Optimizing Bus Stop Distribution In a Certain Area

Ping Zhang, Yi Ren

Abstract

Bus is one of the most publication transportations, which plays a vital role in people's life. Hence, how to set bus stop becomes pivotal to the bus system. For this project, I considered a certain area with different living locations and one common destination. In order to minimize the whole cost to the destination, the bus stops have to be well distributed. Taking time that is needed to the destination as the cost, I successfully obtained the bus stop distribution by using MultiStart in Matlab to search global minimum point.

1. Introduction

Public transportation can greatly facilitate people's life in most of countries. From the perspective of environment, public transportation is more environment-friendly, which can reduce the emission of Carbon Dioxide and other toxic gases due to its large hold volume, compared to cars. Politically, the transportation funding budget is competitive increasingly these days[1]. The optimal distance between bus stops seems significant for it can not only decrease the budget, minimizing the cost, but also enhance the accessibility to bus transportation.

A number of efforts have been made to optimize the distance between bus stops. The first related study was simply based on demand distribution, mainly considering bus patron's convenience, safety and access time, as well as the efficiency of the bus company[2].Furth and Rahbee employed dynamic programming and geographic modeling to study the optimal bus stop distance[3]. Firstly, they determined a geographic model to recapitulate the bus demand distribution via observing existing bus stops. Then, a dynamic programming algorithm was applied to obtain the optimal bus stop distance. A model has been built by Saka, based on the fundamental relationships among velocity, uniform acceleration/deceleration and displacement, average bus operating speed, etc., to look for optimal distance between bus stops[4]. Ángel developed a bi-level optimization model to get bus stops in order to minimize the social cost of the transport system[5].

Intuitively speaking, closely designed bus stops can provide a short distance for people, however, increases the time cost and fuel cost for the bus company. Nevertheless, large distance distributed bus stops will potentially induce the decrease of the bus company's profit as a result of the loss of some passengers and also increases the cost of passengers.

This paper built a model to finalize the optimum bus stops on a 2-D map considering the population density distribution. In the model, the whole cost is divided into three parts:

1. Access cost: time that passengers need to walk to the nearest bus stop.
2. Riding cost: time in the bus.
3. Final cost: time that passengers need to walk to the destination.

To optimize the bus stop distribution, assumptions are made as follow:

1. There are four living locations and one common destination.
2. All people want to go to the same destination.
3. Passengers can go to the nearest bus stop.
4. There is only one bus route, running in a certain direction.
5. There are five bus stops in this area.
6. The area is defined as $x \in [-1, 1]$, $y \in [-1, 1]$.
7. The population density function is based on Clark model.

2. Design Problem

2.1 Problem Statement

The dense distribution of bus stops will no doubt facilitate people's life, enhancing the accessibility to bus and thus reducing the number of cars on the road. However, this, in turn, adds more burdens to government budget or decreases the profits of the bus company. Also, frequent acceleration/deceleration due to closely spaced bus stops, will not only cause damage to buses, but also reduce the degree of satisfaction of taking bus. Although a sparse distribution of bus stops can save money for the bus company or the government, it may lose current and potential passengers, which in turn, decreases the profit of the bus company; and increase the cost of users,

Traditionally, the bus stop models mainly focus on one -dimension model, which may omit some details about true story[1]. Therefore, this model concentrates on two-dimension model that can better mimic the true situation.

2.2 Mathematical Model

2.2.1 Notations

Matrix S : the coordinate matrix of bus stops, 5 by 2;

Matrix l: the living location matrix, 4 by 2;

Matrix P_0: population density at the center of living locations, 4 by 1;

Matrix r_0: characteristic radii of four living locations, 4 by 1;

C_a : the access cost;

C_r : the riding cost;

C_f : the final cost;

C : the total cost;

P(x,y): population density function;

V : average velocity of the bus;

v : average velocity of people walking, around 1.2 m/s.

2.2.2 Objective function

First, I just take infinitesimal, $\Delta A(x, y)$, as study object. Hence, for this $\Delta A(x, y)$, three costs are as follow:

- **Access cost**

Access cost is time that is needed to the nearest bus stop from $\Delta A(x, y)$,

$$[\Delta C_a, idx] = \frac{\min|S(i,:) - \Delta A(x, y)|}{v}$$

idx returns the bus stop number, which is used to compute riding cost.

- **Riding cost**

Riding cost refers to the time in the bus, which can be given as follow:

$$\Delta C_r = \frac{\sum_{i=idx}^{4} |S(i+1,:) - S(i,:)|}{V}$$

- **Final cost**:

Final cost is the time from last stop to destination, which is:

$$\Delta C_f = \frac{norm(S(5,:))}{v}$$

- **The number of people in**

According to Clark model[6], $P_i = P_0 \exp(-r / r_0)$. Here, r_0 is city characteristic radius; P_0 is population density in city center. The number of people in $\Delta A(x, y)$ is:

$$\Delta P = (\sum_{i=1}^{4} P_{0i} \exp(-r_i / r_{0i}))dxdy, \text{ where } r_i = norm(l(i,:) - \Delta A(x,y))$$

- **The cost of ΔA(x, y)**

Hence, the cost of $\Delta A(x, y)$ is given by:

$$\Delta C = \Delta P * (\Delta C_a + \Delta C_r + \Delta C_f)$$

The whole cost will be:

$$C=\int_{-1}^{1}\int_{-1}^{1}\left(\frac{\min|S(i,:)-\Delta A(x,y)|}{v}+\frac{\sum_{i=idx}^{4}|S(i+1,:)-S(i,:)|}{V}+\frac{norm(S(5,:))}{v}\right)\left(\sum_{i=1}^{4}P_{0i}\exp(-r_i/r_{0i})\right)dxdy$$

Here, the whole cost is the function of bus stop coordinates.

2.2.3 Constraints and parameters

There is no extra constraint except upper bound and lower bound.

-1 <= x, y <= 1;

v = 4.32;

V = 72;

P_0 = [0.8, 1, 0.8, 2]';

r_0 = [0.1, 0.1, 0.1, 0.1]';

l =

0.4330	0.2500
-0.2500	0.4330
-0.2828	0.2828
-0.4000	-0.6928

3. Design optimization and discussion

Using "fmincon" with upper bound and lower bound, I got the optimal result as follow:

S =

0.4309	0.2579
-0.2352	0.4616
-0.3296	0.2301
-0.3878	-0.6590
0.0000	0.0000

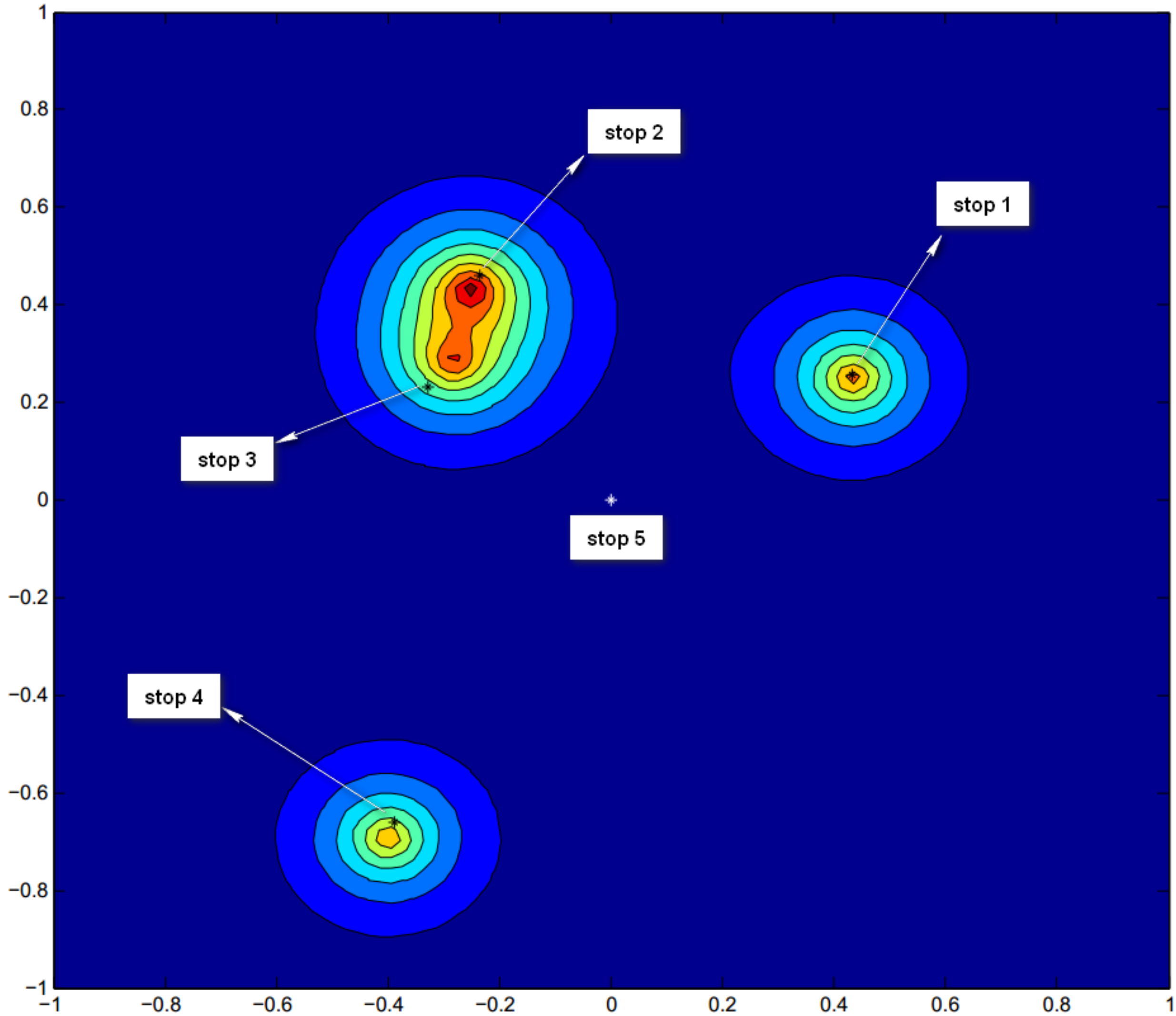

Figure-1: optimal result with local solution

Figure 1 shows the bus stop distribution on the contour plot of population density function. As expected, there is a bus stop near the destination and other bus stops locate near the living locations. However, there is a large shift for bus stop (stop 3) compared to living location 3. As for the whole cost, the Fval is 0.0154, compared to cost, 0.0156, which was computed by taking living locations as variables for the whole cost. From this perspective, I found a local minimum point. Next, in order to find the global minimum point, I employed "MultiStart" slover. S matrix was obtained and displayed as:

S =

-0.3969	-0.6907
0.4141	0.2283
-0.2248	0.4718
-0.2616	0.2643
-0.0004	-0.0002

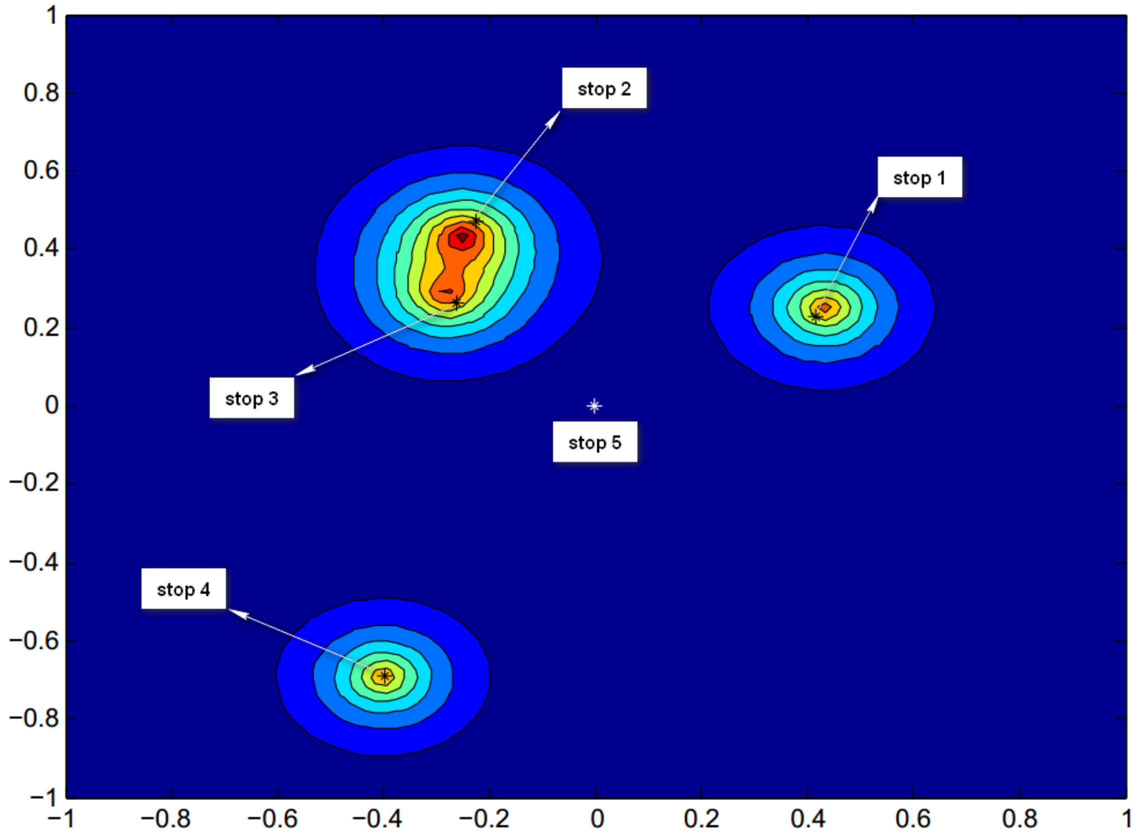

Figure-2: optimal solution with global solution

Figure-2 displays a better result compared to local solution (Figure 1). And also, for the Fval(global solution) equals 0.0150, which is smaller than Fval(local solution, 0.0154).

As predicted, the bus stops should locate near living location and destination. Because the population density of living locations are largest and bus stop near destination can minimize the final cost, although slightly increase the riding cost. This is due to the perturbation of other living locations' population density.

In order to test the robustness of the algorithm, the coordinates of living locations were changed to:

l =

0.4330	0.2500
-0.2828	0.2828
-0.4000	-0.6928
0.2500	-0.4330

Figure 3 demonstrates that the algorithm is able to respond to the changed living locations and gives the optimal distribution of bus stops.

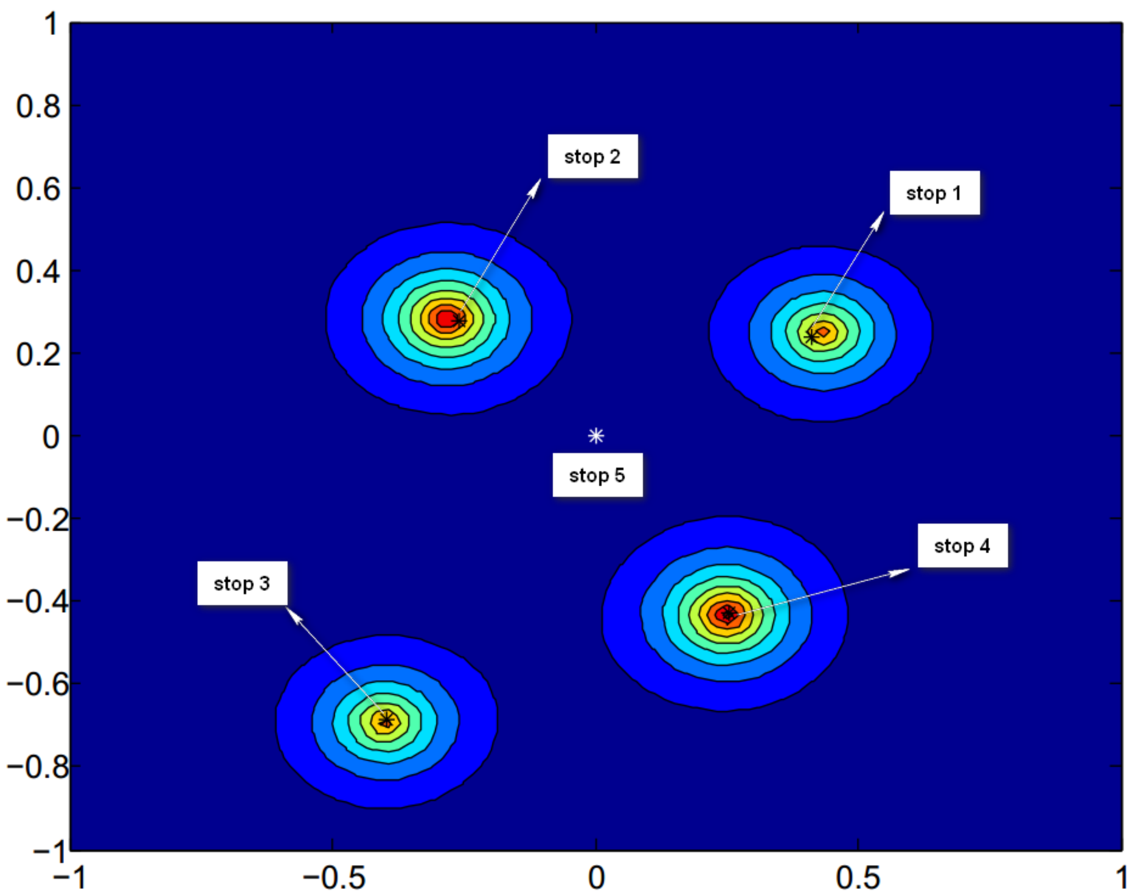

Figure-3: optimal solution with changed living locations

Furthermore, the number of living locations was also taken into consideration. To test the feasibility, one living location was added to the original system. And the Figure-4 shows a good result given by the algorithm.

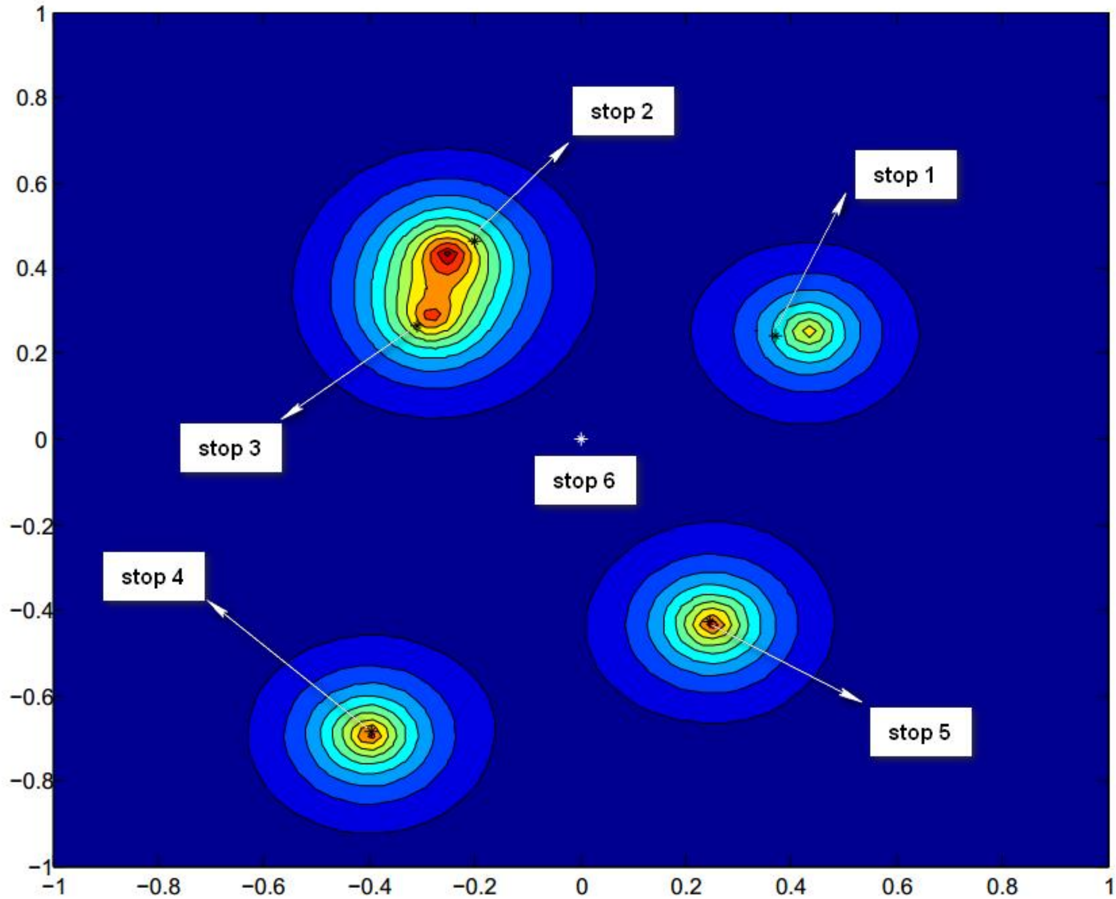

Figure-4: optimal result with five living locations

These two tests have demonstrated the robustness of the algorithm, which can deal with changed living locations and more living locations. This implies that the algorithm can be applied to more complex systems. For the future work, the bus route and waiting time in the bus stops should also be added to the system in order to better mimic the real word and give a more realistic solution to the bus system.

4. Acknowledgment

I really appreciate Max's help during this semester.

References

1. Huan Li, R.B., *Optimal Bus Stop Spacing for Minimizing Transit Operation Cost.* ***Traffic and Transportation Studies***, 2008.
2. Fitzpatrick K, P.D., Hall K, *FINDINGS FROM A SURVEY ON BUS STOP DESIGN.* ***Journal of Public Transportation***, 1997. **1**.
3. Furth P, a.R.A., *Optimal bus stop spacing through dynamic programming and geographic modeling.* ***Transportation Research Record 1731***, 2000.
4. A, S., *Model for determining optimum bus-stop spacing in urban areas.* ***Journal of Transportation Engineering***, 2001. **127**.
5. Ángel Ibeas, L.d.O., Borja Alonso, Olivia Sainz, *Optimizing bus stop spacing in urban areas.* ***Transportation Research Part E***, 2009.
6. Clark, C., *Urban Population Densities.* ***Journal of Royal Statistics Society, Series A***, 1951. **114**.