ME 555

Design Optimize
of the wind power generator

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ABSTRACT

This project is going to optimize the kite machine by the wind force in the stratosphere to generate the maximum power. The net power generated will be sent to the air compressor that designed by another classmate via controller. The overall maximum power is affected by the friction it produces during operation. As the velocity increase, the output work will increase. However, the friction work will increase more significantly. Therefore, the velocity (displacement) needed to be optimize. Whether to rewind less ropes to save more energy or rewind more ropes to get more work for the next cycle is also a trade-off. Thus, there are some variable values that should be determined to get the best solution.

Contents

1. INTRODUCTION

Due to the lack of source on earth, people become more and more emphasize on energy which is renewable in recent years .Wind power is one of them.

There are lots of windmill today is used, but there are some drawbacks of the windmill: they cannot rise above 500 feet, and there are no constant wind flows there. The flabellum is heavy and expensive and is not cost effective enough. The high of the machine is fixed. Therefore, it's hard to maintain it because engineers have to climb on it. The kite machine could solve these problems because the high of the flabellum is light and easy to control.

In this project, the power of constant wind in stratosphere is used to be the main forces which push the kite up. During this the process, the rope wrapped on the flywheel will be taken by the kite to the sky. Thus, the flywheel will keep rotating, and the potential energy will transform to the kinetic energy which storage into the flywheel but also loss energy from mechanic and air friction. When the rope is running out of, the maximum high is reach, and the flywheel will release its energy to rewind the rope. During this period, I will reduce the kite's area and thus, reduce the potential energy needed to rewind the kite. The net energy minus the loss from air and mechanic will transit to the controller (power generator). The kite machine is assumed to be a system, therefore, the work within the system is conservation. That means the total work within the system is equal to zero.

The Design objection is to get the maximum power transiting to controller. To achieve this goal, the maximun velosity , the kite area should be shrinked and the length the rope rewind each time should be optimized.

2. DESIGN PROBLEM

2.1 Problem Statement

Due to the friction force. The system's accerlation is variable. The friction will increase when the velocity increase. The displacement is proportional to the velocity, thus, if the velocity increase, the work will increase. However, the friction work will also increase. In addition, the displacement is the function of time, if the displacement increase, that means the time increase. Nonetheless, our objective is the get the maximun power. Therefore, If the time increase, the power will decrease. Hence, we have to optimize the best length(time) to get the maxinum output.

Though the displacement need to be optimized, the maximum displacement is limit due to the budget. Suppose we could have at most 8400m long rope, and we release the kite from 8000m. Hence, the maximum length of the rope is 400m. Due to the limit length, the system is designed to round more than one round to get more power. However we limit it to three to see the optimize solutions for the limit cycle.

We are not only optimizing the kite rise displacement, but also how much the kite rewind. From the energy aspect, if the rope rewind less, the system could save more potential energy and thus generate more output power. On the other side, we will get less potential energy from next round. In this case, the times of the cycle is limited. Therefore, the length is the track-off that sould be considered.

2.2 Mathematical Models

2.2.1 Notation

symbol	introduction	unit
F_{rise}	the force from kite when it rise	(N)
F_{back}	the force from kite when it goes back	(N)
F_{output}	the force given to controller	(N)
Ffriction	the force from bearing friction	(N)
A_{rise}	The kite’s area when it rise	(m^2)
A_{back}	The kite’s area when it goes back	(m^2)
$Area_{min}$	Minimum kite area that could keep kites in shape and avoid ropes twisting together	(m^2)
S_n	the length the kite rise at the n round	(m)
S_{n_2}	The length the kite goes back at the n cycle	(m)
V_{rise}	the instant velocity when the kite rise	(m/s)
V_{back}	the instant velocity when the kite goes back	(m/s)
V_{max}	The maximum velocity	(m/s)
tn	the time the kite rise at the n round	(s)
t_{n2}	the time kite goes down at the n round	(s)
m	the mass of the flywheel	(kg)
W_r	the work the kite rise	(J)
W_b	The power the kite goes back	(J)
P_{wind}	The pressure of the wind	(N/m^2)
C	The coefficient of the friction force	

2.2.2 Objective function

The objective is to get the maximum total output power.

h1: $F(t_1,t_2) = \sum_1^3 \frac{\int_0^{sn1}(\text{Foutput}*\text{dsn}) + \int_0^{sn2}(\text{Foutput}*\text{dsn\_2})}{tn1+tn2}$

2.2.3 Subject function

The force due to friction from bearing is proportional to the velocity of the flywheel:

$F_{friction} = C*V$

*C is the coefficient

The force by wind when the kite goes down:

$F_{back} = Pw_{ind} * A_{back}$

* A_{rise} is the shrink area of kite when it rise.

The velocity when the kite rises

$V_{rise} = ((F_{rise} - F_{output})/C) - ((F_{rise} - F_{output})/C)*\exp(-C*t/m)$

*Velocity is the function of the time

The velocity when the kite goes back

$V_{back} = Vrise_{Max} - (((F_{back} + F_{output})/C) - ((F_{back} + F_{output})/C)*\exp(-C*t2/m))$

The displacement when the kite rises:

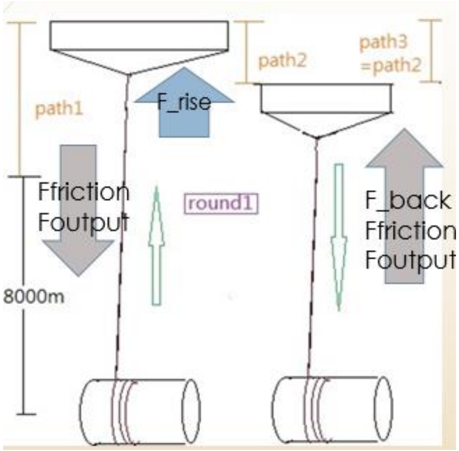
$Sn - \int_0^{t1} v1(t)dt = 0$

The displacement when the kite goes back:

$Sn\_2 - \int_0^{t2} (v2(t))dt = 0$

Assume the kite machine is a system, therefore, the total work within the system is equal to zero.

=>Sum of the positive work - sum of the negative work = Wpositive - Wnegative = 0



For the first round

$W_{positive} = F\_{_{rise}} * path1$ (kite up)

$W_{negative} = \int F_{friction} * dpath1 + F_{output} * path1$ (rise)

$+F\_{_{back}} * path2 + \int F_{friction} * dpath2 + F_{output} * path2$ (back)

Total output work:

$Foutput * S_1 + Foutput * dS_{1\_2} = F_{rise} * S_1 - \int F_{friction} * d_{S1} - F_{back} * S_{1\_2} - \int F_{friction} * dS_{1\_2}$

(*Set $Path1=S_1$, $path2 = S_{1\_2}$)

For the whole round

Total output power (Total output work divided by total time):

$F_{output} * S_n + F_{output} * dS_{n\_2} / (t_{n1} + t_{n2}) =$

$[F_{rise} * S_n - \int F_{friction} * dS_1 - F_{back} * S_{n\_2} - \int F_{friction} * dS_{n\_2}] / (t_{n1} + t_{n2})$

(*n from 1 to 3)

2.2.4 Phycisal Constraints

physical constraints:

$-[F_{back} * Sn - \int F_{friction} * dS1 - F_{back} * Sn\_2 - \int F_{friction} * dSn\_2] / (tn1 + tn\_2) \leqq 0$

The total output power should greater than or equal to zero

$-t_n \leqq 0$

The total time could not be negative.

$-A_{back} \leqq 0$

The reduced kite area could not be negative.

2.2.5 Practical constrains*:*

$F_{back} - F_{rise} \leqq 0$

The force when the kite goes up should greater than the force when the kite goes back by wind or the system could not generate output power.

$S_{n\_2} - S_{1\_1} \leqq 0$

The displacement for each iteration should not greater than the path1, or the kite will go below the original release point and below stratosphere.

$S_{1\_1} - 400 \leqq 0$

Assume the maximum rope we could get is 8000+400(m). That means the S1 should shorter than or equal to 400m.

$Pw_{ind} * Area_{min} - F_b \leqq 0$

Assume there is a minimum kite area that could keep kites in shape and avoid ropes twisting together.

$-Sn \leqq 0$

All the displacement should not be negative.

2.2.6 Design variables, State variables and Parameters

Design variable

symbol	introduction	unit
A_{back_1}	The kite's area when it goes back at the 1 round	(m^2)
A_{back_2}	The kite's area when it goes back at the 2 round	(m^2)
A_{back_3}	The kite's area when it goes back at the 3 round	(m^2)
A_{back_4}	The kite's area when it goes back at the 4 round	(m^2)
S_1	the length the kite rise at the 1 round	(m)
S_{1_2}	The length the kite goes back at the 1 round	(m)
S_2	the length the kite rise at the 2 round	(m)
S_{2_2}	The length the kite goes back at the 2 round	(m)

S_3	the length the kite rise at the 3 round	(m)
S_{3_2}	The length the kite goes back at the 3 cycle	(m)

PS. Though I use "time" as the variable in code, what I could optimize is the length of the rope which is the function of time. (So as the A_{back})

State variable

symbol	introduction	unit
F_{back}	the force from kite when it goes back	(N)
$F_{friction}$	the force from bearing friction	(N)
V_{rise}	the instant velocity when the kite rise	(m/s)
V_{back}	the instant velocity when the kite goes back	(m/s)
V_{max}	The maximum velocity	(m/s)
t_n	the time the kite rise at the n round	(s)
t_{n2}	the time kite goes down at the n round	(s)
W_r	the work the kite rise	(J)
W_b	The power the kite goes back	(J)

Parameter

symbol	introduction	unit
F_{rise}	the force from kite when it rise	(N)
F_{output}	the force given to controller	(N)
A_{rise}	The kite's area when it rise	(m^2)
m	the mass of the flywheel	(kg)
P_{wind}	The pressure of the wind	(N/m^2)
C	The coefficient of the friction force	

2.2.7 Summary model

Objective function

f : $[F_{rise}*S_n- \int F_{friction}*dS1-F_{back}*Sn\_2- \int F_{friction}*dS_{n\_2}]/(t_{n1}+t_{n2})$

Subjective function

h1 : $F_{friction}- C*V = 0$

h2: $F_{back} - Pw_{ind} *A_{back} = 0$

h3: $V_{rise} - ((F_{rise} -F_{output})/C) - ((F_{rise} - F_{output})/C)*exp(-C*t/m) =0$

h4: $V_{back} - Vrise_{Max}-(((F_{back} +F_{output})/C) - ((F_{back} +F_{output})/C)*exp(-C*t2/m)) =0$

h5: $Sn - \int_0^{t1} v1(t)dt = 0$

h6: $Sn\_2 - \int_0^{t2}(v2(t))dt = 0$

h7: $Foutput*S1+Foutput*dS1\_2 - F_{rise}*S1- \int F_{friction}*dS1-F_{back}*S1\_2- \int F_{friction}*dS1\_2 = 0$

g1: $-[F_{back}*Sn- \int F_{friction}*dS1-F_{back}*Sn\_2- \int F_{friction}*dSn\_2]/(tn1+tn\_2) \leqq 0$

g2: $-t_n \leqq 0$

g3: $-A_{back} \leqq 0$

g4: $F_{back} -F_{rise} \leqq 0$

g5: $Sn\_2-S1\_1 \leqq 0$

g6: $S1\_1-400 \leqq 0$

g7: $Pw_{ind} *Area_{min}- F_b \leqq 0$

3. Model Analysis

F	$(\int F_{out11}ds1+ \int F_{out12}ds2)/t_{11}+t_{12} + (\int F_{out21}ds21+\int F_{out22}ds22)/t_{21}+t_{22} + (\int F_{out31}ds31+\int F_{out32}ds32)/t_{31}+t_{32}$
H1	$F_{friction}- C*V = 0$
H2	$F_{back} - Pw_{ind} *A_{back} = 0$
H3	$-t< 0$
H4	$Pw_{ind} *Area_{min}- F_b \leqq 0$
H5	$S_n - \int_0^{t1} v1(t)dt = 0$
H6	$S_n\_2 - \int_0^{t2}(v2(t))dt = 0$
H7	$Foutput*S1+Foutput*dS1\_2 - F_{rise}*S1- \int F_{friction}*dS1-F_{back}*S1\_2- \int F_{friction}*dS1\_2 = 0$
G1	$-[F_{rise}*Sn- \int F_{friction}*dS1-F_{back}*Sn\_2- \int F_{friction}*dSn\_2]/(tn1+tn\_2) \leqq 0$
G2	$-t_n \leqq 0$
G3	$-A_{back} \leqq 0$
G4	$F_{back} -F_{rise} \leqq 0$
G5	$S_{n\_2}-S_{1\_1} \leqq 0$
G6	$S_{1\_1}-400 \leqq 0$
G7	$Pw_{ind} *Area_{min}- F_b \leqq 0$

3.1Monotonicity Test Table:

	t1	t2	F_b1	t3	F_b2	t4	F_b3
H1	-	-		-		-	
H2			+		+		+
H3	-	-		-		-	
H4			+		+		+
H5	-	-		-		-	
H6		-		-		-	
H7	?	?	-	?	-	?	-
G1	?	?	-	?	-	?	-
G2	-	-		-		-	
G3			-		-		-
G4			+		+		+
G5	-	?		?		?	

G6	+						
G7			-		-		-

All the H equation must active. However, it is hard to know which inequality constrains (G equation) are active because there are many unknown trend on the table.

G3 could not be active due to G7.

3.2 Design Parameters

F_{rise}	The force by wind when the kite rise	364*4 = 1456 N	N
$Area_{min}$	Minimum kite area that could keep kites in shape and avoid ropes twisting together.	0.5	m^2
P_{wind}	The pressure of the wind	1.29* 20*20*sin45° = 364 20m/s is the average speed of wind in stratosphere	N/m^2
m	The mass of the flywheel	20	kg
F_{output}	The force give to the power generator	400	N
n	Number of rounds	3	
C	The friction force coefficient	20	

3.3 Optimization analysis (Method and Results)

Fmincon in matlab is used to solve the optimize problem

Result:

variable	T1	T2	F_{back1}	T4	F_{back2}	T6	F_{back3}
Start point	1	1	1	1	1	1	1
Value	8.5756	9.8012	182	6.0741	182	3.7921	182

- Equal to

S_1	S_{1_2}	S_{2_2}	S_{3_2}	A_{back1}	A_{back2}	A_{back3}
400	261.289	172.149	115.446	0.5	0.5	0.5

result	exitflag	$\lambda\_\ F_{back1}\leq 0$	$\lambda\_\ F_{back2}\leq 0$	$\lambda\_\ F_{back3}\leq 0$	$\lambda\_\ S1-400\leq 0$
1.4386e+04	1	6.0990	3.4483	1.7642	0.1869

The maximum length and the minimum shrink kite areas' constrains are active. We may conclude that in this feasible region, the displacement is longer the better, and the area is smaller the better.

3.4 Sensitivity analysis

If we could receive a longer rope which is 600m long (that means the maximum displacement could be as long as 600m), we could try if the result change or not.

Result:

variable	T1	T2	F_{back1}	T4	F_{back2}	T6	F_{back3}
Start point	1	1	1	1	1	1	1
Value	9.7035	11.4187	182	7.0961	182.0000	4.4028	182.0000

\> Equal to

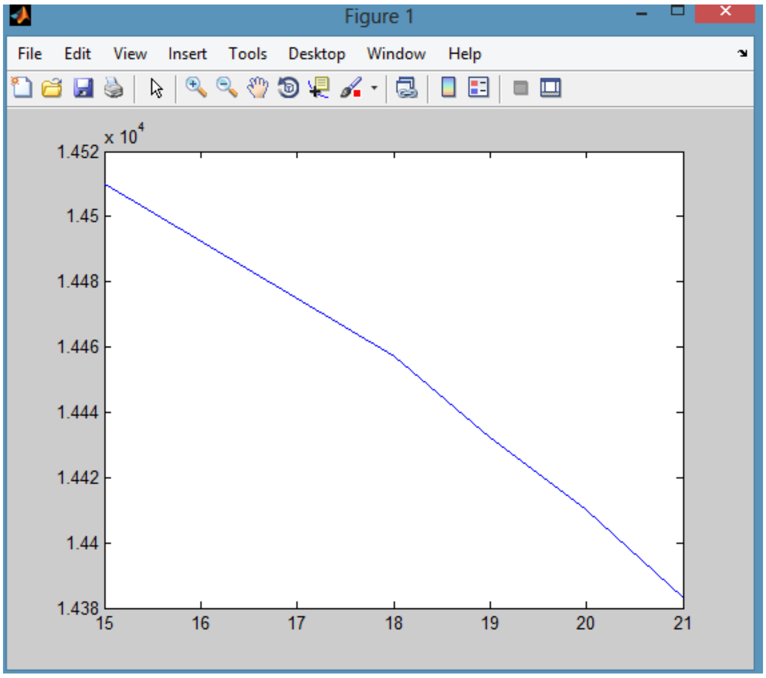
S_1	S_{1_2}	S_{2_2}	S_{3_2}	A_{back1}	A_{back2}	A_{back3}
459.5475	299.6858	196.7810	131.0128	0.5	0.5	0.5

result	exitflag	$\lambda\_\ F_{back1}\leq 0$	$\lambda\_\ F_{back2}\leq 0$	$\lambda\_\ F_{back3}\leq 0$	$\lambda\_\ S1-400\leq 0$
1.4391e+04	1	6.2981	3.6245	1.9131	0

We could see that the maximum displacement increase, and the optimize value also increase. However, the maximum displacement is not the longer the better. The upper constrain for the rope is not active. I think it is because as the displacement increase (means the velocity increase), the friction increase more significantly (it is the function of velocity square). So here is the trade-off.

3.5 Parametric analysis

We change the mass of the flywheel to see if the result change or not:



The velocity and friction are the function of mass, and change the mass will change these two value too. The result shows that when the mass increase, the power output will decrease. However, there is a lower limit of the flywheel. If it's too light, to receive the same amount of kinetic energy it will have very high angular velocity that it's material or bearing may not stand.

4. Discussion

There are two type of variable we would like to optimize, displacement and reduced kite area. Let's talk about the displacement first.

At the beginning, when the time increase, the velocity and the displacement increase, also the positive work. However, the friction work also increase. Though comparing with the work done by wind, the parameter of the friction work is small, the friction if the square function of time. Therefore, as the time keep increase, the sum of the positive work minus the friction work will decrease at some point. In addition, the power is inversely proportional to the time. Hence, the time(displacement) is not as larger as better.

In this project, I suppose that I could have at most 8400m long rope to set the upper constrain to the displacement. We could see that the upper displacement constrain is active. So we do the sensitivity analysis to see if the displacement increase, the result increase and as long as better. The result shows that to get the maximum power, the displacement is about 460m. As the displacement go beyond the 460, the power will decrease. For this result, we could also conclude that if we use a very long rope at the beginning, the reason will not be better than the 460m long rope.

I also set some constrains that displacement for each iteration should not greater than the path1, or the kite will go below the original release point and below stratosphere. The result shows that none of this constrains actives, and the rewind length keep decreasing. I think it means that to save energy, the rewind length should not exceed the originally length naturally.

For reduced kite area, the result shows that it always tough the lower constrains. Because the force is the resistant force within the system, it's reasonable that it's value is as small as possible. However, it still has a lower constrains. If the area shrink to zero, all the rope will twist together, and also hard to expand for the second round.

5. ACKNOWLEDGMENTS

Thanks to instructor of ME 555, MAX, and my classmate Kang-Hsien to clarify the problem and discuss this topic with me.

6. REFERENCES

Essential Concepts of Bearing Technology, Fifth Edition; page182-187;by Tedric A. Harris, Michael N. Kotzalas, June 2006,CRC Press
“kitegen”website : http://www.kitegen.com/en/(referenceits machine design)

7. Code

Objective

```
fun = @(x)-(1456*path_1(x)-frictionwork_path1(x)...
    -x(3)*path_2(x)-frictionwork_path2(x)...
    +1456*path_2(x)-frictionwork_path3(x)...
    -x(5)*path_4(x)-frictionwork_path4(x)...
    +1456*path_4(x)-frictionwork_path5(x)...
    -x(7)*path_6(x)-frictionwork_path6(x))/(x(1)+x(2)+time(x)+x(4)+time_2(x)+x(6));



x0=[1 1 1 1 1 1 1];


A = [];
B = [];
```

```
Aeq = [];
Beq = [];

lb=[];
ub=[];


c=@(x)non_1(x);

ceq=@(x)non_2(x);

nonlcon=@(x)deal(c(x),ceq(x));

[x,fval] = fmincon(fun,x0,A,B,Aeq,Beq,lb,ub,nonlcon)
```

```
function N = non_1(x)

    A = 0.5*364;
    N =[path_2(x)-path_1(x);...
        path_4(x)-path_1(x);...
        path_6(x)-path_1(x);...
        A-x(3);...
        A-x(5);...
        A-x(7);...
        path_1(x)-400];
```

```
function N = non_2(x)

    N =[(1456*path_1(x)-frictionwork_path1(x)...
    -x(3)*path_2(x)-frictionwork_path2(x)...
    -400*path_1(x)-400*path_2(x));
    (1456*path_2(x)-frictionwork_path3(x)...
    -x(5)*path_4(x)-frictionwork_path4(x)...
    -400*path_2(x)-400*path_4(x));
    (1456*path_4(x)-frictionwork_path5(x)...
    -x(7)*path_6(x)-frictionwork_path6(x)...
    -400*path_4(x)-400*path_6(x))];
```

```
function out = path_1(x)

fr = 1456;
f = 400;
C = 20;
m = 20;
cc=x(1);
syms t
v = ((fr-f)/C) - ((fr-f)/C)*exp(-C*t/m);
S = ((fr-f)/C)*(cc+exp(-cc))-((fr-f)/C)
out=S;
```

```
function out = path_2(x)

c2 = x(2);
F2 = x(3);

syms t2
fr = 1456;
f = 400;
C = 20;
m = 20;
v = ((fr-f)/C)-((fr-f)/C)*exp(-C*x(1)/m);
S =(v*c2-(((F2+f)/C)*(c2+exp(-c2))-(F2+f)/C));
out=S;
```

```
function out =path_4(x)

c4 = x(4);
F3 = x(5);
syms t3
fr = 1456;
f = 400;
C = 20;
m = 18;
v = ((fr-f)/C)-((fr-f)/C)*exp(-C*time(x)/m);
```

```
S =(v*c4-(((F3+f)/C)*(c4+exp(-c4))-(F3+f)/C));
out=S;
```

```
function out =path_6(x)

c4 = x(6);
F4 = x(7);
syms t3
fr = 1456;
f = 400;
C = 20;
m = 18;
D = C/m;
v = ((fr-f)/C)-((fr-f)/C)*exp(-C*time_2(x)/m);
S = (v*c4-(((F4+f)/C)*(c4+exp(-c4))-(F4+f)/C));
out=S;
```

```
function out =frictionwork_path1(x)

cc=x(1);
fr = 1456;
f = 400;
C = 20;
m = 20;
syms t
B = 52.8;
D = C/m;
SS = (B^2*C*cc)/2 - (B^2*C*(exp(-2*D*cc) - 4*exp(-D*cc)+ 3))/(4*D);
out=SS;
```

```
function out =frictionwork_path2(x)

c2 = x(2);
F2 = x(3);
syms t2
fr = 1456;
f = 400;
C = 20;
```

```
m = 20;
B = ((F2+f)/C);
D = C/m;
v = ((fr-f)/C) - ((fr-f)/C)*exp(-C*c2/m);
S4 = ((C*((2*(- B^2 + v*B))/D + B^2/(2*D) + c2*(B^2 - 2*B*v + v^2) - (B^2*exp(-2*D*c2))/(2*D)
- (2*exp(-D*c2)*(- B^2 + v*B))/D))/2);

out=S4;
```

```
function out =frictionwork_path3(x)

syms t2
fr = 1456;
f = 400;
C = 20;
m = 20;
B = 52.8;
D = C/m;
S4 = ((B^2*C*time(x))/2 - (B^2*C*(exp(-2*D*time(x)) - 4*exp(-D*time(x))+ 3))/(4*D));
out=S4;
```

```
function out =frictionwork_path4(x)

c3 = x(4);
F3 = x(5);
syms t3
fr = 1456;
f = 400;
C = 20;
m = 20;
B = ((F3+f)/C);
D = C/m;
v = ((fr-f)/C) - ((fr-f)/C)*exp(-C*time(x)/m);
W = ((C*((2*(- B^2 + v*B))/D + B^2/(2*D) + c3*(B^2 - 2*B*v + v^2) - (B^2*exp(-2*D*c3))/(2*D)
- (2*exp(-D*c3)*(- B^2 + v*B))/D))/2);

out=W;
```

```
function out =frictionwork_path5(x)
```

```
syms t2
fr = 1456;
f = 400;
C = 20;
m = 20;
B = 52.8;
D = C/m;
S4 = ((B^2*C*time_2(x))/2 - (B^2*C*(exp(-2*D*time_2(x)) - 4*exp(-D*time_2(x))+ 3))/(4*D));
out=S4;
```

```
function out =frictionwork_path6(x)

c4= x(6);
F4 = x(7);
syms t3
fr = 1456;
f = 400;
C = 20;
m = 20;
B = ((F4+f)/C);
D = C/m;
v = ((fr-f)/C) - ((fr-f)/C)*exp(-C*time_2(x)/m);
W = ((C*((2*(- B^2 + v*B))/D + B^2/(2*D) + c4*(B^2 - 2*B*v + v^2) - (B^2*exp(-2*D*c4))/(2*D)
- (2*exp(-D*c4)*(- B^2 + v*B))/D))/2);
out=W;
```

```
function out =time(x)

fr = 1456;
f = 400;
C = 20;
m = 20;
syms a
f = 400;
fr = 1456;
C = 20;
m = 20;
B = ((fr-f)/C)*(a+exp(-a))-(fr-f)/C;
aa = double(solve(B==path_2(x),a));
```

```
out=aa;
```

```
function out =time_2(x)
syms a t
f = 400;
fr= 1456;
C= 20;
m = 20;
aa = double(solve(((fr-f)/C)*(a+exp(-a))-(fr-f)/C == path_4(x),a));
out=aa;
```

```
function out =time_3(x)

fr = 1456;
f = 400;
C = 20;
m = 20;
syms a
f = 400;
fr = 1456;
C = 20;
m = 20;
B = ((fr-f)/C)*(a+exp(-a))-(fr-f)/C;

aa = double(solve(B==path_6(x),a));
out=aa;
```
