MFG 555 Final Project Report
Optimal Profit of a Dial-Readout Scales

Team 6

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1. Introduction

This project is to produce a successful and profitable scale using conjoint analysis, which would optimize the profits according to consumer's needs and Engineering optimization in multiple ways. The initial analysis would separate the system into Marketing subsystem and Engineering subsystem and then conduct both optimal results. Then, we will conjoin the system of consumer needs and engineering design to iteratively the target variables and finally obtained the 'sweet point' profit of the scale. The system integration analysis would consider the system as a whole thing by conjoint the subsystems using linking variables and boundary constraints, and optimize the all the targets together. Comparison between disjoint analysis and conjoint analysis will be conducted at the end of this report.

As for the scale, five product characteristics-weight capacity, aspect ratio, platform area, tick mark gap, and number size-plus price were adopted because of their relevance to consumers and designers as well as their prevalence in online purchase descriptions and pictures. Respectively, they were represented by z_1, z_2, z_3, z_4 and z_5. Optimal values for each of these 5 product characteristics were found first for Marketing Planning Subsystem and Engineering Design Subsystem separately by creating the mathematical model and applying fmincon optimization tool in Matlab. Then Analytical Target Cascading, ATC will be used to iteratively minimize the gap between two subsystems.

The whole system is modeled as in Figure 1.1. It shows how to do the two subsystems link with each other. Customer Preference Survey provides the ideal characteristic values to both subsystems. Subsystem 1 is aimed to find the engineering possible characteristic values that are closest to the target. Subsystem 2 is to balance the demands and price to maximize the profit.

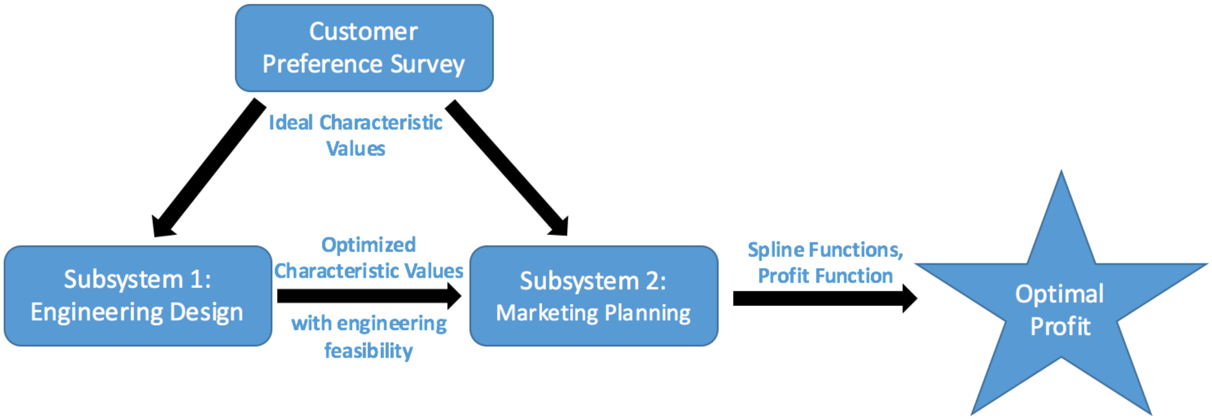

Figure 1.1 Subsystems Interaction

2. Subsystem-level Optimization

1) Optimal Profits of Marketing Planning Subsystem (Chi Qiu)

To find the optimal profits of the Marketing Planning subsystem, a mathematical problem was generated to represent the model. The objective function with respect to marketing characteristics, z_M and price, p as well as all the constraints are listed below.

Maximize $\Pi - \| w^{\circ}(z_M - z_E) \|_2^2$
with respect to z_M, p
subject to $z_{min} \leq z_M \leq z_{max}$
$p_{min} \leq p_M \leq p_{max}$
where $\Pi = q\,(p - c_V) - c_1$
$q = se^{v}(1 + e^{v})^{-1}$
$v = \sum_{k=1}^{k-1} \Psi_k(z_k) + \Psi_k(p)$

Table 2.1 Summary of Notation, System Variables and Parameters

‖ ‖	Vector norm	p	Selling price
	Term-by-term vector	z_M	Vector of product characteristics targets set by marketing
c_1	Investment Cost	z_E	Vector of product characteristics achieved by engineering
c_v	Variable cost per product	β	Part-worth coefficient
v	Deterministic component of utility	**Π**	Profit
s	Size of the entire market	Ψ	Spline function to interpolate part-worths

First of all, the objective function needs to be created in Matlab. The part-worth coefficients for all 5 of the product characteristics and price are estimated by taking the results of the data from the choice-based conjoint survey. By using the spline interpolation function of the part-worth, we are able to represent each of the product attribute and price with an equation. Then the utility, v_j was calculated using Equation 4, the demand, q is calculated by Equation 3, and profit is calculated by using Equation 2. At this point, the main objective function, Equation 1 is not considered since this can only be done after we combining two subsystems together. Also, no variable constraints are considered for this subsystem.

Next step, by applying fmincon in Matlab, the initial result of Marketing Planning subsystem is obtained and listed in Table 2.2, Page 6. The code that used to create the objective function for the Marketing Planning subsystem are provided in *MarObjecfun.m*. In the script, x(1)-(6) represent the 5 product attributes and price. $y_1 \sim y_6$ represent the part-worth values for the 5 product attributes and price. yy_1~ yy_6 represent the spline interpolation functions. v is utility, q is demand. Also we assume that the total market is 5 Million \$, the cost is 3\$, the Investment is 1 Million \$ and the utility for competitor's product is 1.

MarCallfun.m includes the script used to call the fmincon. x_0 is the initial points, which was picked as the lower bounds of all the variables. Since no linear/nonlinear constraints is associated with the system, blank is remained for A, B, Aeq and Beq. lb and ub represent the lower bound and upper bound of the variables.

Table 2.2 Initial Marketing Plan Optimization Result

Description	Units	Initial Marketing Plan
z_1 (Weight Capacity)	lbs	283
z_2 (Aspect Ratio)	-	0.946
z_3 (Platform Area)	in.^{2}	124.4
z_4 (Tick Mark Gap)	in.	0.136
z_5 (Number Size)	in.	1.75
p (Price)	\$	28.51
Profit	\$	80.49 M

As a results, the profit of the marketing planning subsystem is around 1 million \$ which is off from what it is supposed to be. Further work is necessary to check all the details in the Matlab code to get a better result.

2) Optimization of Engineering Design Subsystem (Wenjing Xu)

a. Objective Function of Engineering Subsystem

Traditionally, the engineering design optimization focuses on choosing value of design variables that maximize product performance. However, the best engineering design might not as need as customers' desire. So to make a "customer-ready" prototypes, we need to minimize the gap between engineering optima and customer's preference.

minimize $\| w^{\circ}(z_M - z_E) \|_2^2$
w.r.t. x
s.t. $g(x) \leq 0, h(x) \leq 0$
$x_{min} \leq x \leq x_{max}$
where z_M = r(x)

z_M: Marketing optimization result;
z_E: Engineering optimization result;

In the objective function, z_M are the characteristic values that acquired from the marketing survey, z_E are represented by variables, the optimal z_E value should be both engineering feasible and the closest to marketing target.

b. Design Variables, Parameters and Constraints of Engineering Subsystem

Table 2.3 lists all the design variables in the Engineering Design subsystem with the meaning of each variable.

Table 2.3 Engineering Design Variables

Name	Description
x_1	Length from base to force on long lever
x_2	Length from force to spring on long lever
x_3	Length from base to force on short lever
x_4	Length from force to joint on short lever
x_5	Length from force to joint on long lever
x_6	Spring constant
x_7	Distance from base edge to spring
x_8	Length of rack
x_9	Pitch diameter of pinion
x_{10}	Length of Pivot's horizontal arm
x_{11}	Length of Pivot's vertical arm
x_{12}	Dial diameter
x_{13}	Cover length
x_{14}	Cover width

Table 2 lists all the design parameters of the Engineering Design subsystem.

Table 2.4 Engineering Design Parameters

Name	Description	Value	Units
y_1	Gap between base and cover	0.30	in
y_2	Minimum distance between spring and base	0.50	in
y_3	Internal thickness of scale	1.90	in
y_4	Minimum pinion pitch diameter	0.25	in
y_5	Length of window	3.00	in
y_6	Width of window	2.00	in
y_7	Distance between top of cover and window	1.13	in
y_8	Number of lbs measured per tick mark	1.00	lbs
y_9	Horizontal Distance between spring and pivot	1.10	in
y_{10}	Length of tick mark and gap to number	0.31	in
y_{11}	Number of lbs that number length spans	16.00	lbs
y_{12}	Aspect ratio of number (length/width)	1.29	-
y_{13}	Minimum Allowable Distance of lever at base to centerline	4.00	in

Eight mathematical constraint functions g(x) were developed based on geometric and mechanical relationships to ensure that the design variable vector X represents a meaningful and feasible design. All the constraint functions in the Engineering Design subsystem are shown in Table 2.5.

Table 2.5 Engineering Subsystem Constraints g(x)

Constraints	Description
$x_{12} \leq x_{14}$-$2y_1$	The dial diameter x_{12} must be small enough to fit inside the base widthwise, measured as the cover width x_{14} minus the gap y_1 between the cover and the base on both sides.
$x_{14} \leq x_{13}$-$2y_1$- x_7- y_9	The dial G must fit lengthwise inside the scale base (x_{13}+$2y_1$) with sufficient room for the spring plate (x_7+ y_9).
(x_4+ x_5) $\leq x_{13}$-$2y_1$	The length of the short levers (x_4+ x_5) must be small enough to fit inside the base lengthwise (x_{13}-$2y_1$).
$x_5 \leq x_2$	The position along the long lever of the short joint x_5 must be within the bounds of the long length x_2.
$x_8 \geq (x_{13}$-$2y_1)$ - ($\frac{x_{12}}{2}$+ y_7)- x_7- x_9- x_{10}	The length x_8 of the rack must be long enough to span the space between the pivot lever and the pinion.
x_7+ y_9 + x_{11} + $x_8 \leq x_{13}$ -$2y_1$	The fully extended position, the end of the rack (x_7+ y_9 + x_{11} + x_8) must fit inside the scale body lengthwise (x_{13} -$2y_1$).
$(x_1 + x_2)^2 \leq (x_{13} - 2y_1 - x_7)^2 + (\frac{x_{14}-2y_1}{2})^2$	The two levers connect to the top edge of the base rather than the side. Therefore, the lever length $(x_1 + x_2)$ is limited by the width dimension of the scale body $(x_{14} - 2y_1)$.
$(x_1 + x_2)^2 \geq (x_{13} - 2y_1 - x_7)^2$+ ${y_{13}}^2$	The long levers must be long enough $(x_1 + x_2)$ to attach to the top edge of the base at a minimum distance y_{13} from the centerline.

c. Response Functions r(x)

The response functions r(x) are defined in Table 2.6. The five product characteristics $z_1 \sim z_5$ are calculated in terms of related design variables x and parameters y assuming the scale is made up of rigid bodies (except for the spring) and using standard static force and moment balancing.

Table 2.6 Engineering Respond Functions

Response function	Explanation
$z_1 = \frac{4\pi x_6 x_9 x_{10}(x_1 + x_2)(x_3 + x_4)}{x_{11}(x_1(x_3 + x_4) + x_3(x_1 + x_5))}$	Weight capacity
$z_2 = \frac{x_{13}}{x_{14}}$	Aspect ratio, length of the cover divided by its width
$z_3 = x_{13}x_{14}$	The area of the scale cover
$z_4 = \pi\frac{x_{12}}{z_1}$	The arc length of the gap between 1-lb interval tick marks
$z_5 = \frac{(2\tan(\frac{\pi y_{11}}{z_1}))(\frac{x_{12}}{2} - y_{10})}{(1 + \frac{2}{y_{12}}\tan(\frac{\pi y_{11}}{z_1}))}$	The number of length, a measure of overall printed number size.

d. Initial Engineering Optimization Result

Apply SQP algorithm to the objective function in the Matlab file *EngCallfun.m*, with all the variables and parameters, claimed in *EngObjective.m* and constraints in *EngConstraint.m.* A set of z_E values are obtained, as well as the minimum difference between the Marketing plan and Engineering design. The initial Engineering optimization results are included in Table 2.7.

Table 2.7 Initial Engineering Optimization Result

Description	Units	Result
z_1 (Weight Capacity)	lbs	287
z_2 (Aspect Ratio)	-	1.68
z_3 (Platform Area)	in.^{2}	124.2
z_4 (Tick Mark Gap)	in.	0.037
z_5 (Number Size)	in.	0.38
f (difference between Marketing Plan and Engineering Design)	-	2.43

3) Subsystem Sequential Optimization

After acquiring the Engineering feasible characteristic values, $z_1 \sim z_5$, we can find corresponding customer satisfaction extent value to optimize the price using fmincon, and calculate the profit with the optimized price. See append file *SeqObjecfun.m* and *SeqCallfun.m.* The optimal price for sequential optimization is $26.18, and the profit is $ 60.77 million.

3. System-level Optimization

1) System-level Description

For system-level optimization, SQP algorithm was implemented. However, instead of using sequential optimization to have the optimal solution of the marketing subsystem as the target for the engineering subsystem and calculate the optimal and feasible solution, the system-level optimization was defined as All-In-One (AIO) system without system decomposition. Both of the subsystems would formulate the objective function together for the system-level optimizer. Figure 3.1 shows a flowchart that represents the relationship between subsystems and system optimizer.

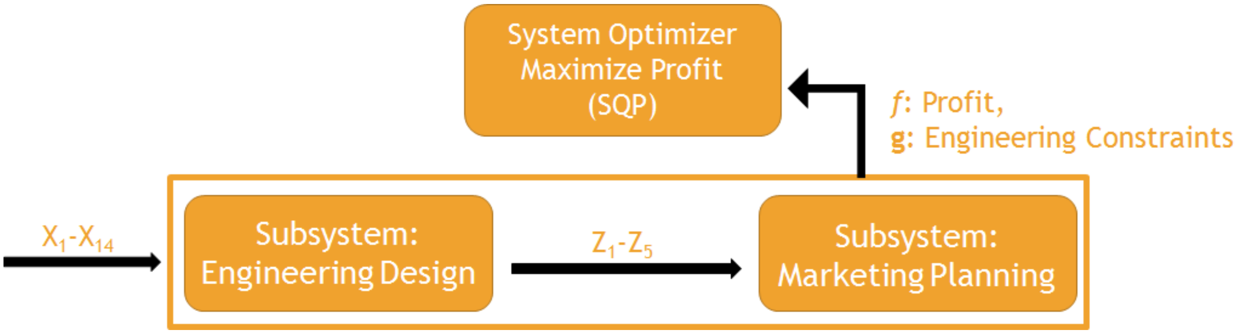

Figure 3.1: This flowchart represents how the system-level optimization works.

System inputs are $x_1 \sim x_{14}$, the local design variables of the scale. In the Engineering Design subsystem, $x_1 \sim x_{14}$ were formulated to the coupling variables $z_1 \sim z_5$ through the response functions listed in Table 2.6, P. 6. Then through the marketing planning subsystem, $z_1 \sim z_5$ together with customer survey data, we can generate spline functions and iteratively find the biggest Beta values, which represents the customer's satisfaction degree. Also, by applying the probability and demand functions, the profit function will be sent to the system-level optimizer along with the Engineering design constraints. By using the SQP algorithm, the optimal solution can be obtained.

2) System-level Variables, Parameters and Constraints

Variables $x_1 \sim x_{14}$ are same as Engineering Design variables, while price is added as the 15th variable x_{15} listed in Table 3.1. The five characteristics

Table 3.1 AIO Design Variables

Name	Description	Type
x_1	Length from base to force on long lever	Local design variable
x_2	Length from force to spring on long lever	Local design variable
x_3	Length from base to force on short lever	Local design variable
x_4	Length from force to joint on short lever	Local design variable
x_5	Length from force to joint on long lever	Local design variable
x_6	Spring constant	Local design variable
x_7	Distance from base edge to spring	Local design variable

x_8	Length of rack	Local design variable
x_9	Pitch diameter of pinion	Local design variable
x_{10}	Length of Pivot’s horizontal arm	Local design variable
x_{11}	Length of Pivot’s vertical arm	Local design variable
x_{12}	Dial diameter	Local design variable
x_{13}	Cover length	Local design variable
x_{14}	Cover width	Local design variable
x_{15}	Price	Local design variable
z_1	Weight Capacity	Coupling variable
z_2	Aspect Ratio	Coupling variable
z_3	Platform Area	Coupling variable
z_4	Tick Mark Gap	Coupling Variable
z_5	Number Size	Coupling Variable

Table 3.2 listed the system-level parameters, which are exactly the same as the engineering design parameters.

Table 3.2 AIO Design Parameters

Name	Description	Value	Units
y_1	Gap between base and cover	0.30	in
y_2	Minimum distance between spring and base	0.50	in
y_3	Internal thickness of scale	1.90	in
y_4	Minimum pinion pitch diameter	0.25	in
y_5	Length of window	3.00	in
y_6	Width of window	2.00	in
y_7	Distance between top of cover and window	1.13	in
y_8	Number of lbs measured per tick mark	1.00	lbs
y_9	Horizontal Distance between spring and pivot	1.10	in
y_{10}	Length of tick mark and gap to number	0.31	in
y_{11}	Number of lbs that number length spans	16.00	lbs
y_{12}	Aspect ratio of number (length/width)	1.29	-
y_{13}	Minimum Allowable Distance of lever at base to centerline	4.00	in

In Table 3.3, the first eight constraints are same as in the Engineering Design Subsystem. The last ten constraints are the boundary of the five characteristics.

Table 3.3 AIO Constraint Functions

Constraints	Description
$x_{12} \leq x_{14}-2y_1$	The dial diameter x_{12} must be small enough to fit inside the base widthwise, measured as the cover width x_{14} minus the gap y_1 between the cover and the base on both sides.
$x_{14} \leq x_{13}-2y_1-x_7-y_9$	The dial G must fit lengthwise inside the scale base ($x_{13}+2y_1$) with

	sufficient room for the spring plate (x_7+ y_9).
$(x_4+ x_5) \leq x_{13}$-2y_1	The length of the short levers (x_4+ x_5) must be small enough to fit inside the base lengthwise (x_{13}-2y_1).
$x_5 \leq x_2$	The position along the long lever of the short joint x_5 must be within the bounds of the long length x_2.
$x_8 \geq (x_{13}$-2y_1) - ($\frac{x_{12}}{2}$+ y_7)- x_7- x_9- x_{10}	The length x_8 of the rack must be long enough to span the space between the pivot lever and the pinion.
x_7+ y_9 + x_{11} + $x_8 \leq$ x_{13} -2y_1	The fully extended position, the end of the rack (x_7+ y_9 + x_{11} + x_8) must fit inside the scale body lengthwise (x_{13} -2y_1).
$(x_1 + x_2)^2 \leq (x_{13} - 2y_1 - x_7)^2 + (\frac{x_{14}-2y_1}{2})^2$	The two levers connect to the top edge of the base rather than the side. Therefore, the lever length ($x_1 + x_2$) is limited by the width dimension of the scale body ($x_{14} - 2y_1$).
$z_1 \geq 200$	The lower bound of weight capacity is 200lbs.
$z_2 \leq 400$	The upper bound of weight capacity is 200lbs.
$z_3 \geq 0.75$	The lower bound of aspect ratio is 0.75.
$z_4 \leq 1.33$	The upper bound of aspect ratio is 1.75.
$z_5 \geq 100$	The lower bound of platform area is 100 in2.
$z_6 \leq 140$	The upper bound of platform area is 140 in2.
$z_7 \geq 0.625$	The lower bound of tick mark gap is 0.625in.
$z_8 \leq 0.1875$	The upper bound of tick mark gap is 0.1875in.
$z_9 \geq 0.75$	The lower bound of number size is 0.75.
$z_{10} \leq 1.75$	The upper bound of number size is 1.75.

3) System-level Optimization Results

1. AIO Optimization for One Year

For the one-year AIO optimization, all the variables, parameters and functions are plugged in *AIOobjecfun.m*, all the constraints are set in *AIOconstraints.m.* And then, apply SQP algorithm to optimize the target variables $z_1 \sim z_5$ and price x_{15} iteratively. The final profit is calculated using the most optimal values of target variables. Results are listed below in Table 3.4.

Table 3.4 AIO Optimization Result for One Year

Description	Units	β	AIO - 1
z_1 (Weight Capacity)	lbs	0.1350	250
z_2 (Aspect Ratio)	-	0.2821	1.00
z_3 (Platform Area)	in.^{2}	0.0364	131.4
z_4 (Tick Mark Gap)	in.	0.1438	0.116
z_5 (Number Size)	in.	0.2915	1.34
x_{15} (Price)	$	-0.5666	27.19

Profit	$	-	69.13 M

b. AIO Optimization for Five Year

However, one-year optimization is not representative enough to support the long term profit since customer preference and price are fluctuating over time. To alleviate the uncertainty, we extended the period of profit from one year to five years to enable a robust design, which will reduce variation without eliminating the causes of variation.

To model the marketing fluctuation, we randomly change the value of β from the customer survey within the range of plus or minus 10%. With the new β values for 5 years, we can generate new spline functions and calculate the profit for each year just as the procedure of one-year AIO Optimization. After that, the objective function is changed to the summation of the profits for the five years, which means the optimization is applied to the profits for five years integratively. In Table 3.5, the obtained $z_1 \sim z_5$ represents the robust design feature and price x_{15} stands for a corresponding insensitive price. (The Matlab codes for optimizing five year profit are in folder AIO – 5.)

Table 3.5 AIO Optimization Result for five Year

Description	Units	β	AIO - 5
z_1 (Weight Capacity)	lbs	0.1350	254
z_2 (Aspect Ratio)	-	0.2821	0.99
z_3 (Platform Area)	in.^{2}	0.0364	140
z_4 (Tick Mark Gap)	in.	0.1438	0.119
z_5 (Number Size)	in.	0.2915	1.37
x_{15} (Price)	$	-0.5666	27.13
Profit	$	-	68.65 M

4) Comparison

Table 3.5 Comparison between Sequential Optimization, AIO - 1 Year and AIO - 5 Year.

		Disjoint		Joint	
Description	Units	Initial Marketing Plan	Sequential Optimization	AIO - 1 Year	AIO - 5 Year
z_1 (Weight Capacity)	lbs	283	287	250	254

z_2 (Aspect Ratio)	-	0.946	1.680	1.000	0.990
z_3 (Platform Area)	in^2	124.4	124.2	131.4	140.0
z_4 (Tick Mark Gap)	in.	0.136	0.037	0.116	0.119
z_5 (Number Size)	in.	1.75	0.38	1.34	1.37
x_{15} (Price)	$	28.04	26.18	27.19	27.13
Profit	$	80.49 M	73.68 M	69.13 M	68.63 M

From the comparison results in Table 3.6, the profit from the initial marketing plan is the largest since all the β, part worth coefficients, were picked at the maximum, which means the highest customer preference was assumed. Then by applying engineering constraints to the model, the results was decreased since some of the β can't be at the maximum anymore. Speaking of the system-level optimization, two subsystems were integrated together and the optimizer uses SQP algorithm to solve the model. Unlike optimizing separately, in the AIO model, two subsystems don't optimize anything but help the system to formulate the final objective function. After that, we expanded the profit model from 1 year to 5 years and optimized the average profit for the scale with different customer preference curves, which were generated randomly. The result of AIO system for 5 years was close but with some difference to the 1-year model.

5) Post Analysis

In the SQP algorithm, the Lagrange Multiplier λ are used to evaluate the effectiveness and importance of each constraint. According to the KKT conditions, λ must be zero or greater since all the constraints are inequality constraints. The bigger the λ, the more important the constraint. And if $\lambda = 0$, the corresponding constraint is inefficient.

Table 3.7 λ Values for AIO - 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
λ Values for Lower Bound Constraints																	
0	0	0	0	0	0	5.4740	0	0	0	0	0	0	0	0			
λ Values for Upper Bound Constraints																	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
λ Values for Unequal Constraints																	
0	5.4740	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Obviously from Table 3.7, most constraints are ineffective. The first effective constraints in Lower Bound column with $\lambda_7 = 5.4740$ means x_7, the distance from base edge to spring should be shorter than 0.5 in. The second effective constraints in Unequal Constraints column with $\lambda_2 = 5.4740$ means the second constraint $x_{14} \leq x_{13}$-2y_1- x_7- y_9, which means The length x_8 of the rack must be long enough to span the space between the pivot lever and the pinion is effective for the AIO optimization.

Table 3.8 λ Values for AIO - 5

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
λ Values for Lower Bound Constraints																	
0	0	0	0	0	0	4.5796	0	0	0	0	0	0	0	0			
λ Values for Upper Bound Constraints																	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
λ Values for Unequal Constraints																	
0	4.5796	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3.8 shows the λ Values for five-year AIO optimization in a similar way. In comparison, the effective λs are relatively less important. The changes in λ are the reason for different optimization results.

Reference:

[1] Jeremy J. Michalek, Fred M. Feinberg, and Panos Y. Papalambros. (2005) Linking Marketing an Engineering Product Design Decisions via Analytical Target Cascading.