour approach that can be used to determine feasibility of active constraints with reduced computational cost. In the presence of many probabilistic inequality constraints, computational costs can be reduced if probabilistic values are computed only for constraints that are known to be active or likely active. Extensions of monotonicity analysis to probabilistic constraints are used to construct active set strategies that reduce the computations associated with handling inequality constraints, similarly to the deterministic case. Such a strategy is demonstrated on the SLP algorithm above, along with numerical examples.

The above method is shown to be also applicable to problems previously difficult to solve, namely, problems with non-normal random variables and joint constraint reliabilities. These problems are critical when dealing with environmental design. A case study is presented where the relationship between vehicle emissions and air quality is considered. The study calculates speed limits that jointly satisfy air quality standards for NO<sub>x</sub> and CO within a desired reliability.

As a first effort to extend these methods to multilevel systems, we consider the stochastic extension of the analytical target cascading (ATC) formulation. A linearization technique is employed for estimating the propagation of uncertainties throughout the problem hierarchy. The proposed methodology is applied to a piston-ring/cylinder-liner engine subassembly design problem. This bi-level application of ATC is promising for future extension to multilevel problems.