ME 555 Design Optimization Project Final Report

Bus Stop Selection and Routing Problem

by

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Abstract

To design the public transportation system, the design of the public bus transportation system is an important part. In order to design the public bus transportation system, we generally need to choose bus stops, determine the routes and the schedule for the system. This project tries to simultaneously solve three problems in the public bus design problems. As this is the first step of designing the bus route, to simplify this problem, this project neglects the different destination of the passenger, the variation of demand in different time of the day, and the travel time on the bus. This project applies the formulation method that is developed by Schittekat et al to simultaneously assign the passengers, select the bus stop and generate the route. This project first reformulated the formulation by adding a second objective that is the total distance the passengers have to walk to the bus stop and one more constraint to make sure the bus go back to the depot. Then, this project comes up with a specific problem with 6 potential sites and 40 passengers to assign. Thirdly, the project reforms the decision variables and the constraints to make it possible to put the problem into a Matlab solver. By doing all these things, this project gets result for this specific problem with both one bus and two buses.

Content

1. INTRODUCTION

This project is motivated by the interest of solving one of the urban planning problems: the transportation system design. This project specifically concentrates on the public bus design, focusing on the bus stop selection and routing problem. This project neglects the variation of demand in different time of the day. Rather, this project would give certain amount of demand in each area during analyzing the problem.

The Public Bus Transportation Problem (PBTP) is very similar to the School Bus Routing Problem (SBRP), which falls into the class of location-routing problems (LRPs). LRP includes determining the location of facilities (in SBRP, bus stops) serving more than one passenger and the optimal set of routes for a fleet of vehicles (Min et al., 1998) [1][2]. The vehicle routing problem (VRP) and the location-routing problem (LRP) are NP-hard problems. There is no known polynomial time algorithm that guarantees to solve the problem with an optimal solution for every specific problem. Heuristic Algorithm is preferred in most studies regarding the SBRP[1][2][3][4]. Schittekat et al. proposed an Integer Programming formulation for a SBRP, and try to solve it by using cutting plane algorithm to obtain lower bounds of the problem and metaheuristic optimization approach [4].

In the SBRP, most studies consider the sub-problems bus stop selection and routing separately and sequentially. The strategy is to first determine a set of bus stops for a school and assign students to these stops, and then generates the routes for the selected stops. However, since the students are assigned without taking into consideration their effect on generating routes, this approach tends to generate excessive routes [1]. However, the Integer Programming formulation that Schittekat et al. proposed for a SBRP considers the bus stop selection and generating the routes at the same time. This project applied this method and a few modification when formulated the problem.

In the SBRP, most studies only consider the single-school (one destination) problem [1][4]. The normally considered objective in this problem is to reduce the number of bus, the number of different routes, total distance accumulated among all routes, travel time on the bus, and walking distance. The constraints are usually the capacity limit of the bus, the travel time limit on the bus, the limit of walking distance, and every route must have at least one stop [1][3][4][5][6]. As mentioned above, the Public Bus Transportation Problem is very similar to the SBRP. In this project, the objectives and the constraints are the same as above. The PBTP should have multiple destinations, which falls in the multi-school problem category. However, in the beginning, this project would only consider one destination.

2. DESIGN PROBLEM

2.1 Problem Statement

This project intends to minimize the cost of the transportation system while provides the satisfactory service. To achieve this goal, while design the transportation system, this project aims at reducing the number of bus, the number of different routes, total distance accumulated among all routes, travel time on the bus, and walking distance. The priority of the objectives is given by the effect it has on the cost. The constraints of this project should be the capacity limit of the bus, the travel time limit on the bus, the limit of walking distance, and every route must have at least one stop, minimum number of people to create a route.

To formulate this problem, this project needs data from the people, destination, and the bus. The data from the people includes the location (address) of their homes, the destination, and the type of the student (general or handicapped). The data from the destination should contain the location of the destination. Normally the schedule of the destination is also wanted if the project plans to do scheduling for the bus. The data from the bus contain its capacity and location [1].

2.2 Mathematical Model

2.2.1 Notations

M_{ij}	Cost of traversing arc ij
MC_{ik}	Cost of walking to stop i for passenger k
K	Number of buses
C	Capacity of the buses
V	Set of all potential stops
E	Set of all arcs between stops
S	Set of all people
S_{li}	binary variable that indicates whether student l can walk to stop i or not
X_{ijk}	Number of times vehicle k traverses arcs
Y_{ik}	1 if vehicle k visits stop i, 0 otherwise
Z_{ik}	1 if people l is picked up by vehicle k at stop i, 0 otherwise

2.2.2 Objective Function

For the objective, this project aims at (1) reducing the number of bus, (2) the number of different routes, (3) total distance accumulated among all routes, (4) travel time on the bus, and (5) walking distance. However, due to the complexity of the model, this project only aims at minimize (3) the total distance accumulated among all routes and (5) walking distance. This project considers from the company’s perspective and give the total distance a higher weight, that is to view this as the first objective and under this premise, this problem try to find the least total walking distance for the passengers.

$$\min 400000 * (\sum_{i\in V}\sum_{j\in V} M_{ij}\sum_{k=1}^{K} X_{ijk}) + \sum_{i\in V}\sum_{k\in K} MC_{ik}\sum_{k=1}^{K} Z_{ijk}$$

2.2.3 Constraints

In summary, as mentioned above, the constraints of this problem should be (1) the capacity limit of the bus, (2) the travel time limit on the bus, (3) the limit of walking distance, (4) and every route must have at least one stop, (5) minimum number of people to create a route. These are also considered as the physical constraint. This project only formulates the (1) capacity limit of the bus and (2) limit of walking distance. As the (4) constraint is satisfied whenever the route is generated. For now, the project intends to carry everyone on the road, and therefore we currently don't consider (5). For the (2) constraint, it's rather difficult for this model to track the time each person spent on the bus now.

Physical Constraint:

$$\sum_{i\in V}\sum_{l\in S} Z_{ilk} \le C, \forall k = 1, \dots, K$$

This constraint is to ensure that the capacity of the bus is not exceeded.

$$\sum_{k=1}^{K} Z_{ilk} \le S_{li}, \forall l \in S, \forall i \in V$$

This constraint ensures the general walking distance limit for each person.

Practical Constraint:

$$\sum_{k=1}^{K} Y_{1k} = K$$

This constraint is to ensure the number of buses start from the depot is equal to the number of buses available

$$\sum_{j\in J} X_{ijk} = \sum_{j\in J} X_{jik} = Y_{ik}, \qquad \forall i \in V, k = 1, \dots, K$$

This constraint is to ensure that if stop I is visited by vehicle k, then one arc should be traversed by vehicle k entering stop i and leaving stop i.

$$\sum_{k=1}^{K} Y_{ik} \leq 1, \forall i \in V \setminus \{1\}$$

This constraint is to ensure that all stops are visited by different buses no more than once, except the stop corresponding to the depot.

$$Z_{ilk} \leq Y_{ik}, \forall i, l, k$$

This constraint ensures that person l is not picked up at stop I b vehicle k if vehicle k does not visit stop i.

$$\sum_{i \in V} \sum_{k=1}^{K} Z_{ilk} = 1, \forall l \in S$$

This constraint ensures that all people are picked up for one time.

$$\mathrm{X_{ijk}} + \mathrm{X_{jik}} \leq 1$$

This constraint ensures no circle in two stations.

$$Y_{ik} \in \{0,1\}, \forall i \in V, k = 1, \dots, K$$
$$X_{ijk} \in \{0,1\}, \forall i, j \in V | i \neq j$$
$$Z_{ilk} \in \{0,1\}, \forall i, j \in V | i \neq j$$

These three constraints ensure that all decision variables are binary.

2.2.4 Design Variables and Parameters

2.2.4.1 Design Variables

X_{ijk}	Number of times vehicle k traverses arcs
Y_{ik}	1 if vehicle k visits stop i, 0 otherwise
Z_{ik}	1 if people l is picked up by vehicle k at stop i, 0 otherwise

2.2.4.2 Parameters

M_{ij}	Cost of traversing arc ij (This can be formulated from the length of the arc ij)
MC_{ik}	Cost of walking to stop i for passenger k (This is calculated as the distance between passenger k and stop i)
K	Number of buses (This is a given number, start with 1)
C	Capacity of the buses (This is a given number, the same for all buses)
V	Set of all potential stops (This is several potential location sites, given randomly)
E	Set of all arcs between stops (This is possible routes in the graph)
S	Set of all people (This is the population that this project is going to cover)
S_{li}	binary variable that indicates whether student l can walk to stop i or not (This can be expressed by the relationship between the walking distance and the limit walking distance)

2.2.5 Summary Model

$$\min 400000 * (\sum_{i\in V}\sum_{j\in V} M_{ij}\sum_{k=1}^{K} X_{ijk}) + \sum_{i\in V}\sum_{k\in K} MC_{ik}\sum_{k=1}^{K} Z_{ijk}$$

s.t.

$$\sum_{k=1}^{K} Y_{1k} = K \quad (1)$$

$$\sum_{j\in J} X_{ijk} = \sum_{j\in J} X_{jik} = Y_{ik}, \forall i \in V, \forall k = 1, \dots, K \quad (2)$$

$$\sum_{k=1}^{K} Y_{ik} \leq 1, \forall i \in V \setminus \{1\} \quad (3)$$

$$\sum_{k=1}^{K} Z_{ilk} \leq S_{il}, \forall l \in S, \forall i \in V \quad (4)$$

$$\sum_{i\in V}\sum_{l\in S} Z_{ilk} \leq C, \forall k = 1, \dots, K \quad (5)$$

$$Z_{ilk} \leq Y_{ik}, \forall i, l, k \quad (6)$$

$$\sum_{i\in V}\sum_{k=1}^{K} Z_{ilk} = 1, \forall l \in S \quad (7)$$

$$X_{ijk} + X_{jik} \leq 1 \quad (8)$$

$$Y_{ik} \in \{0, 1\}, \forall i \in V, k = 1, \dots, K \quad (9)$$

$$X_{ijk} \in \{0, 1\}, \forall i, j \in V | i \neq j \quad (10)$$

$$Z_{ilk} \in \{0, 1\}, \forall i, j \in V | i \neq j \quad (11)$$

2.3 Model Analysis

2.3.1 Stage One Problem

As this problem is relatively hard to solve, the problem was simplified by only considering having one bus. This is the stage one problem.

The stage one problem can be expressed as:

$$\min 400000 * (\sum_{i\in V}\sum_{j\in V} M_{ij}\sum_{k=1}^{K} X_{ijk}) + \sum_{i\in V}\sum_{k\in K} MC_{ik}\sum_{k=1}^{K} Z_{ijk}$$

s.t.

$$Y_{1k} = 1 \quad (1)$$

$$X_{ijk} + X_{jik} \leq 1 \quad (2)$$

$$\sum_{j\in J} X_{ijk} = \sum_{j\in J} X_{jik} = Y_{ik}, \forall i \in V, k = 1, \dots, K \quad (3)$$

$$\sum_{k=1}^{K} Y_{ik} \leq 1, \forall i \in V \setminus \{1\} \quad (4)$$

$$\sum_{k=1}^{K} Z_{ilk} \leq S_{il}, \forall l \in S, \forall i \in V \quad (5)$$

$$Z_{ilk} \leq Y_{ik}, \forall i, l, k \quad (6)$$

$$\sum_{i\in V}\sum_{k=1}^{K} Z_{ilk} = 1, \forall l \in S \quad (7)$$

$$Y_{ik} \in \{0, 1\}, \forall i \in V, k = 1, \dots, K \quad (8)$$

$$X_{ijk} \in \{0, 1\}, \forall i, j \in V | i \neq j \quad (9)$$

$$Z_{ilk} \in \{0, 1\}, \forall i, j \in V | i \neq j \quad (10)$$

The decision variable Y_{ik} can be eliminated by the constraint $\sum_{j \in J} X_{ijk} = \sum_{j \in J} X_{jik} = Y_{ik}$, the constraint becomes as:

$$\sum_{j \in J} X_{ijk} = \sum_{j \in J} X_{jik}, \forall i \in V, k=1,\ldots, K \quad (1)$$

$$\sum_{j \in J} X_{ijk} \leq 1, \forall i \in V \setminus \{1\} \quad (2)$$

$$\sum_{k=1}^{K} Z_{ilk} \leq S_{il}, \forall l \in S, \forall i \in V \quad (3)$$

$$Z_{ilk} \leq \sum_{j \in J} X_{ijk}, \forall i, l, k \quad (4)$$

$$\sum_{i \in V} \sum_{k=1}^{K} Z_{ilk} = 1, \forall l \in S \quad (5)$$

$$\sum_{j \in J} X_{1jk} = 1 \quad (6)$$

$$X_{ijk} + X_{jik} \leq 1 \quad (7)$$

$$X_{ijk} \in \{0, 1\}, \forall i, j \in V | i \neq j \quad (8)$$

$$Z_{ilk} \in \{0, 1\}, \forall i, j \in V | i \neq j \quad (9)$$

2.3.1.1 Specific Problem Analysis

For a specific problem as the Figure 1 shows (the coordinates of the stops and passengers are in the appendix):

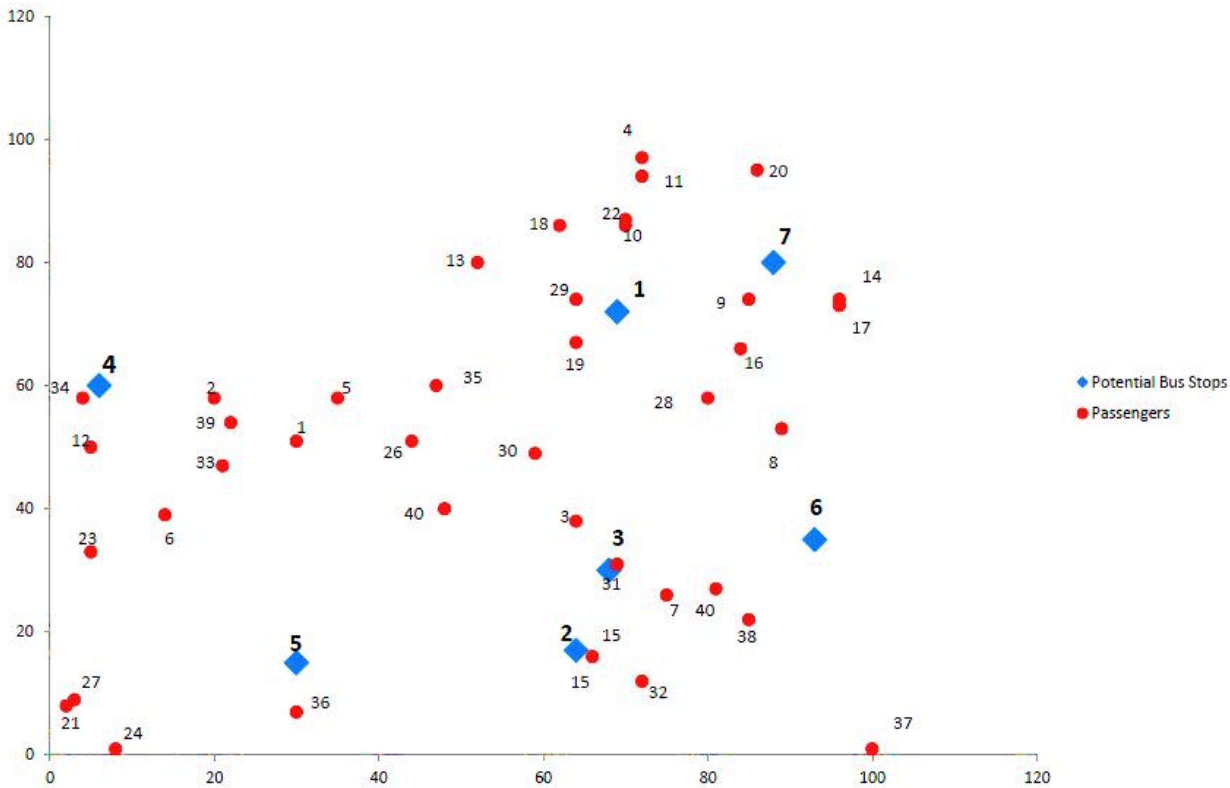

Figure 1

This project assumes the depot is stop 1. This problem has 40 passengers to assign to the bus stops and 6 potential bus stops to be selected. In order to put this problem into the bintprog solver, we need to reshape the decision variables Xijk and Zilk into one vector. Now we call this vector P. The first 49 elements of vector P are from Xijk, the next 240 elements of vector P are from Zilk. Now, in order to discuss this problem in a smaller size, we consider P1 as the vector for Xijk, and P2 as the vector for Zilk. P1 and P2 will be merged in the end. As the matrixes used in this process are huge matrixes, they are listed in the appendix.

For the first constraint:

$$\sum_{j\in J} X_{ijk} = \sum_{j\in J} X_{jik}, \forall i\in V, k=1,\ldots, K \qquad (1)$$

The expression becomes:

P1’*A=P1’*B

Where A is a 49*7 matrix, and B is a 49*7 matrix.

For the second constraint:

$$\sum_{j\in J} X_{ijk} \leq 1, \forall i \in V \setminus\{1\} \qquad (2)$$

it can simply be expressed as:

P1’*D $\leq \mathbf{1}$

Where D is just A without the first column (49*6 matrix).

For the third constraint:

$$\sum_{k=1}^{K} Z_{ilk} \leq S_{il}, \forall l \in S, \forall i \in V \qquad (3)$$

it can be simply expressed as:

P2 $\leq$ E

Where E is a vector that corresponds to Sli (240*1 vector).

For the fourth constraint:

$$Z_{ilk} \leq \sum_{j\in J} X_{ijk}, \forall i, l, k \qquad (4)$$

it can be expressed as:

$\mathrm{P2} \leq (\mathrm{P1}' * \mathrm{D} * \mathrm{F})'$

Where F is a 6*240 matrix.

For the fifth constraint:

$$\sum_{i\in V} \sum_{k=1}^{K} Z_{ilk} = 1, \forall l \in S \qquad (5)$$

it can be expressed as:

(P2'*G)’=$\mathbf{1}$

Where G is:

g is a 40*40 identical matrix, and G is a 240*40 matrix: G = $\begin{bmatrix} g \\ g \\ g \\ g \\ g \\ g \end{bmatrix}$

The sixth constraint:

$$\sum_{j\in J} X_{1jk} = 1 \qquad (6)$$

It can be expressed as N'*P1=1, where N is a vector the same as the first column of A.

The seventh constraint:

$$X_{ijk} + X_{jik} \leq 1 \qquad (7)$$

It can be expressed as:

O'*P1≤ **1**

Where O is a 49*21 matrix

The constraint eight and nine (the binary constraints) are automatically satisfied using bintprog solver.

Now we merge P1 and P2 into P. It's easy to see that P1'=P'× H^1, and P2'= P'× H^2. $H^1 = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$, where h_1 is a 49*49 identical matrix, h_2 is a 240*49 all-zero matrix. $H^2 = \begin{bmatrix} h_3 \\ h_4 \end{bmatrix}$, where h_3 is a 49*240 all-zero matrix, h_4 is a 240*240 identical matrix.

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The constraints can be expressed as:

$$\text{P}' \times H^1 \times (A - B) = \mathbf{0} \qquad (1)$$

$$\text{P}' \times H^1 \times D \leq \mathbf{1} \qquad (2)$$

$$\text{P}' \times H^2 - E \leq \mathbf{0} \qquad (3)$$

$$\text{P}'(H^2 - H^1 * \text{D} * \text{F}) \leq \mathbf{0} \qquad (4)$$

$$\text{P}' \times H^2 \times G = 1 \qquad (5)$$

$$\text{P}' \times H^1 \times \text{N} = 1 \qquad (8)$$

$$\text{P}' \times H^1 \times \text{O} \leq \mathbf{1} \qquad (9)$$

Let R= $\begin{bmatrix} (A - B) & \mathbf{0} \\ \mathbf{0} & \text{G} \end{bmatrix}'$,

I= $[H^2 - H^1 * \text{D} * \text{F}]'$

J= $\mathbf{0}$

K= $\begin{bmatrix} D & \mathbf{0} \\ \mathbf{0} & \text{h4} \end{bmatrix}'$,

$L=\begin{bmatrix}\mathbf{1}\\ \mathbf{E}'\end{bmatrix}$

The standard form is

Min M'×P

$$\begin{bmatrix}N & \mathbf{0}\\ & R\end{bmatrix}\times P=\begin{bmatrix}\mathbf{1}\\ \boldsymbol{a}\\ \mathbf{b}\end{bmatrix}$$

a is a 7*1 all-zero vector, **b** is a 40*1 all-one vector

$$\begin{bmatrix}O' & \mathbf{0}\\ & I\\ & K\end{bmatrix}\times P\le\begin{bmatrix}\boldsymbol{d}\\ J\\ L\end{bmatrix}$$

d is a 21*1 all-one vector

2.3.2 Stage Two Problem

The stage-two problem can be expressed as:

$$\min 400000*\left(\sum_{i\in V}\sum_{j\in V}M_{ij}\sum_{k=1}^{K}X_{ijk}\right)+\sum_{i\in V}\sum_{k\in K}MC_{ik}\sum_{k=1}^{K}Z_{ijk}$$

s.t.

$Y_{1k}=2$ (1)

$X_{ijk}+X_{jik}\le 1$ (2)

$\sum_{j\in J}X_{ijk}=\sum_{j\in J}X_{jik}=Y_{ik},\forall i\in V,k=1,\dots,K$ (3)

$\sum_{k=1}^{K}Y_{ik}\le 1,\forall i\in V\setminus\{1\}$ (4)

$\sum_{k=1}^{K}Z_{ilk}\le S_{il},\forall l\in S,\forall i\in V$ (5)

$Z_{ilk}\le Y_{ik},\forall i,l,k$ (6)

$\sum_{i\in V}\sum_{k=1}^{K}Z_{ilk}=1,\forall l\in S$ (7)

$Y_{ik}\in\{0,1\},\forall i\in V,k=1,\dots,K$ (8)

$X_{ijk}\in\{0,1\},\forall i,j\in V|i\ne j$ (9)

$Z_{ilk}\in\{0,1\},\forall i,j\in V|i\ne j$ (10)

$\sum_{i\in V}\sum_{l\in S}Z_{ilk}\le C,\forall k=1,\dots,K$ (11)

The decision variable Y_{ik} can be eliminated by the constraint$\sum_{j\in J}X_{ijk}=\sum_{j\in J}X_{jik}=Y_{ik}$, the constraint becomes as:

$\sum_{j\in J}X_{ijk}=\sum_{j\in J}X_{jik}$, ∀i∈V, k=1,..., K (1)

$\sum_{j\in J}X_{ijk}\le 1,\forall i\in V\setminus\{1\}$ (2)

$\sum_{k=1}^{K}Z_{ilk}\le S_{il},\forall l\in S,\forall i\in V$ (3)

$Z_{ilk} \leq \sum_{j \in J} X_{ijk}, \forall i, l, k$ (4)

$\sum_{i \in V} \sum_{k=1}^{K} Z_{ilk} = 1, \forall l \in S$ (5)

$\sum_{j \in J} X_{1jk} = 2$ (6)

$X_{ijk} + X_{jik} \leq 1$ (7)

$X_{ijk} \in \{0, 1\}, \forall i, j \in V | i \neq j$ (8)

$Z_{ilk} \in \{0, 1\}, \forall i, j \in V | i \neq j$ (9)

$\sum_{i \in V} \sum_{l \in S} Z_{ilk} \leq C, \forall k = 1, \dots, K$ (In this problem, $C = 25$) (10)

This problem is very similar to the stage-one problem, except for we have two buses, and we have twice the number of variable as in the stage-one problem. In order to put this problem into the bintprog solver, we need once again to reshape the decision variables Xijk and Zilk into one vector. Now we call this vector P. The first 98 elements of vector P are from X_{ijk}, the next 480 elements of vector P are from Z_{ilk}. Now, in order to discuss this problem in a smaller size, we consider P1 and P2 as the vectors for X_{ij1} and X_{ij2}, and P3 and P4 as the vectors for Z_{il1} and Z_{il2}. P1, P2, P3 and P4 will be merged in the end. In the stage-two problem, we use the same matrixes that has generated in stage-one problem.

For the first constraint:

$\sum_{j \in J} X_{ijk} = \sum_{j \in J} X_{jik}$, ∀i∈V, k=1,..., K (1)

The expression becomes:

P1'*A=P1'*B

P2'*A=P2'*B

Where A is a 49*7 matrix, and B is a 49*7 matrix.

For the second constraint:

$\sum_{j \in J} X_{ijk} \leq 1, \forall i \in V \setminus \{1\}$ (2)

it can simply be expressed as:

P1'*D+ P2'*D $\leq \mathbf{1}$

Where D is just A without the first column (49*6 matrix).

For the third constraint:

$\sum_{k=1}^{K} Z_{ilk} \leq S_{il}, \forall l \in S, \forall i \in V$ (3)

it can be simply expressed as:

P3 ≤ E

P4 ≤ E

Where E is a vector that corresponds to Sli (240*1 vector).

For the fourth constraint:

$$Z_{ilk} \leq \sum_{j \in J} X_{ijk}, \forall i, l, k \quad (4)$$

it can be expressed as:

$P3 \leq (P1' * D * F)'$

$P4 \leq (P2' * D * F)'$

Where F is a 6*240 matrix

For the fifth constraint:

$$\sum_{i \in V} \sum_{k=1}^{K} Z_{ilk} = 1, \forall l \in S \quad (5)$$

it can be expressed as:

(P3'*G)'=**1**

(P4'*G)'=**1**

Where G is:

g is a 40*40 identical matrix, and G is a 240*40 matrix: $G = \begin{bmatrix} g \\ g \\ g \\ g \\ g \\ g \end{bmatrix}$

The sixth constraint:

$$\sum_{j \in J} X_{1jk} = 2 \quad (6)$$

It can be expressed as N'*P1+ N'*P2=2, where N is a vector the same as the first column of A.

The seventh constraint:

$$X_{ijk} + X_{jik} \leq 1 \quad (7)$$

It can be expressed as:

O'*P1≤ **1**

O'*P2≤ **1**

Where O is a 49*21 matrix

The constraint eight and nine are automatically satisfied using bintprog solver.

The tenth constraint:

$$\sum_{i \in V} \sum_{l \in S} Z_{ilk} \leq C, \forall k = 1, \ldots, K \quad (10)$$

It can be expressed as:

P3'*Q≤ C

P4’*Q≤ C

Q is an240*1 all one vector.

Now we merge P1, P2, P3 and P4 into P. It’s easy to see that P1’=P’× H^1, P2’= P’× H^2, P3’= P’× H^3 and P4’= P’× H^4. $H^1 = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$, where h_1 is a 49*49 identical matrix, h_2 is a 529*49 all-zero matrix. $H^2 = \begin{bmatrix} h_3 \\ h_4 \\ h_5 \end{bmatrix}$, where h_3 is a 49*49 all zero matrix, h_4 is a 49*49 identical matrix, h_5 is a 480*49 all-zero matrix. $H^3 = \begin{bmatrix} h_3 \\ h_3 \\ h_6 \\ h_5 \end{bmatrix}$, where h_6 is a 240*240 identical matrix. $H^2 = \begin{bmatrix} h_3 \\ h_3 \\ h_5 \\ h_6 \end{bmatrix}$.

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The constraints can be expressed as:

$P' \times H^1 \times (A - B) = \mathbf{0}$ (1)

$P' \times H^2 \times (A - B) = \mathbf{0}$

$P' \times H^1 \times D + P' \times H^2 \times D \leq \mathbf{1}$ (2)

$P' \times H^3 - E \leq \mathbf{0}$ (3)

$P' \times H^4 - E \leq \mathbf{0}$

$P'(H^3 - H^1 * \mathrm{D} * \mathrm{F}) \leq \mathbf{0}$ (4)

$P'(H^4 - H^2 * \mathrm{D} * \mathrm{F}) \leq \mathbf{0}$

$P' \times H^3 \times G + P' \times H^4 \times G = 1$ (5)

$P' \times H^1 \times \mathrm{N} + P' \times H^2 \times \mathrm{N} = 2$ (6)

$P' \times H^1 \times \mathrm{O} \leq \mathbf{1}$ (7)

$P' \times H^2 \times \mathrm{O} \leq \mathbf{1}$

$P' \times H^3 \times Q \leq \mathrm{C}$ (10)

$P' \times H^4 \times Q \leq \mathrm{C}$

I=$[H^3 - H^1 * \mathrm{D} * \mathrm{F}]'$

J=$[H^4 - H^2 * \mathrm{D} * \mathrm{F}]'$

The standard form is

Min M’×P

$$\begin{bmatrix} N & N & \mathbf{0} & \mathbf{0} \\ (A-B)' & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (A-B)' & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathrm{G} & \mathrm{G} \end{bmatrix} \times \mathrm{P} = \begin{bmatrix} \mathbf{2} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix}$$

$$\begin{bmatrix} D & D & \mathbf{0} & \mathbf{0} \\ & H^{3'} & & \\ & H^{4'} & & \\ \mathbf{0} & \mathbf{0} & \mathrm{Q}' & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathrm{Q}' \\ \mathbf{0} & \mathbf{0} & \mathrm{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathrm{J} \\ O & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & O & \mathbf{0} & \mathbf{0} \end{bmatrix} \times \mathrm{P} \leq \begin{bmatrix} \mathbf{1} \\ E \\ \mathrm{E} \\ C \\ \mathrm{C} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix}$$

3. DESIGN OPTIMIZATION

3.1 Stage-one Problem Result

3.1.1 Bus Selection and Route Selection

The result for the bus selection and the route selection is shown as follows:

	1	2	3	4	5	6	7
1	0	0	0	1	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	1
4	0	0	0	0	1	0	0
5	0	0	1	0	0	0	0
6	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0

This shows the route should be 1-4-5-3-7-1

3.1.2 Assignment of Passengers

The assignment of the passengers to bus stops is shown in the following table.

	2	3	4	5	6	7
1	0	0	1	0	0	0
2	0	0	1	0	0	0
3	0	1	0	0	0	0
4	0	0	0	0	0	1
5	0	0	1	0	0	0
6	0	0	1	0	0	0

7	0	1	0	0	0	0
8	0	0	0	0	0	1
9	0	0	0	0	0	1
10	0	0	0	0	0	1
11	0	0	0	0	0	1
12	0	0	1	0	0	0
13	0	0	0	0	0	1
14	0	0	0	0	0	1
15	0	1	0	0	0	0
16	0	0	0	0	0	1
17	0	0	0	0	0	1
18	0	0	0	0	0	1
19	0	0	0	0	0	1
20	0	0	0	0	0	1
21	0	0	0	1	0	0
22	0	0	0	0	0	1
23	0	0	1	0	0	0
24	0	0	0	1	0	0
25	0	1	0	0	0	0
26	0	1	0	0	0	0
27	0	0	0	1	0	0
28	0	0	0	0	0	1
29	0	0	0	0	0	1
30	0	1	0	0	0	0
31	0	1	0	0	0	0
32	0	1	0	0	0	0
33	0	0	1	0	0	0
34	0	0	1	0	0	0
35	0	1	0	0	0	0
36	0	0	0	1	0	0
37	0	1	0	0	0	0
38	0	1	0	0	0	0
39	0	0	1	0	0	0
40	0	1	0	0	0	0

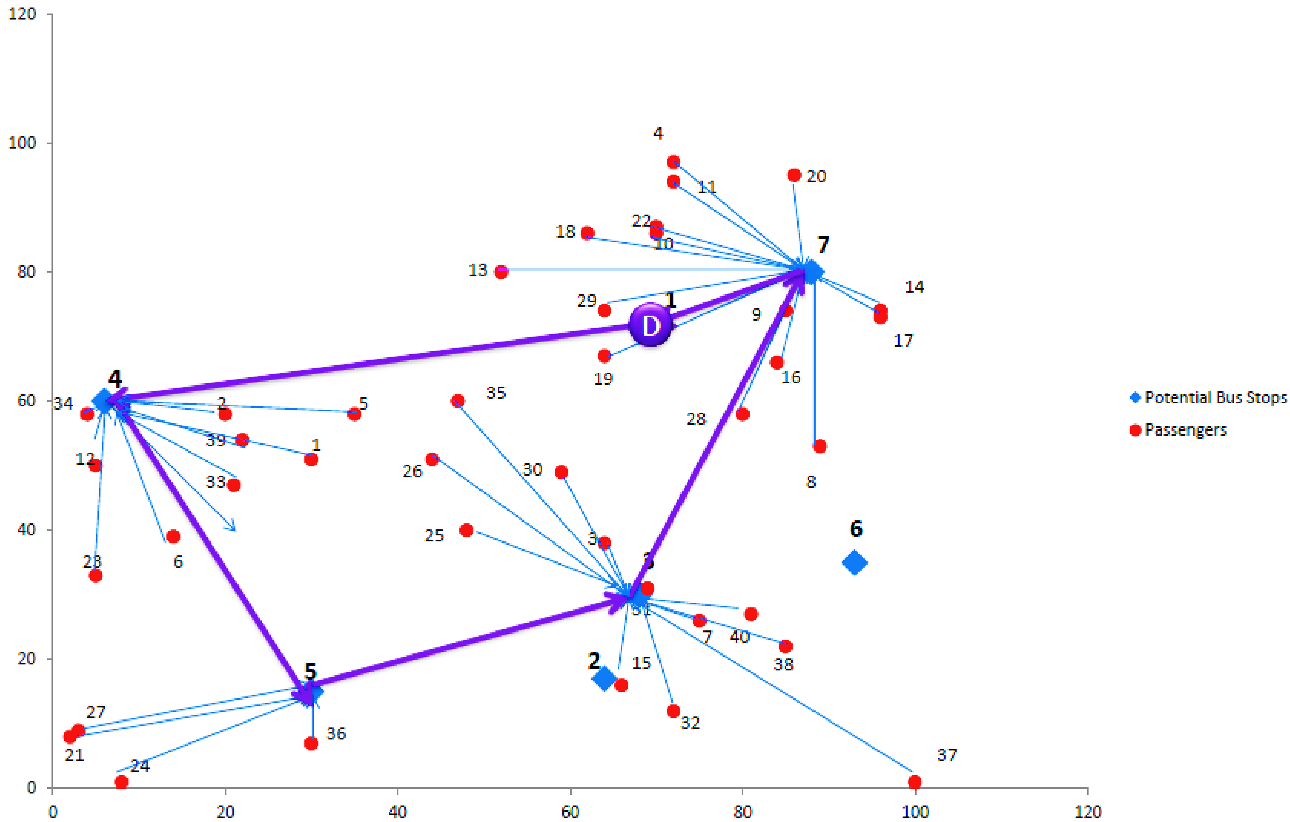

Figure 2

3.2 Stage-two Problem Result

3.2.1 Bus Selection and Route Selection

The result for the bus selection and the route selection is shown as follows:

k=1	1	2	3	4	5	6	7
1	0	0	0	0	1	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0
5	0	0	0	1	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0

This shows the route should be 1-5-4-1

k=2	1	2	3	4	5	6	7
1	0	0	0	0	0	1	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1
7	1	0	0	0	0	0	0

This shows the route should be 1-6-7-1

3.2.2 Assignment of Passengers

The assignment of the passengers to bus stops is shown in the following tables.

k=1	2	3	4	5	6	7
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	1	0
4	0	0	0	0	0	1
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	1	0
8	0	0	0	0	1	0
9	0	0	0	0	0	1
10	0	0	0	0	0	1
11	0	0	0	0	0	1
12	0	0	0	0	0	0
13	0	0	0	0	0	1
14	0	0	0	0	0	1
15	0	0	0	0	1	0
16	0	0	0	0	0	1
17	0	0	0	0	0	1
18	0	0	0	0	0	1
19	0	0	0	0	0	1
20	0	0	0	0	0	1
21	0	0	0	0	0	0
22	0	0	0	0	0	1
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0	0	0	0	0	0
27	0	0	0	0	0	0

28	0	0	0	0	0	1
29	0	0	0	0	0	1
30	0	0	0	0	1	0
31	0	0	0	0	1	0
32	0	0	0	0	1	0
33	0	0	0	0	0	0
34	0	0	0	0	0	0
35	0	0	0	0	0	0
36	0	0	0	0	0	0
37	0	0	0	0	1	0
38	0	0	0	0	1	0
39	0	0	0	0	0	0
40	0	0	0	0	1	0

k=2	**2**	**3**	**4**	**5**	**6**	**7**
1	0	0	1	0	0	0
2	0	0	1	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	1	0	0	0
6	0	0	1	0	0	0
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	1	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	1	0	0
22	0	0	0	0	0	0
23	0	0	1	0	0	0
24	0	0	0	1	0	0
25	0	0	0	1	0	0
26	0	0	0	1	0	0
27	0	0	0	1	0	0

28	0	0	0	0	0	0
29	0	0	0	0	0	0
30	0	0	0	0	0	0
31	0	0	0	0	0	0
32	0	0	0	0	0	0
33	0	0	1	0	0	0
34	0	0	1	0	0	0
35	0	0	1	0	0	0
36	0	0	0	1	0	0
37	0	0	0	0	0	0
38	0	0	0	0	0	0
39	0	0	1	0	0	0
40	0	0	0	0	0	0

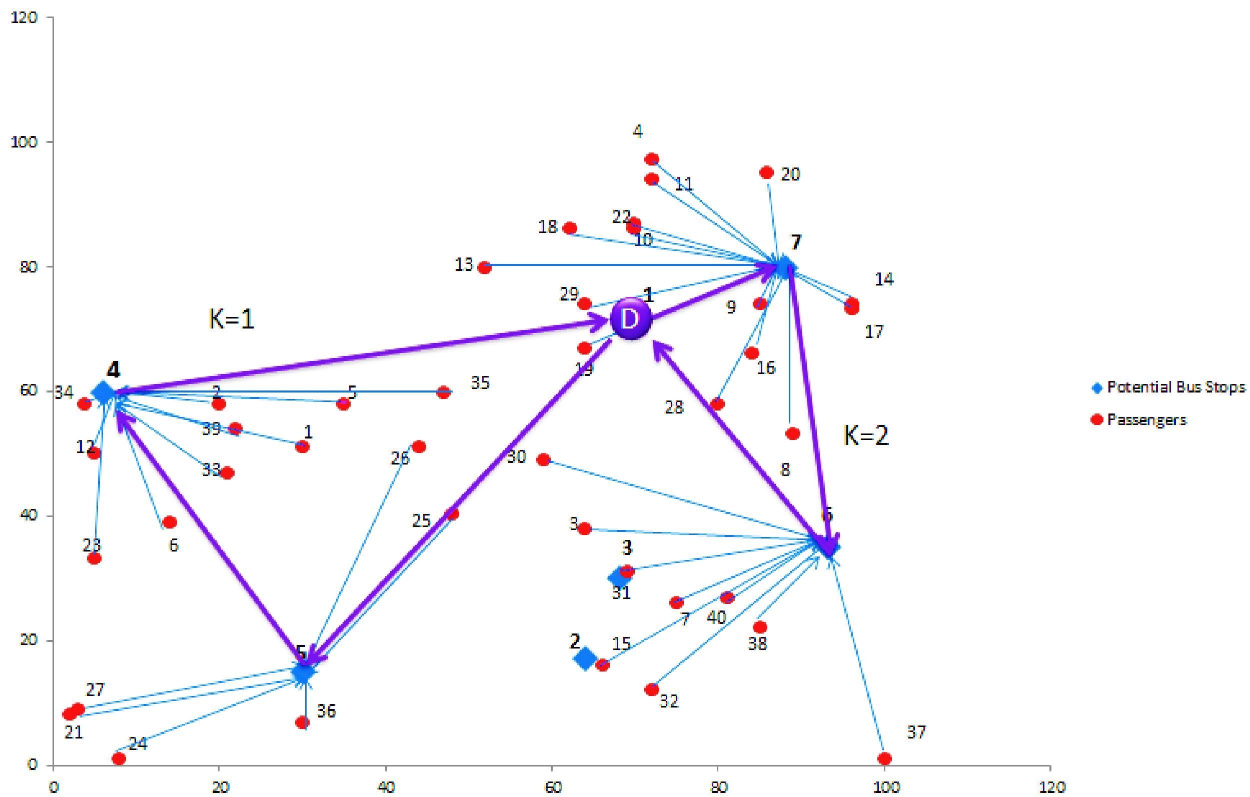

Figure 3

4. DISCUSSION

4.1 Stage-One Problem

For the stage-one problem, four stops have been selected apart from the depot. The four stops are 3, 4, 5 and 7. 12 passengers are assigned to bus stop 3, 9 passengers are assigned by bus stop 4, 4 passengers are assigned to bus stop 5 and 15 passengers are assigned to bus stop 7. The result can also be been seen from Figure 2, Pg. 16.

4.2 Stage-Two Problem

For the stage-tw0 problem, two stops have been selected apart from the depot for each of the route. For route one (k=1), the two stops are 6 and 7. 10 passengers are assigned to bus stop 6 and 14 passengers are assigned to bus stop 7. For route one (k=2), the two stops are 4 and 5. 10 passengers are assigned to bus stop 4 and 6 passengers are assigned to bus stop 5. The result can also be been seen from Figure 3, Pg. 19.

4.3 Future Directions

Currently, the project has only solved the most basic and basic problems in this public bus design problem. Future steps can be taken to firth solve the problem with time window (travel time limit) with considering the different destinations of the passengers. An even further step should be taken to solve the scheduling problem of the buses by considering the variation of the demand in the day. Also, for this project, as it only solves one specific problem. Further work can be done to modify the Matlab Code so that it could solve different problems with only the basic information that is the coordinates of the stops and passengers, the bus capacity.

Acknowledgements

The mathematical model of this project largely depends on the model in the study of Schittekat et al.: A Mathematical Formulation for a school bus routing problem [4].

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Appendix

1. Coordinates of the potential bus stops and the passengers

Bus Stop	x	y
1	69	72
2	64	17
3	68	30
4	6	60
5	30	15
6	93	35
7	88	80

Passenger	x	y
1	30	51
2	20	58
3	64	38
4	72	97
5	35	58
6	14	39
7	75	26
8	89	53
9	85	74
10	70	86
11	72	94
12	5	50
13	52	80
14	96	74
15	66	16
16	84	66

17	96	73
18	62	86
19	64	67
20	86	ss
21	2	8
22	70	87
23	5	33
24	8	1
25	48	40
26	44	51
27	3	9
28	80	58
29	64	74
30	59	49
31	69	31
32	72	12
33	21	47
34	4	58
35	47	60
36	30	7
37	100	1
38	85	22
39	22	54
40	81	27

2 Matlab Code

The result for routing and stop selection is in VS matrix.

The result for routing and stop selection is in VC matrix.

2.1 For stage one problem:

```
clear all;
clc;
tic

stopm=[69   72
64  17
68  30
6   60
30  15
93  35
88  80];
```

```
demandm=[30 51
20  58
64  38
72  97
35  58
14  39
75  26
89  53
85  74
70  86
72  94
5   50
52  80
96  74
66  16
84  66
96  73
62  86
64  67
86  95
2   8
70  87
5   33
8   1
48  40
44  51
3   9
80  58
64  74
59  49
69  31
72  12
21  47
4   58
47  60
30  7
100 1
85  22
22  54
81  27];

for i=1:7
    for j=1:7
    c(i,j)=((stopm(i,1)-stopm(j,1))^2+(stopm(i,2)-stopm(j,2))^2)^0.5;
    if c(i,j)==0
        c(i,j)=10000;
    else
        c(i,j)=c(i,j);
    end
```

```
    end
end



for i=1:40
    for j=2:7 % station 1 is the start point. If someone is able to walk to the start point, 
this person would not be on this map
    e(i,j)=((demandm(i,1)-stopm(j,1))^2+(demandm(i,2)-stopm(j,2))^2)^0.5;
    if e(i,j)<50
        ex(i,j)=e(i,j);
        e(i,j)=1;
    else
        ex(i,j)=e(i,j);
        e(i,j)=0;
    end

    end
end

c1=[c(1,:),c(2,:),c(3,:),c(4,:),c(5,:),c(6,:),c(7,:)]';
c2=[ex(:,2);ex(:,3);ex(:,4);ex(:,5);ex(:,6);ex(:,7)];
m=[4000000*c1;c2];
h=zeros(49, 240);
h1=eye(240, 240);
H1=[h; h1];
h2=eye(49, 49);
h3=zeros(240, 49);
H2=[h2; h3];
D=[zeros(1,6);eye(6);zeros(1,6);eye(6);zeros(1,6);eye(6);zeros(1,6);eye(6);zeros(1,6);eye(6);
zeros(1,6);eye(6);zeros(1,6);eye(6)];
F=[ones(1,40),zeros(1,200);zeros(1,40),ones(1,40),zeros(1,160);zeros(1,80),ones(1,40),zeros(1
,120);zeros(1,120),ones(1,40),zeros(1,80);zeros(1,160),ones(1,40),zeros(1,40);zeros(1,200),on
es(1,40)];



flag=H1-H2*D*F;
I=flag';
J=zeros(240, 1);
A=[eye(7);eye(7);eye(7);eye(7);eye(7);eye(7);eye(7)];
B=[ones(1,7),zeros(1,42);zeros(1,7),ones(1,7),zeros(1,35);zeros(1,14),ones(1,7),zeros(1,28);z
eros(1,21),ones(1,7),zeros(1,21);zeros(1,28),ones(1,7),zeros(1,14);zeros(1,35),ones(1,7),zero
s(1,7);zeros(1,42),ones(1,7)]';
g=eye(40,40);
G=[g; g; g; g; g; g];

g1=zeros(49, 40);
g2=zeros(240, 7);
```

```
g3=zeros(7, 1);
g4=ones(40, 1);

g12=[1  0   0   0   0   0   0   1   0   0   0   0   0   0   1   0   0   0   0   0   0   1   0
0   0   0   0   0   1   0   0   0   0   0   0   1   0   0   0   0   0   0   1   0   0   0   0
0   0];
g13=zeros(1,240);
R=[g12 g13];
S=[A-B g1; g2 G]';
Aeq=[R;S];
Beq=[1;g3; g4];
%lb=zeros(289, 1);
%ub=ones(289, 1);

g5=zeros(49,240);
g6=zeros(240,6);
g7=eye(240,240);
K=[D g5; g6 g7]';
g8=ones(6, 1);
E=[e(:,2);e(:,3);e(:,4);e(:,5);e(:,6);e(:,7)];
L=[g8; E];
g9=[0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0
0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
```

```
0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   1
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   1
0];
g10=zeros(21, 240);
g11=ones(21,1);
O=[g9 g10];
Ain=[I;K;O];
Bin=[J;L;g11];
[x, fval]=bintprog(m,Ain,Bin,Aeq,Beq);

toc

for i=1:7
    for j=1:7
        VS(j,i)=x((i-1)*7+j);
    end
```

```
end

for i=1:6
    for j=1:40
        VC(j,i)=x((i-1)*40+j+49);
    end
end
```

2.2 For stage-two problem

```
clear all;
clc;
tic
stopm=[69   72
64  17
68  30
6   60
30  15
93  35
88  80];
demandm=[30 51
20  58
64  38
72  97
35  58
14  39
75  26
89  53
85  74
70  86
72  94
5   50
52  80
96  74
66  16
84  66
96  73
62  86
64  67
86  95
2   8
70  87
5   33
8   1
48  40
44  51
3   9
80  58
```

```
64  74
59  49
69  31
72  12
21  47
4   58
47  60
30  7
100 1
85  22
22  54
81  27];

for i=1:7
    for j=1:7
    c(i,j)=((stopm(i,1)-stopm(j,1))^2+(stopm(i,2)-stopm(j,2))^2)^0.5;
    if c(i,j)==0
        c(i,j)=10000;
    else
        c(i,j)=c(i,j);
    end

    end
end



for i=1:40
    for j=2:7 % station 1 is the start point. If someone is able to walk to the start point,
this person would not be on this map
    e(i,j)=((demandm(i,1)-stopm(j,1))^2+(demandm(i,2)-stopm(j,2))^2)^0.5;
    if e(i,j)<50
        ex(i,j)=e(i,j);
        e(i,j)=1;
    else
        ex(i,j)=e(i,j);
        e(i,j)=0;
    end

    end
end

c1=[c(1,:),c(2,:),c(3,:),c(4,:),c(5,:),c(6,:),c(7,:)]';
c2=[ex(:,2);ex(:,3);ex(:,4);ex(:,5);ex(:,6);ex(:,7)];
m=[4000000*c1;4000000*c1;c2;c2];



h1=eye(49, 49);
```

```
h2=zeros(529, 49);
H1=[h1; h2];
h3=zeros(49,49);
h4=eye(49, 49);
h5=zeros(480,49);
H2=[h3;h4;h5];
h6=zeros(98, 240);
h7=eye(240, 240);
h8=zeros(240,240);
H3=[h6; h7;h8];
h9=zeros(338,240);
h10=eye(240,240);
H4=[h9;h10];
D=[zeros(1,6);eye(6);zeros(1,6);eye(6);zeros(1,6);eye(6);zeros(1,6);eye(6);zeros(1,6);eye(6);
zeros(1,6);eye(6);zeros(1,6);eye(6)];
F=[ones(1,40),zeros(1,200);zeros(1,40),ones(1,40),zeros(1,160);zeros(1,80),ones(1,40),zeros(1
,120);zeros(1,120),ones(1,40),zeros(1,80);zeros(1,160),ones(1,40),zeros(1,40);zeros(1,200),on
es(1,40)];

flag1=H3-H1*D*F;
I1=flag1';
flag2=H4-H2*D*F;
I2=flag2';
J=zeros(240, 1);
A=[eye(7);eye(7);eye(7);eye(7);eye(7);eye(7);eye(7)];
B=[ones(1,7),zeros(1,42);zeros(1,7),ones(1,7),zeros(1,35);zeros(1,14),ones(1,7),zeros(1,28);z
eros(1,21),ones(1,7),zeros(1,21);zeros(1,28),ones(1,7),zeros(1,14);zeros(1,35),ones(1,7),zero
s(1,7);zeros(1,42),ones(1,7)]';
g=eye(40,40);
G=[g; g; g; g; g; g];

%% Equality Constraint
g1=zeros(49, 7);
g2=zeros(49, 40);
g21=zeros(7,240);
g3=zeros(14, 1);
g4=ones(40, 1);

N=[1    0    0    0    0    0    0    1    0    0    0    0    0    0    1    0    0    0    0    0    0    1    0
0    0    0    0    0    1    0    0    0    0    0    0    1    0    0    0    0    0    0    1    0    0    0    0
0    0];
g13=zeros(1,240);
R=[N N g13 g13];
S=[(A-B)' g1' g21 g21;g1' (A-B)' g21 g21];
Aeq=[R;S;g2' g2' G' G'];
Beq=[2;g3; g4];

%% Inequality Constraint
g5=zeros(49,240);
```

```
g6=zeros(240,6);
g7=eye(240,240);
K=[D g5];
E=[e(:,2);e(:,3);e(:,4);e(:,5);e(:,6);e(:,7)];
g9=[0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0
0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   1
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0
0
```

```
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   1   0   0   0   0   0   0   0   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0
0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   1
0];
g10=zeros(21, 529);
g11=ones(21,1);
O=[g9 g10];
Ain=[D' D' g6' g6'; H3'; H4';zeros(1,98) ones(1,240) zeros(1,240);zeros(1,338) ones(1,
240);I1;I2;g9 zeros(21,529); zeros(21,49) g9 zeros(21,480)];
Bin=[ones(6,1);E;E;25;25;g13';g13';g11;g11];

%% Optimization

[x, fval]=bintprog(m,Ain,Bin,Aeq,Beq);
toc
for i=1:7
    for j=1:7
        VS1(j,i)=x((i-1)*7+j);
    end
end

for i=1:7
    for j=1:7
        VS2(j,i)=x((i-1)*7+j+49);
    end
end

for i=1:6
    for j=1:40
        VC1(j,i)=x((i-1)*40+j+98);
    end
end
for i=1:6
    for j=1:40
        VC2(j,i)=x((i-1)*40+j+338);
```

```
    end
end
```

3 Huge Matrixes

A:

1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
1	0	0	0	0	0	0

0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1

B:

1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	1	0	0	0	0	0
0	1	0	0	0	0	0
0	1	0	0	0	0	0
0	1	0	0	0	0	0
0	1	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	1	0	0	0	0
0	0	1	0	0	0	0
0	0	1	0	0	0	0
0	0	1	0	0	0	0
0	0	1	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0

0	0	0	0	1	0	0
0	0	0	0	1	0	0
0	0	0	0	1	0	0
0	0	0	0	1	0	0
0	0	0	0	1	0	0
0	0	0	0	1	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	1	0
0	0	0	0	0	1	0
0	0	0	0	0	1	0
0	0	0	0	0	1	0
0	0	0	0	0	1	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
0	0	0	0	0	0	1
0	0	0	0	0	0	1
0	0	0	0	0	0	1
0	0	0	0	0	0	1
0	0	0	0	0	0	1
0	0	0	0	0	0	1

D:

0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0

0	0	0	0	0	1
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

E:

[1	1	0	1	0	1	1	1	1	0	0	1
0	0	1	1	1	0	1	1	1	1	1	0
0	0	0	1	1	1	1	1	1	0	0	0
0	0	1	0	0	0	1	0	0	0	0	1
1	0	0	0	1	0	0	0	0	0	0	1
0	1	1	1	0	1	0	1	1	0	0	1
1	1	1	1	1	1	0	0	1	1	0	0
0	0	1	1	1	1	0	0	0	0	0	1
1	0	1	0	1	1	1	0	1	0	1	1
0	1	0	0	0	1	1	1	0	1	1	0
0	0	0	0	0	0	1	1	0	1	0	1

1	0	1	0	0	0	1	1	0	1	1	1
1	0	1	1	1	1	0	0	0	1	1	1
0	0	0	0	0	1	1	1	1	0	0	1
1	1	0	0	1	1	0	1	0	0	0	0
1	1	1	1	1	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	0	1	1	1
0	1	0	1	0	1	1	0	1	0	0	0
1	1	1	1	0	1	0	1	1	1	1	0
1	1	0	1	1	0	0	0	1	0	0	0]’

F:

1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
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1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
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1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0

1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
1	0	0	0	0	0
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0	1	0	0	0	0
0	1	0	0	0	0

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0	1	0	0	0	0
0	1	0	0	0	0
0	1	0	0	0	0
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0	0	1	0	0	0
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0	0	1	0	0	0
0	0	1	0	0	0
0	0	1	0	0	0
0	0	1	0	0	0

0	0	1	0	0	0
0	0	1	0	0	0
0	0	1	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
0	0	0	1	0	0
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