



Optimal Design of a Residential Wind Turbine

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1 ABSTRACT

A residential wind turbine can offset a significant percentage of the electricity of a single family home. In order to make wind turbines more accessible to this market, the payback period of a wind turbine is minimized. If a consumer can recuperate the cost of the device in 5 – 10 years, it becomes a viable alternative to the traditional grid hookup. In the course of this paper, three separate subsystems of a typical residential wind turbine design are optimized separately with the ultimate objective of reducing payback period. These subsystems are: the aerodynamic design of the blades, the power generation system design, and the structural support tower design. Using this method, global minima for each separate subsystem were found, and a payback period of 163.6 years was obtained. A full system optimization was then performed, and a payback period of 6.2 years was obtained. The results for a full system optimization were better because the flow of information from one subsystem to the others was not allowed in the individual subsystem optimizations. The final payback period could not be proved to be a global minimum because a gradient based solver may not be sufficient to overcome the multimodal nature associated with the power generation subsystem.

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3 MOTIVATION

The purpose of this project is to optimize the design of a wind turbine for a residential application. A wind turbine converts wind energy into electrical energy, and the specific design being considered is an upwind facing, horizontal axis, three-blade turbine. This wind turbine will be relatively inexpensive and will attempt to offset a part of the energy consumption costs of a single family home in Grand Haven, MI, which is a location of high wind quality on Lake Michigan's coast, according to the Michigan Department of Energy, Labor, and Economic Growth (MDELEG). The ultimate goal of this project is to minimize the cost payback period (in years) of the wind turbine design in order to make wind technology more accessible to the average consumer family.

Current wind turbines on the market cost between \$12,000 and \$55,000 (Shevory, 2007), and have a payback period of between 5 and 10 years (Cook, 2008). My team anticipated that we would be able to find a wind turbine design that has a payback period of less than 10 years, and an initial cost within the range of typical small wind turbines.

There are three subsystems: The structural design, the blade design, and the power generation design. It is clear that each part of the design is separate, yet will have important interactions with the other parts, making this a natural breakdown of the design problem. The purpose of this report is to present four separate optimization studies. First is presented a study for the optimization of the wind turbine blade design, which was completed by Manan Thakkar. Second is presented a study for the optimization of the electrical power generation system, completed by Kurt Cunningham. Third is shown a study for the optimization of the structural tower subsystem, completed by Kevin Schaefer. Finally, a full system optimization study is presented. All four studies are completed with the end-user in mind, but this paper shows that a full system optimization will have better overall results than separate system optimization designs.

4 PROBLEM STATEMENT

The goal of this project is to minimize the payback period (PP) of a wind turbine for a residential application. Equation 4.1 below shows how the payback period is calculated. C_{gen} is the total cost of the power generation components, C_{blade} is the cost of the blades, and C_{struct} is the cost of the supporting structure. Ω is the cost recuperation rate in \$/yr, and is related to the total power that the system can

output over the course of a year. If this system sells the power directly back to the grid, the cost recuperation rate can be calculated.

$$PP = \frac{C_{gen} + C_{blade} + C_{struct}}{\Omega} \quad [\text{Eq. 4.1}]$$

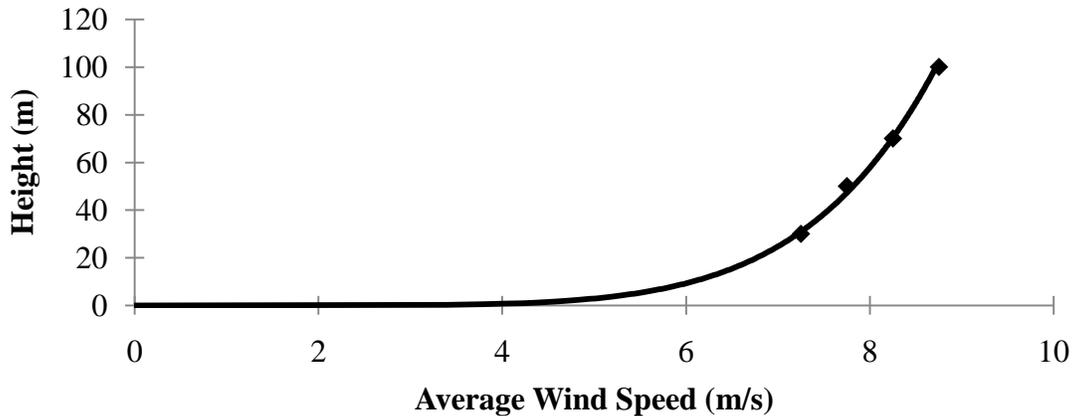
The Michigan DELEG has wind speed data at different elevations throughout the state, which can be used to create a model of the average wind speed (V_{avg}) as a function of height (h) at the chosen location (Michigan DELEG, 2010). Because the height of the tower is a critical parameter throughout this paper, this is an important first step in order to vary the height of our final wind turbine design. The raw data collected from the DELEG is shown in Table 4.1. Using this data and the assumption that the wind speed at the ground is always equal to zero (a basic fluid dynamics assumption), a best fit exponential line is found using Excel. This best fit line is shown in Figure 4.1 along with the raw data, and the equation for the meta-model is shown in Equation 4.1.

Table 4.1: Raw Data of Average Wind Speed from DELEG [3]

Tower Height	Average Wind Speed in Grand Haven, MI
30	7.25
50	7.75
70	8.25
100	8.75

$$V_{avg} = 4.222h^{.1575} \quad [\text{Eq. 4.1}]$$

Figure 4.1: Effect of Tower Height on Average Wind Speed



Each of the subsystem designs includes tower height (equivalent to wind speed) as a parameter, but the overall optimization study has tower height as a variable. Each of our optimization studies, therefore, does not optimize for the best tower height, and therefore the individual system optimizations do not result in the optimal design. During full system optimization, tower height is a variable, and so the optimal design is different. For all of the following studies, the wind speed is assumed to be unobstructed by upwind obstacles.

5 AERODYNAMIC DESIGN OF THE BLADES – Manan Thakkar

The blades are the first component of the turbine that come in contact with the wind and hence are the most important. The oncoming wind at high velocities strikes the blades and rotates them because they are inclined at an angle to the wind direction. This rotation is caused by a lift force generated by the wind on the blades. Thus the blades need to be designed for maximum lift force generation. As the wind travels over the blade surface, some of its energy is lost as friction in the form of drag and hence they must be designed for minimum drag. The blades are connected to the actual power generation system which then converts the wind energy transferred to it into useful power.

Problem statement

Subsystem: The first main objective of this subsystem is to select an appropriate material for the blades depending on the requirements. The other and more important objective is to minimize the weight of the blades and at the same time maximize their aerodynamic efficiency.

Assumptions

Before starting any kind of analysis, it is necessary to lay down all the assumptions made in order to simplify the mathematical model.

- The cost of the blade has a direct relation to only the weight and material of the blade
- 3 blades will be utilized on the wind turbine
- 2 airfoil sections will be used on the blade. The root airfoil will span 0.45 times the total blade length and the tip airfoil will span the remaining blade length. These lengths have been specified by the National Renewable Energy Laboratory (NREL)
- Pitch angle is constant throughout the span of blade
- The chord of the airfoils remains constant throughout blade span
- The wind velocity is constant in the region near the blades and is calculated as the average wind velocity

5.1 Nomenclature

A_b	=	cross section area of the blade airfoil, m^2
A_s	=	swept area of the blades, m^2
c	=	chord of the airfoil, m.
C_L	=	co-efficient of lift
C_D	=	co-efficient of drag
F_L	=	lift force generated by the blades, N
F_D	=	drag force on the blades, N
g	=	acceleration due to gravity = 9.81 m/s^2
g_{blade}	=	material cost of the blades, $\$/\text{kg}$
h	=	tower height, m. (parameter)
L	=	length of blade, m.
M	=	torque generated by the blades, Nm.
N	=	blade rpm
P	=	power produced by the turbine, W
P_{max}	=	maximum power that could be produced, W
T	=	thrust force exerted by the wind, N
V_0	=	actual wind velocity, m/s
V_{avg}	=	average wind velocity, m/s
V_t	=	tangential velocity of the blades at the tip, m/s
v_{blade}	=	volume of the blade, m^3
w	=	weight of the turbine blades, kg
W	=	non-dimensionalized weight
α	=	local angle of attack, degrees
λ	=	tip speed ratio
φ	=	pitch angle, degrees
ρ_{mat}	=	density of the blade material, kg/m^3
ρ_{air}	=	density of air = $1.225 \text{ kg}/\text{m}^3$
ω	=	angular velocity of the blades, r/s

Variables: α , λ , L , c

Note: g_{blade} is not utilized in the blade subsystem but only in the overall system integration

5.2 Mathematical Model

5.2.1 Blade Material Selection

The material selection of the blade is a discrete procedure and depends only on the requirements of a small scale horizontal axis wind turbine. Material requirements can be summarized as follows:

- Since weight of the blades needs to be minimized the most important requirement is that the material needs to be light weight. Now since the weight has a direct relation with the material density, a material with lower density will have low weight. Thus a low density material needs to be selected.
- Since the blades will be experiencing high wind velocities they will need to withstand bending and torsion loads. Bending occurs along the length of the blade and torsional loads occur because the blades have finite width. Torsion tends to twist the blade along an axis passing through its length. The material thus needs to be able to withstand these loads without change in properties.
- Since the turbine may not rotate continuously and remain stationary for long periods of time, the blades will be subject to loads due to self- weight. The material needs to withstand these self-weight loads, again without change in properties.

Keeping in mind all these requirements, the material selected is smooth *CFOAM 25 Structural Carbon Foam* (Touchstone Research Laboratory) for the core. It is covered outside with a similar density carbon sheet to reduce roughness. Its properties are as follows (MatWeb):

Table 5.1: Material properties of CFOAM 25

Material	CFOAM 25 Structural Carbon Foam
Density	500 kg/m ³
Tensile strength	5 MPa
Compressive strength	15 MPa
Bending strength	5 MPa
Modulus of elasticity	827 MPa

5.2.2 Objective Function

The main objective of this optimization is to minimize the weight of the blades so that the cost is minimized. Another objective which conflicts with the one already mentioned is to maximize the aerodynamic efficiency of the blades so that maximum power is extracted from the wind.

The total blade weight is a function of its cross section area, A_b and span (or length), L of the blade. It is also a function of the material density, ρ_{mat}

$$w = 3 v_{blade} \rho_{mat} \quad [\text{Eq. 5.1}]$$

The two airfoils that are going to be used are standard available wind turbine blade airfoils designated by NREL S822 (in the tip region) and NREL S823 (in the root region) (Somers).

For NREL S822, $A_b = 1.7964c^2$

For NREL S823, $A_b = 2.1862c^2$

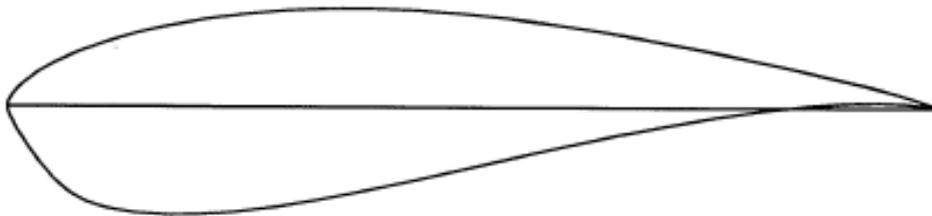
The cross-section area varies linearly along the blade span and is calculated using mathematical principles as

$$A_x = 1.7964 c^2 \left(\frac{0.2169}{L} x + 1 \right) \quad [\text{Eq. 5.2}]$$

Figure 5.1: NREL S822 and S823 airfoils



(a) S822.



(b) S823.

This area function is integrated over the blade length to obtain the blade volume.
Therefore the weight of the blades is

$$w = \frac{3(1.9913 c^2 L)}{v} \rho_{mat} \quad [\text{Eq. 5.3}]$$

In order for both the terms in the objective function to have the same units, we non-dimensionalize the weight by its upper limit i.e. 300 kg.

$$W = \frac{3(1.9913 c^2 L) \rho_{mat}}{300} \quad [\text{Eq. 5.4}]$$

The aerodynamic efficiency is given as

$$\eta = \frac{P}{P_{max}} \quad [\text{Eq. 5.5}]$$

$$\eta = \frac{3 F_L \lambda V_0}{(0.5 \rho_{air} A_s V_0^3)} \text{ where } A_s = \pi L^2 \quad [\text{Eq. 5.6}]$$

Finally, the overall objective function is given as

$$f = \frac{3 (1.9913 c^2 L) \rho_{mat}}{300} - \frac{3 F_L \lambda V_0}{(0.5 \rho_{air} A_s V_0^3)} \quad [\text{Eq. 5.7}]$$

5.2.3 Reasons for Using NREL S822 and NREL S823 Airfoils

The airfoils that are being used in the optimization are standard ones prescribed by the National Renewable Energy Laboratory (NREL) designated as S822 and S823. These airfoils are designed specifically for small, horizontal axis wind turbines with tower height up to 12 meters. Main reasons for using them are:

- High profile co-efficient of lift:

This is essential because when C_L is high, lift force generated by the blades is high and since power generated is directly related to lift force, the blades generate more power for higher lift force

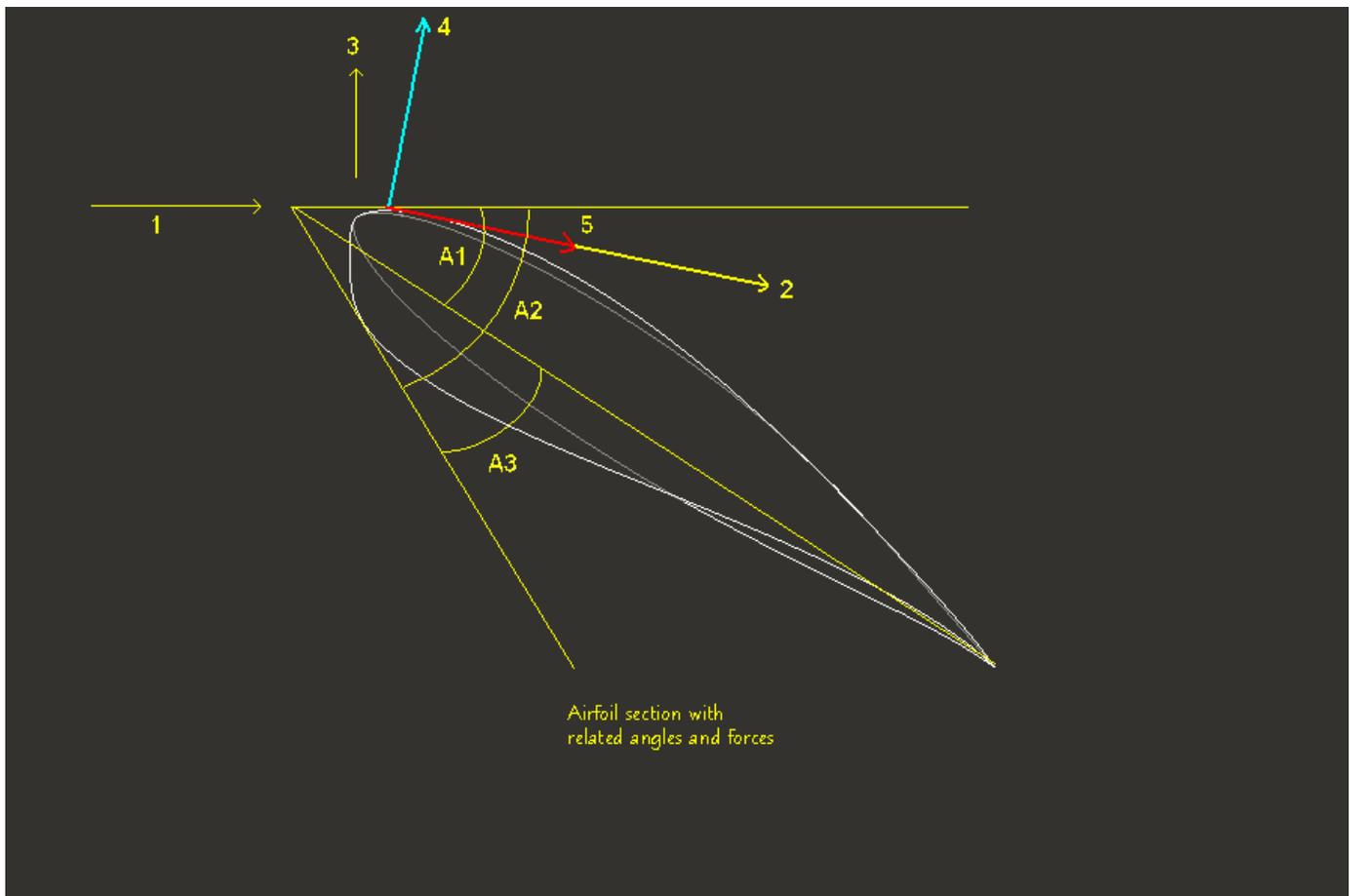
- Low profile co-efficient of drag:

The shape of these airfoils enables them to have a low C_D . Drag force is caused due to friction on the blade surface and results in energy dissipation as heat. This leads to wastage of precious wind energy and thus must be minimized

- Very little boundary layer separation:

Separation of boundary layer leads to a dramatic increase in C_D and decrease in C_L . It is directly related to pressure gradient on the airfoil surface. These airfoils maintain a favorable pressure gradient and thus avoid separation of the boundary layer (up to a certain angle of attack)

Figure 5.2: Airfoil section with various angles and forces



Legend for figure 5.2:

1	-	V_0
2	-	V_{avg}
3	-	$L\omega$
4	-	F_L
5	-	F_D
A1	-	α
A2	-	φ
A3	-	$\varphi - \alpha$

5.2.4 Constraints

- Constraint on weight: since we want the blades to have positive finite weight we chose the weight range as $0 \leq w \leq 300$ kg for small wind turbine blades. Also since the weight has been non-dimensionalized by its upper bound, we have a constraint as $0 \leq W \leq 1$
- constraint on angle of attack: $0^\circ \leq \alpha \leq 15^\circ$ to avoid boundary layer separation
- constraints on C_L and C_D :

For S822 airfoil, $0.1 \leq C_L \leq 1.6$ and $0.2 \leq C_D \leq 0.8$

For S823 airfoil, $0.2 \leq C_L \leq 1.9$ and $0.4 \leq C_D \leq 1.0$

To get the equations for C_L and C_D in terms of α , a small simulation was run in the airfoil analysis software, *XFOIL*. The 2 airfoils were analyzed for different angles of attack at their design Reynolds number (as specified by NREL). There were about 45 data points generated for each airfoil. Thus equations for C_L and C_D were obtained using a linear-least squares curve fit using a 5th degree polynomial. They are as follows:

$$\text{S822: } C_L = 1.0525 + 0.2081\alpha - 0.0341\alpha^2 + 0.0036\alpha^3 - 0.0002\alpha^4$$

$$C_D = 0.0192 - 0.0107\alpha + 0.0060\alpha^2 - 0.0007\alpha^3$$

$$\text{S823: } C_L = 1.2196 + 0.1171\alpha + 0.0041\alpha^2 - 0.0012\alpha^3 + 0.0001\alpha^4$$

$$C_D = 0.0171 - 0.0084\alpha + 0.0026\alpha^2 - 0.0003\alpha^3$$

- The 2 airfoil sections will generate a different amount of lift force. Thus the total lift force on the blade will be the sum of these individual lift forces.

$$\text{In general, } F_L = 0.5 \times C_L \times \rho_{air} \times (c \times L) \times V_0^2$$

$$\text{Therefore, } F_{L,822} = 0.5 \times C_{L,822} \times \rho_{air} \times (c \times L_{822}) \times V_0^2$$

$$F_{L,823} = 0.5 \times C_{L,823} \times \rho_{air} \times (c \times L_{823}) \times V_0^2$$

$$\text{And thus, } F_{L,total} = F_{L,822} + F_{L,823}$$

[Eq. 5.8]

Also assume that the lift force acts on the blade tip to generate maximum torque.

Here we also need to put a constraint on the maximum lift force that can be generated. This constraint is obtained from P_{max} as

$$F_{L,total} \leq \frac{P_{max}}{3 \lambda_{max} V_0} \text{ where } \lambda_{max} = \text{upper bound of } \lambda = 8 \quad [\text{Eq. 5.9}]$$

This constraint is needed because otherwise the algorithm will give an efficiency value of 1, which is only theoretically possible

The Drag force is given by

$$F_{D,822} = 0.5 \times C_{D,822} \times \rho_{air} \times (c \times L_{822}) \times V_0^2$$

$$F_{D,823} = 0.5 \times C_{D,823} \times \rho_{air} \times (c \times L_{823}) \times V_0^2$$

And thus, $F_{D,total} = F_{D,822} + F_{D,823}$

where $L_{822} = 0.55 L$ and $L_{823} = 0.45 L$ (Tangler and Somers)

- constraint on the tower height: $h \leq 12$ m. from zonal laws in the USA. Since tower height is only a parameter, its value will be constant for a given optimization
- constraint on blade span: $0 \leq L \leq 8$ m. for a small wind turbine
- constraint on tip speed ratio: since the blade tip speed must be equal to at least the free stream wind velocity in order to generate lift, we have $1 \leq \lambda \leq 8$
- constraint on the airfoil chord: we also assume that the airfoil chord has a finite value and thus for a small scale wind turbine

$$0.1 \leq c \leq 2$$

- constraint on average velocity: the average wind velocity is given by, $V_{avg} = 4.22 \times h^{0.1575}$
 - constraint on Torque: the torque generated by one blade is given by, $M = F_L \times L$
 - constraint on Power: the total power generated by the blades is given by, $P = 3 \times F_L \times \lambda \times V_0$
 - constraint on the blade efficiency: it is obvious that $0 \leq \eta \leq 1$
 - constraint on blade rpm: we want our residential wind turbine to have a maximum rpm for reasons regarding maximum loads and stress. Thus $N \leq 300$ rpm
 - minimum power to be generated: we want the wind turbine to generate some finite power and therefore, $P \geq 1000$ W
 - constraint on the thrust force: the thrust exerted on the blades is given as, $T = \frac{4}{9} \rho_{air} A_s V_{avg}^2$
 - non-negativity constraint: all variables and parameters must be greater than 0 i.e.
- $$V_{avg}, V_t, M, P, P_{max}, N, T, \omega, \geq 0$$

5.2.5 Summary Model

After defining the objective function and all the constraints, we get the complete model in a summarized form as follows:

Minimize:

$$f = \frac{3 (1.9913 c^2 L) \rho_{mat}}{300} - \frac{3 F_L \lambda V_0}{(0.5 \rho_{air} A_s V_0^3)} \quad [\text{Eq. 5.10}]$$

Subject to:

$$g_{bl-1i} = -w \leq 0, w - 300 \leq 0 \quad [\text{Eq. 5.11}]$$

$$g_{bl-1ii} = -W \leq 0, W - 1 \leq 0 \quad [\text{Eq. 5.12}]$$

For S822

$$g_{bl-2i} = 0.1 - C_L \leq 0 \quad [\text{Eq. 5.13}]$$

$$g_{bl-2ii} = C_L - 1.6 \leq 0 \quad [\text{Eq. 5.14}]$$

$$g_{bl-3i} = 0.2 - C_D \leq 0 \quad [\text{Eq. 5.15}]$$

$$g_{bl-3ii} = C_D - 0.8 \leq 0 \quad [\text{Eq. 5.16}]$$

For S823

$$g_{bl-4i} = 0.2 - C_L \leq 0 \quad [\text{Eq. 5.17}]$$

$$g_{bl-4ii} = C_L - 1.9 \leq 0 \quad [\text{Eq. 5.18}]$$

$$g_{bl-5i} = 0.4 - C_D \leq 0 \quad [\text{Eq. 5.19}]$$

$$g_{bl-5ii} = C_D - 1.0 \leq 0 \quad [\text{Eq. 5.20}]$$

$$g_{bl-6} = F_{L,total} - \frac{P_{max}}{3 \lambda_{max} V_0} \leq 0 \quad [\text{Eq. 5.21}]$$

$$g_{bl-7} = 1000 - P \leq 0 \quad [\text{Eq. 5.22}]$$

$$g_{bl-8i} = -\eta \leq 0 \quad [\text{Eq. 5.23}]$$

$$g_{bl-8ii} = \eta - 1 \leq 0 \quad [\text{Eq. 5.24}]$$

Variables:

$$\xi_{bl-9i} = -\alpha \leq 0 \quad [\text{Eq. 5.25}]$$

$$\xi_{bl-9ii} = \alpha - 15^\circ \leq 0 \quad [\text{Eq. 5.26}]$$

$$\xi_{bl-10i} = 1 - \lambda \leq 0 \quad [\text{Eq. 5.27}]$$

$$\xi_{bl-10ii} = \lambda - 8 \leq 0 \quad [\text{Eq. 5.28}]$$

$$\xi_{bl-11i} = -L \leq 0 \quad [\text{Eq. 5.29}]$$

$$\xi_{bl-11ii} = L - 8 \leq 0 \quad [\text{Eq. 5.30}]$$

$$\xi_{bl-12i} = 0.1 - c \leq 0 \quad [\text{Eq. 5.31}]$$

$$\xi_{bl-12ii} = c - 2 \leq 0 \quad [\text{Eq. 5.32}]$$

$$\xi_{bl-13} = -V_{avg}, -V_t, -M, -P_{max}, -N \leq 0 \quad [\text{Eq. 5.33}]$$

We then have all the equations needed to calculate variables and parameters.

- S822:

$$C_{L,822} = 1.0525 + 0.2081\alpha - 0.0341\alpha^2 + 0.0036\alpha^3 - 0.0002\alpha^4 \quad [\text{Eq. 5.34}]$$

$$C_{D,822} = 0.0192 - 0.0107\alpha + 0.0060\alpha^2 - 0.0007\alpha^3 \quad [\text{Eq. 5.35}]$$

$$F_{L,822} = 0.5 C_{L,822} \rho_{air} (c L_{822}) V_0^2 \quad [\text{Eq. 5.36}]$$

$$F_{D,822} = 0.5 C_{D,822} \rho_{air} (c L_{822}) V_0^2 \quad [\text{Eq. 5.37}]$$

- S823:

$$C_{L,823} = 1.2196 + 0.1171\alpha + 0.0041\alpha^2 - 0.0012\alpha^3 + 0.0001\alpha^4 \quad [\text{Eq. 5.38}]$$

$$C_{D,823} = 0.0171 - 0.0084\alpha + 0.0026\alpha^2 - 0.0003\alpha^3 \quad [\text{Eq. 5.39}]$$

$$F_{L,823} = 0.5 C_{L,823} \rho_{air} (c L_{823}) V_0^2 \quad [\text{Eq. 5.40}]$$

$$F_{D,823} = 0.5 C_{D,823} \rho_{air} (c L_{823}) V_0^2 \quad [\text{Eq. 5.41}]$$

- $F_{L,total} = F_{L,822} + F_{L,823} \quad [\text{Eq. 5.42}]$

- $F_{D,total} = F_{D,822} + F_{D,823} \quad [\text{Eq. 5.43}]$

- $L_{822} = 0.55 L \text{ and } L_{823} = 0.45 L \quad [\text{Eq. 5.44}]$

- $V_{avg} = 4.222 h^{0.1575} \quad [\text{Eq. 5.45}]$

- $M = F_L L \quad [\text{Eq. 5.46}]$

- $P = 3 F_L V_t \quad [\text{Eq. 5.47}]$

- $T = \frac{4}{9} \rho_{air} A_s V_{avg}^2 \quad [\text{Eq. 5.48}]$

5.3 Model Analysis

5.3.1 Monotonicity Analysis

A monotonicity analysis with respect to the design variables will be performed in order to check if the problem is well constrained. The monotonicities of all the variables with respect to the objective function and constraints are shown in the table below.

Table 5.2: Monotonicity table

	α	λ	L	c
f	non-monotonic	-	+	+
g_{bl-9i}	-			
g_{bl-9ii}	+			
g_{bl-10i}		-		
$g_{bl-10ii}$		+		
g_{bl-11i}			-	
$g_{bl-11ii}$			+	
g_{bl-12i}				-
$g_{bl-12ii}$				+

As we can see that the angle of attack, α is non-monotonic with respect to the objective function, f. This may be explained as follows.

We have $f = F_L^-$ and $F_L = C_L^+$

Thus, $f = C_L^-$

But $C_L = 1.0525 + 0.2081\alpha - 0.0341\alpha^2 + 0.0036\alpha^3 - 0.0002\alpha^4$ for S822

and $C_L = 1.2196 + 0.1171\alpha + 0.0041\alpha^2 - 0.0012\alpha^3 + 0.0001\alpha^4$ for S823

Both these equations are non-monotonic in α .

Thus the objective function is non-monotonic with respect to angle of attack, α .

The other variables are monotonic in the objective function and constraints as shown in the above table.

In spite of α being non-monotonic in the objective function, the model is well constrained.

5.4 Optimization Study

5.4.1 Initial Conditions

The optimization needs a starting point which is assumed as follows.

- $\alpha = 2^\circ$
- $\lambda = 6$
- $L = 4$ m.
- $c = 0.5$ m.
Thus $X_0 = [2, 6, 4, 0.5]$
- $h = 10$ m.
- $V_0 = 12$ m/s
- $V_{\text{avg}} = 6.0676$ m/s
- $V_t = V_0 \lambda = 72$ m/s

- $N = \frac{60 V_t}{2 \pi L} = 171.88$ rpm

$$\omega = \frac{2 \pi N}{60} = 18 \text{ r/s}$$

- $w = 2986.95$ kg.
- $W = 9.9565$
- S822, $C_L = 1.3579$, $C_D = 0.0162$
 $F_L = 131.74$ N, $F_D = 1.5717$ N
- S823, $C_L = 1.4622$, $C_D = 0.0083$
 $F_L = 116.07$ N, $F_D = 0.6589$ N
- $F_{L,\text{total}} = 247.81$ N, $F_{D,\text{total}} = 2.231$ N
- $M = 991.25$ Nm.
- $P = 53528$ W = 53.53 kW
- $P_{\text{max}} = 53201$ W = 53.2 kW
- $\eta = 100.64$ %
- $T = 1007.53$ N

(Clearly, this starting point is infeasible)

5.4.2 Optimization

The optimization was carried out in Matlab using the built-in function fmincon for optimization. A number of initial points were selected and they all gave more or less the same results in terms of minimizers and objective function values.

Table 5. 3 Optimization results for various starting points

Initial point X_0	Local minimizer	Objective function, f
[2,6,4,0.5]	[5.4475,3.5838,1.3689,0.1047]	-0.2987
[1,1,1,1]	[5.4475,3.5839,1.3689,0.1047]	-0.2987
[2,2,2,2]	[5.4475,3.5839,1.3690,0.1047]	-0.2987
[5,5,5,1]	[5.4475,5.4697,2.0893,0.1029]	-0.2955
[4,4,5,.25]	[5.4475,5.0705,1.9368,0.1110]	-0.2375

Other quantities at the local minimizer are tabulated as follows

$$X^* = [\alpha^*, \lambda^*, L^* c^*]$$

i.e. $X^* = [5.4475^\circ, 3.5838, 1.3689 \text{ m}, 0.1047 \text{ m}]$

Table 5. 4 Other quantities at the local minimizer

w^*	44.7959 kg.
W^*	0.1493
P^*	2791.31 W
P_{\max}^*	6230.93 W
η^*	44.79 %
$C_{L,822}^*$	1.58
$C_{L,823}^*$	1.87
$F_{L,822}^*$	10.9822 N
$F_{L,823}^*$	10.6529 N
$F_{L,avg}^*$	21.6352 N
V_t^*	43.0057 m/s
ω^*	31.4159 r/s
N^*	300
M^*	29.6167 Nm.
T^*	118 N

From the optimization, we see that the constraint on the maximum average lift force is active as is necessary to maximize the efficiency. Thus, it is satisfied with a strict equality i.e.

$$F_{L,total} = \frac{P_{max}}{3 \lambda_{max} V_0} \text{ where } \lambda_{max} = \text{upper bound of } \lambda = 8$$

Also we must note that $N = 300$ rpm, again for maximum power generation and hence efficiency

5.4.3 Trade-offs and Pareto Analysis

The most interesting part about this problem is that we have 2 conflicting objective functions of minimizing the weight and maximizing the efficiency. As we know that

$$W = \frac{3 (1.9913 c^2 L) \rho_{mat}}{300}$$

ie $W \sim c$ and $W \sim L$

Thus in order to minimize the weight, we need to minimize the airfoil chord and the blade length. And the aerodynamic efficiency is given as

$$\eta = \frac{3 F_L \lambda V_0}{0.5 \rho (\pi L^2) V_0^3}$$

ie $\eta \sim F_L$

We also know that,

$$F_L = C_L 0.5 \rho_{air} (c L) V_0^2$$

ie $F_L \sim c$ and $F_L \sim L$

Thus in order to have a higher efficiency we need to have a high lift force acting on the blade and the lift force increases with c and L .

We need to achieve a compromise between these competing objectives. So we get to see some interesting trade-offs in this problem.

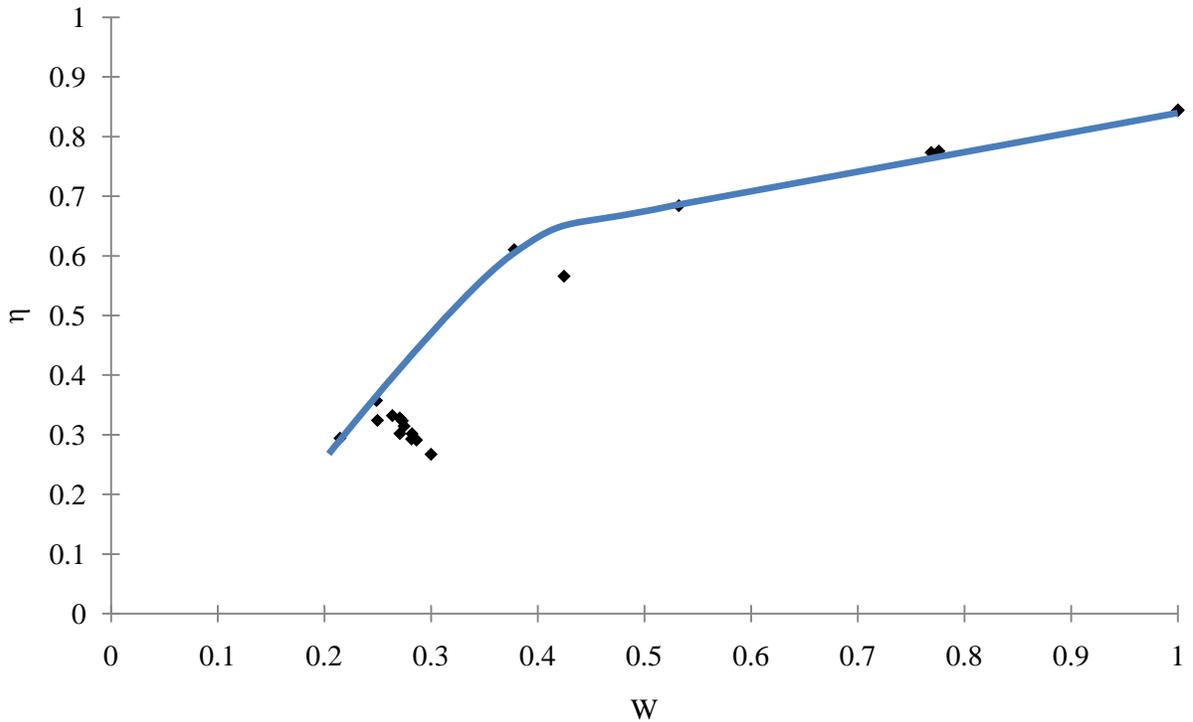
For the Pareto analysis we assign Pareto weight functions to each objective and vary the weight functions. A plot is then made after optimization of the varying objective functions along with the weights. Since Pareto optimal points are ones that show the trade-offs between the two conflicting objective functions (W and η in this case), depending on the requirement, we may select a suitable point as the optimal.

The Pareto analysis is as follows. p_1 and p_2 are the weights assigned to W and η respectively, $0 \leq p_1 \leq 1$. Values of W and η are calculated for different values of the Pareto weights and are summarized in the table below:

Table 5.5: Pareto Data

p_1	$p_1 \times W$	W	$p_2 = p_1 - 1$	$p_2 \times \eta$	η
1	0.2998	0.2998	0	0	0.2673
0.95	0.2371	0.2496	0.05	0.0162	0.3242
0.9	0.2434	0.2706	0.1	0.0302	0.3021
0.85	0.2394	0.2816	0.15	0.0439	0.2929
0.8	0.2289	0.2862	0.2	0.0582	0.2911
0.75	0.2862	0.2145	0.25	0.0735	0.2943
0.7	0.1974	0.282	0.3	0.0905	0.3017
0.65	0.1784	0.2745	0.35	0.11	0.3145
0.6	0.1581	0.2636	0.4	0.1329	0.3322
0.55	0.1368	0.2487	0.45	0.1609	0.3575
0.5	0.1365	0.2729	0.5	0.1617	0.3235
0.45	0.1217	0.2705	0.55	0.1805	0.3282
0.4	0.1698	0.4245	0.6	0.3396	0.566
0.35	0.1322	0.3778	0.65	0.3968	0.6105
0.3	0.1596	0.5321	0.7	0.4789	0.6843
0.25	0.1939	0.7759	0.75	0.5819	0.7759
0.2	0.1537	0.7687	0.8	0.6188	0.7735
0.15	0.15	1	0.85	0.7177	0.8444
0.1	0.1	1	0.9	0.7599	0.8444
0.05	0.05	1	0.95	0.8022	0.8444
0	0	1	1	0.8444	0.8444

Figure 5.3: Pareto Optimal Points



Thus we see a plot of Pareto optimal points obtained from the analysis. Then depending on the requirements, any suitable point may be selected as the optimal. This point will be the best trade-off between the weight and efficiency as is required by the particular application.

For example, for the current residential wind turbine problem we see that most of the points are concentrated in the $0.2 \leq W \leq 0.3$ range (above the curve) i.e. $60 \text{ kg} \leq w \leq 90 \text{ kg}$ corresponding to $0.2 \leq \eta \leq 0.4$ and any one may be selected. These η values are ideal for such small scale horizontal axis wind turbines.

5.5 Parametric Study

A parametric study was performed by varying two parameters and noting the behavior of the objective function and weight, w as well as efficiency η of the blades. The parameters that were varied were the minimum power to be generated by the blades (P_{\min}) and the free stream wind velocity, V_0 . P_{\min} would depend on the particular residential requirement and V_0 may be considered as free stream wind velocity at different times. The study is tabulated as follows:

5.5.1 Parameter - P_{\min}

Table 5.6: Parametric study data

P_{\min}	w	W	P	P_{\max}	η	f
1000	44.7959	0.1493	2791.31	6230.93	0.4479	-0.2987
1500	44.7959	0.1493	2791.31	6230.93	0.4479	-0.2987
2000	44.7959	0.1493	2791.31	6230.93	0.4479	-0.2987
2500	44.7985	0.1493	2791.48	6231.18	0.4479	-0.2987
3000	48.145	0.1605	3000	6537.75	0.4589	-0.2984
3500	56.1692	0.1872	3500	7245.35	0.4831	-0.2958
4000	64.1934	0.2139	4000	7919.92	0.5051	-0.2911
4500	72.2175	0.2407	4500	8566.88	0.5253	-0.2846
5000	80.2417	0.2675	5000	9190.25	0.5441	-0.2766
5500	88.2659	0.2942	5500	9793.15	0.5616	-0.2674
6000	96.29	0.3209	6000	10378	0.5781	-0.2572

Figure 5.4: Variation of blade weight with minimum power to be generated

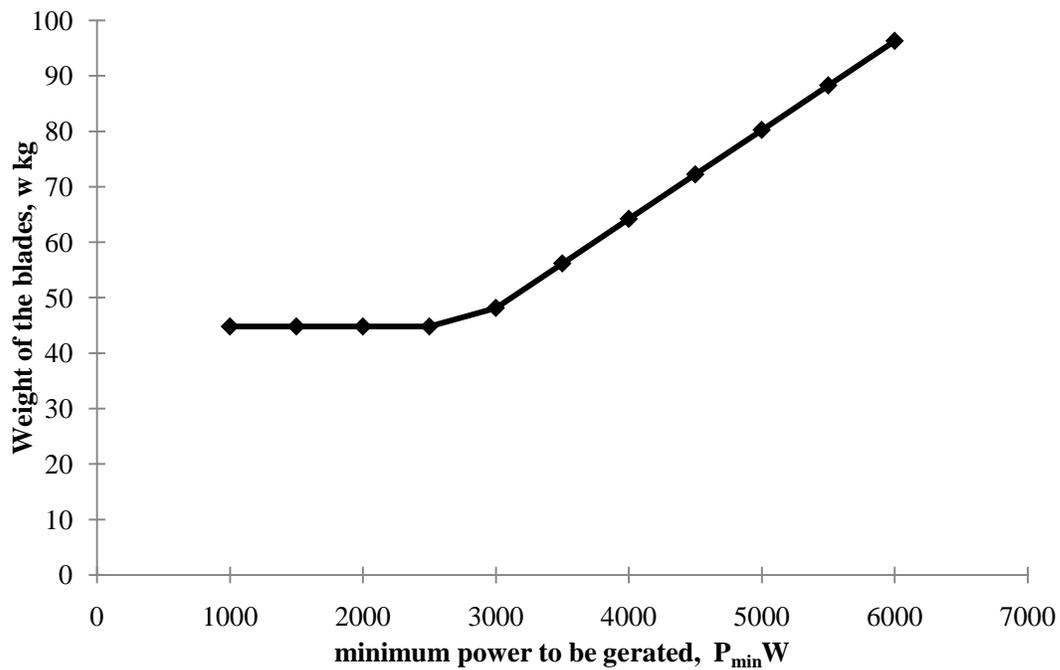


Figure 5.5: Variation of efficiency with minimum power to be generated

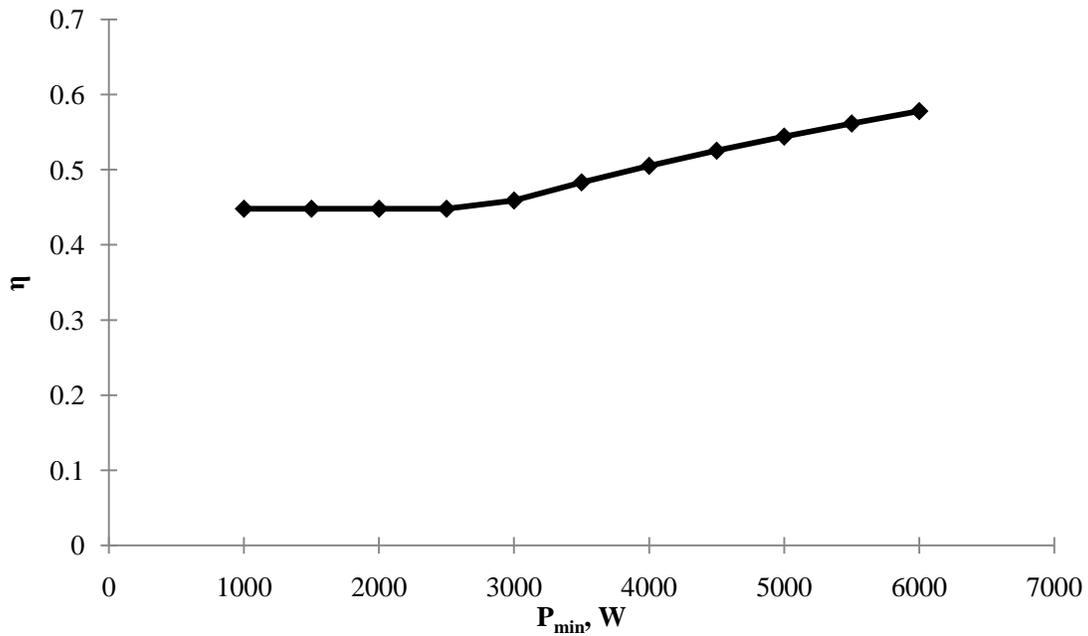
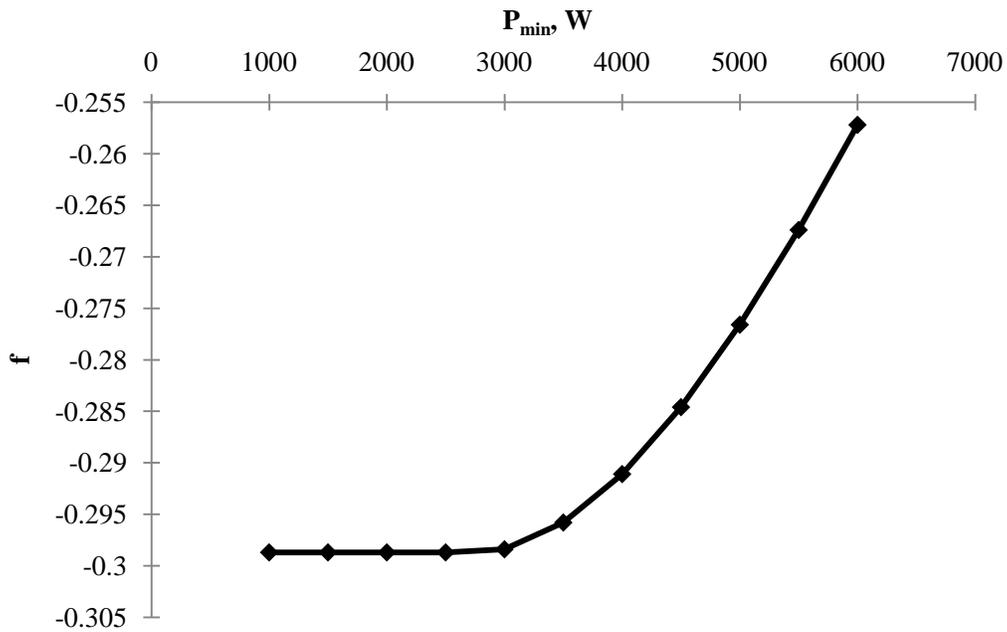


Figure 5.6: Variation in f value with minimum power to be generated



Observations: from the above parametric studies, we see how w , η and f vary with P_{\min} . It is seen that

$$w \sim P_{\min}, \eta \sim P_{\min} \text{ and } f \sim P_{\min}$$

5.5.2 Parameter – V_0

Table 5.7: Parametric study data

V_0	w	W	P	P_{max}	η	f
10	58.873	0.1962	2547.55	4326.41	0.5888	-0.3926
11	51.0566	0.1702	2673.28	5236.77	0.5105	-0.3403
12	44.7959	0.1493	2791.31	6230.93	0.4479	-0.2987
13	39.0664	0.1302	2856.92	7231.3	0.3951	-0.2649
14	39.065	0.1302	3313.23	9031.5	0.3669	-0.2366
15	37.3182	0.1244	3315.71	10137.1	0.3271	-0.2027
16	37.056	0.1235	3693.59	12130.5	0.3045	-0.1809
17	36.741	0.1225	4064.27	14303.7	0.2841	-0.1617
18	39.065	0.1302	5476.97	19195.2	0.2853	-0.1551
19	44.8433	0.1495	8041.23	29747.9	0.2703	-0.1208
20	45.6308	0.1521	9225.64	35925.8	0.2568	-0.1047

Figure 5.7: Variation of blade weight with free stream wind velocity

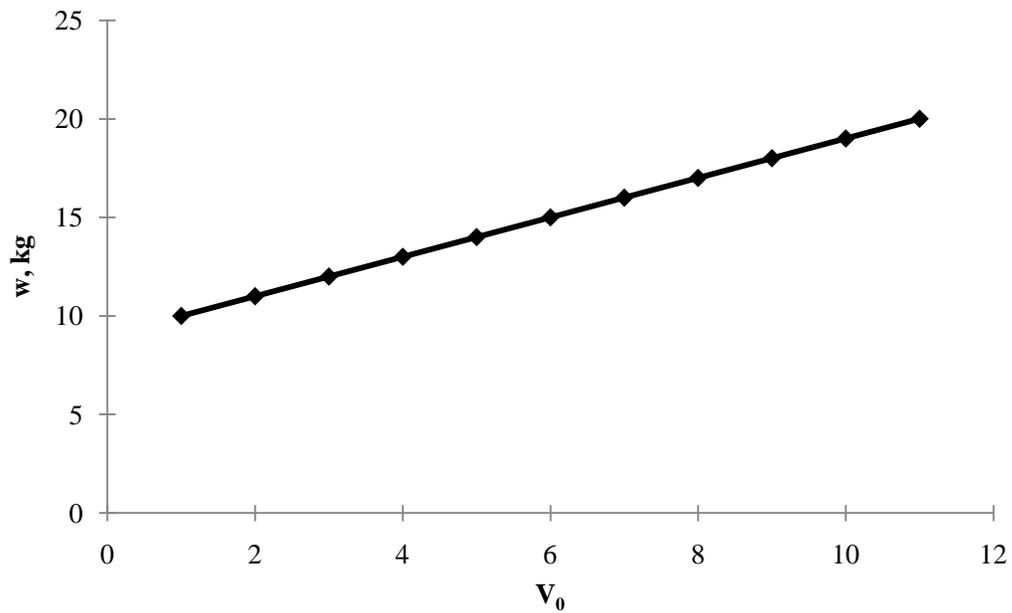


Figure 5.8: Variation of efficiency with free stream wind velocity

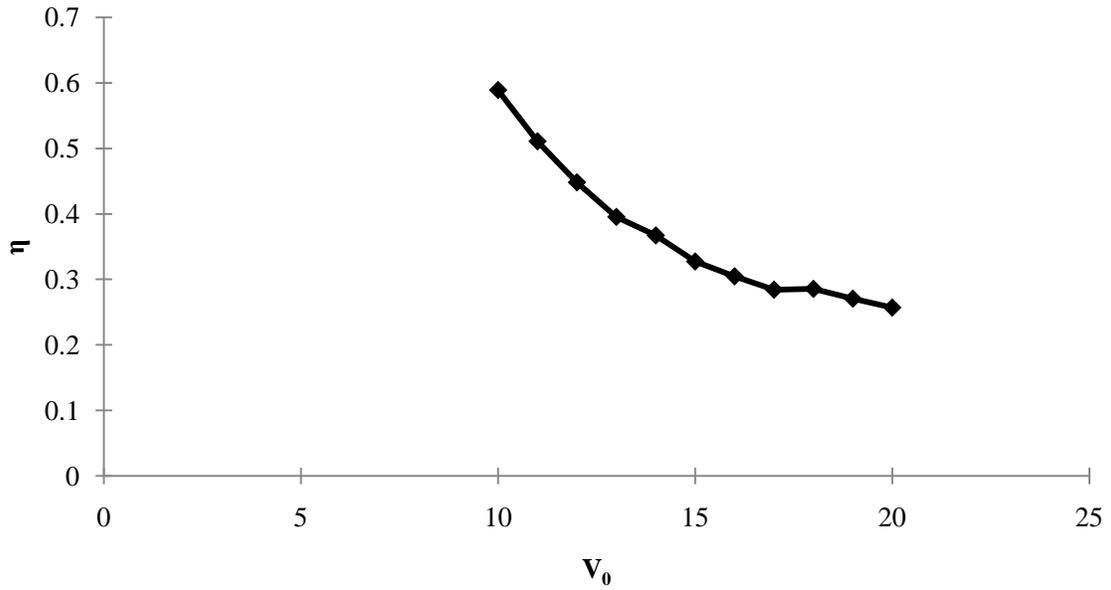
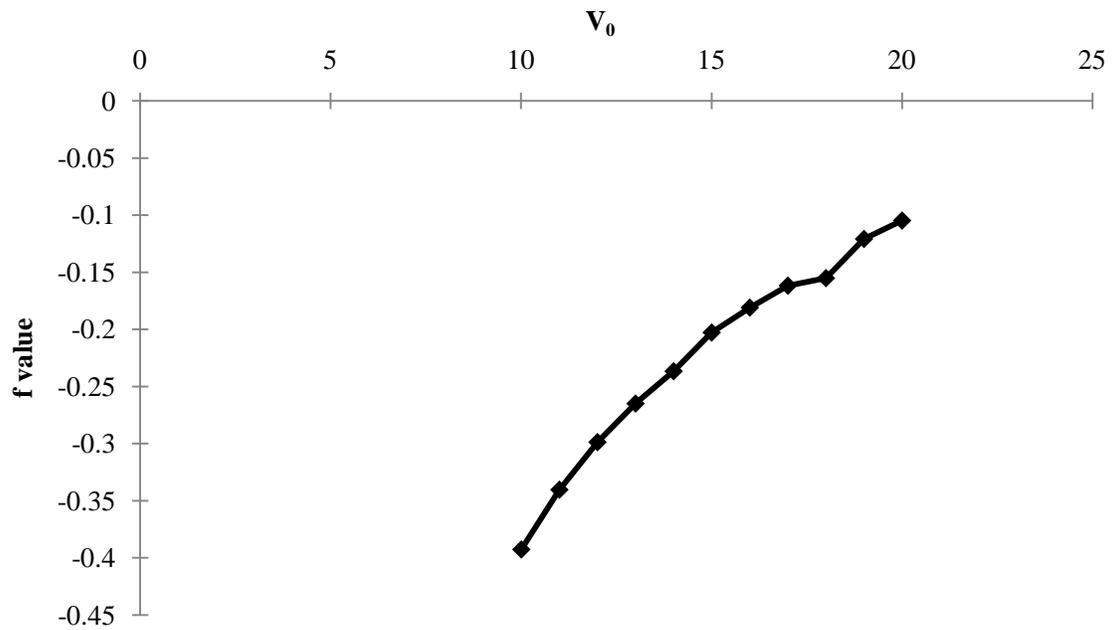


Figure 5.9: variation in f value with free stream wind velocity



Observations: as we can see from the above parametric studies that changing the values of V_0 have a profound effect on w , η and f . We observe that

$$w \sim V_0, \eta \sim \frac{1}{V_0} \text{ and } f \sim V_0$$

Thus the aerodynamic design optimization of the blades of a small scale horizontal axis wind turbine has been performed along with a Pareto analysis of the two competing objective functions and a parametric study by varying P_{\min} and V_0 and checking their effect on weight, efficiency and the overall objective function.

6 POWER GENERATION SYSTEM DESIGN – Kurt Cunningham

This wind turbine will be small scale, meaning that it will most likely have a power output of about 5-15 kW, and will be between 0 and 10 m in height. The zoning laws in Michigan limit the maximum height of an individually owned wind turbine in a residential neighborhood to 35 ft in the state of Michigan, which is approximately 10 m. This is in contrast to a wind-farm scale turbine, whose height can be between 70 and 90 m and will output Megawatts of power at a time. The purpose of the power generation subsystem is to convert the wind power captured by the blades into electrical energy. The power generation system includes a gearbox (from the blade shaft to the generator), an electrical generator, and a rectifier/inverter set that will incur some standard losses in order to sell the generated power back to the grid or use it within a home.

The power generation subsystem needs to be optimized to find the most efficient energy transfer from the blades to the electrical grid, while keeping cost low. This system will minimize the end user payback period by maximizing the electrical power output of the system, thus increasing the cost recuperation rate. As a trade-off, the cost of the generation system must remain low during the optimization in order to keep it affordable. This optimization study will only consider the cost of the generator, so the payback period of this study is not defined as the payback period of the entire wind turbine.

6.1 Nomenclature

$C_{elecgen}$	=	Cost of the generation system, \$
C_{gear}	=	Cost of the materials of the gears, \$
C_{gen}	=	Cost of the whole generation system, \$
C_H	=	Cost of the generation system housing, \$
C_{L1}	=	Coefficient of lift of the base wind turbine blade design
C_{L2}	=	Coefficient of lift of the tip wind turbine blade design

C_{shaft}	=	Cost of the turbine shaft, \$
F_L	=	Lift force provided by a turbine blade, N
E_{tot}	=	Total Energy provided by the generation system, J
G_R	=	Gear ratio of the gearbox
g_{steel}	=	Cost of steel, 1.12 \$/kg
g_{al}	=	Cost of aluminum, 1.57 \$/kg
h	=	Height of the tower, m
k	=	Shape parameter for the Weibull distribution of wind speeds, 2.0
P_{max}	=	Maximum rated power of the generator, kW
R_{LG}	=	Radius of the large gear, m
r_B	=	Radius of the turbine blades, m
T_B	=	Torque output of the turbine blades (distributed with the wind speed dist.), Nm
T_{max}	=	Maximum torque seen by the gearbox, Nm
t_B	=	Thickness of the turbine blades, m
t_G	=	Thickness of the gear set, m
V	=	Velocity of the wind, m/s
V_{avg}	=	Average Velocity of the wind, m/s
$W_{elecgen}$	=	Weight of the generator, kg
W_{gear}	=	Weight of the gearbox, kg
W_H	=	Weight of the generator system housing, kg
W_{shaft}	=	Weight of the turbine shaft, kg
λ	=	Tip-Speed Ratio
λ_s	=	Scaling factor of the wind speed Weibull distribution, m/s
Ω	=	Cost recuperation rate, \$/yr
ρ_{air}	=	Density of air, 1.2 kg/m ³
ρ_{steel}	=	Density of steel, 7805 kg/m ³
ρ_{al}	=	Density of aluminum, 2700 kg/m ³
$\sigma_{max-steel}$	=	Yield Stress of Steel, 1.034×10^9 Pa

6.2 Mathematical Model

This section completely describes the mathematical model used to determine an optimal design for a wind turbine generation system. In order to determine which designs are most optimal, an objective function

must be defined in the context of what would make the payback period for a wind turbine lowest. The objective function for this subsystem optimization problem is shown in Equation 6.1. This equation includes both the cost of the generation system and the cost recuperation rate. In order to minimize f_{gen} , the cost of the generation components must be minimized, while the cost recuperation rate is maximized. By having an objective function of this nature, we are ensuring a minimum contribution to payback period from the generation system design.

$$f_{gen}(P_{max}, G_R, R_{LG}, t_G) = \frac{C_{gen}}{\Omega} \quad [\text{Eq. 6.1}]$$

6.2.1 Calculating the Objective Function

6.2.1.1 Information from the Blade Design

In order to find this objective function, information about what kind of power (torques and speeds) that will be provided by the wind must be known. This information comes from the blade design in the form of a coefficient of lift that the blades exhibit. During the whole system optimization, the coefficient of lift provided by the blades at all wind speeds will be provided by the turbine blade subsystem design. During this optimization, however, the coefficient of lift for the three blade turbine is assumed to be equal to 1. In the blade optimization, the blades have two parts (a base part and a tip part) that could have different coefficients of lift, but for this study, they are the same. The blades are also assumed to have a radius with respect to the central axis (r_B) of 3 meters and a thickness (t_B) of 1 meter. The equation for coefficients of lift as they relate to the force provided by a single blade is shown in Equation 6.2. If one assumes that the force acts halfway along the turbine blades, the torque input into the gearbox at every wind speed can be calculated from Equation 6.3. During the whole system optimization, the length, radius, and coefficients of lift will be outputs of the blade design.

$$C_{L1} + C_{L2} = \frac{F_L}{\frac{1}{2}\rho_a t_B r_B V^2} \quad [\text{Eq. 6.2}]$$

$$T_B = \frac{3}{2} F_L r_B \quad [\text{Eq. 6.3}]$$

Using a constant tip-speed ratio of the blade design, which is the same assumption used in the blade system optimization, the steady-state rotational speed at every wind speed will be given by Equation 6.4.

$$\omega_{ss} = \frac{V\lambda}{r_B} \quad [\text{Eq. 6.4}]$$

6.2.1.2 Design of the Gears

The gearbox is a heavy component of the power generation system of a wind turbine, and the gearbox for use in this application will consist of three pairs of identical gears. Because the wind turns turbine blades slowly (on the order of 1-10 RPM) and generators are made to create power at several thousand RPM, multiple gear reductions are needed, and the gears will be large and heavy due to the large torques that will be transmitted. The cost of the gearbox is directly related to its volume and its weight. Assuming that the gears will be made out of steel ($\rho_{steel} = 7850 \text{ kg/m}^3$, $g_{steel} = 0.875 \text{ \$/kg}$) (CES Edupack, 2009), the equation for the weight of the gearbox is shown in Equation 6.5 and the equation for the cost of the gearbox is shown in Equation 6.6. These equations assume that the gears will be custom machined, and only take into account the cost of materials for the gears themselves. In order to get a more accurate measure of actual cost as it relates to the cost to the consumer, a cost of manufacturing the gearbox will have to be included in the final optimization study.

$$W_{gear} = 3\rho_{steel} \left[R_{LG}^2 + \left(R_{LG} G_R^{1/3} \right)^2 \right] \pi t_G \quad [\text{Eq. 6.5}]$$

$$C_{gear} = g_{steel} W_{gear} \quad [\text{Eq. 6.6}]$$

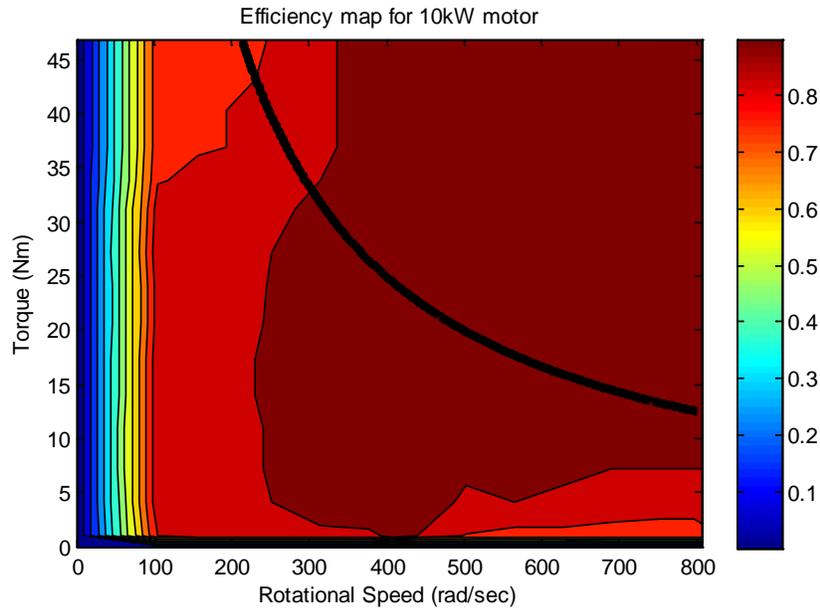
The overall gear ratio of the gearbox is used to calculate the torque and rotational speeds that are applied to the generator. By multiplying the torque provided by the blades by the gear ratio, the torque input into the generator becomes many times smaller. By dividing the rotational speed provided by the blades by the gear ratio, the rotational speed input into the generator becomes many times larger. These characteristics are needed to utilize a typical generator properly.

6.2.1.3 Design of the Generator

Electrical generators can vary in size to accept different ranges of torques and rotational speeds that the gearbox will output. In this design optimization problem, the variable for the size of the generator will be the maximum power output of the generator selected. For each maximum power output, a generator will have a characteristic efficiency map (axes: rad/sec vs. torque). This variation in generator efficiency is based on a linear scaling based on maximum power output of the efficiency points for a 75Kw Unique

Mobility SR218H brushless permanent magnet motor coupled with its companion CA40-300L inverter. A sample efficiency map for the 10 kW scaled version of this generator is shown in Figure 6.1.

Figure 6.1: Efficiency map for a 10kW scaled motor



The input torque and rotational speeds into the motor at each wind speed will be used to look up the efficiency of the generator at each wind speed. Any wind speeds that result in torques and rotational speeds that lie outside of the range of the map or that have a higher power output than the scaled maximum power output variable will have an efficiency of zero. In these cases, the wind turbine will have a lock that will stop the wind turbine from spinning and putting torque on the gearbox. The power that the blades provide is given by multiplying the torque and speed input into the generator. The total electrical power output of the generator at each wind speed is found by multiplying the input power by the efficiency of the generator at that point on the map. This efficiency is multiplied by 0.95 to account for the approximate efficiency of an inverter (needed to sell the energy back to the grid).

The cost of a generator can also be linearly scaled in the same way as the efficiency map using the maximum power output of the generator. The cost of a Unique Mobility SR218H electric generator is \$13,200 at the time of this report, and the weight is 60 kg (Fortin, 2000). See Equation 6.7 for the scaling calculation on cost. The weight of the generator (kg) will be determined in a similar manner when the final optimization is performed (See Equation 6.8).

$$C_{elecgen} = 13200 \frac{P_{max}}{75} \quad [\text{Eq. 6.7}]$$

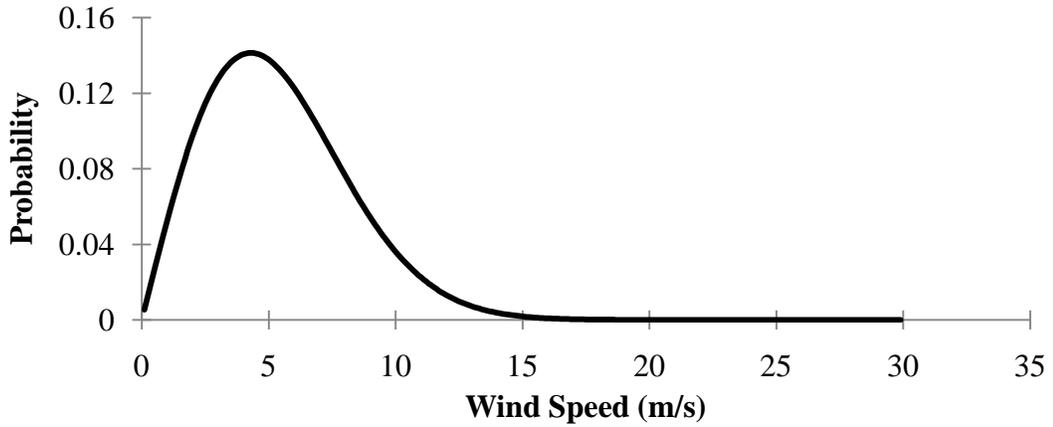
$$W_{elecgen} = 60 \frac{P_{max}}{75} \quad [\text{Eq. 6.8}]$$

6.2.1.4 Wind Speed Probability

Wind speed in a locale can be approximated by a statistical Weibull distribution with a shape factor ($k = 2.0$) [3]. This distribution describes the amount of time during a year that the wind will be blowing at each speed. By setting the Weibull scale factor (λ_s) that is equivalent to the mean wind speed in the area, a distribution can be calculated from Equation 6.9. A sample Weibull distribution for an average wind speed (V_{avg}) of 6.0676 m/s (at $h = 10$ m) is shown in Figure 6.2 below.

$$Prob = \frac{k}{V_{avg}^k} V^{k-1} e^{-\left(\frac{V}{V_{avg}}\right)^k} \quad [\text{Eq. 6.9}]$$

Figure 6.2: Weibull distribution of wind speed with a mean of 6.0676 m/s



6.2.1.5 Cost Recuperation Rate

Each torque/speed combination is given a probability for the amount of time during each year the blades provide that torque and speed. This probability (converted to number of hours in a year) is multiplied by the electrical power output at each wind speed to get the total energy generated as a function of wind speed. In order to get the total energy generated in a single year (in kWh), this function is then integrated over all wind speeds. This amount of energy can be directly converted to kWh, and, assuming an energy cost in cents per kWh, the cost recuperation rate can be calculated from Equation 6.10 (where E_{tot} is the

total energy in kWh each year). The cost recuperation rate is in units of \$/yr. The cost of each kWh on average in MI is 10.43 cents/kWh (State of Nebraska, 2009) at the time of this report.

$$\Omega = 0.1043 * E_{tot} \quad [\text{Eq. 6.10}]$$

6.2.1.6 Generation System Housing

The wind turbine needs a housing to sit on top of the tower. The generator system housing is modeled as a hollow cylinder that envelops the gearbox and the generator. The clearance around the large gear is 0.025 m on all sides, and the thickness of the walls is 0.0025 m. The length of the cylinder is 3 times the thickness of the gears plus an extra 1.25 m for the generator and some clearance room. The calculation for the weight of the housing is shown in Equation 6.11, and the calculation for the cost of the housing is shown in Equation 6.12.

$$W_H = \pi^2 \rho_{al} (3t_G + 1.25) \left[(0.055 + R_{LG} + R_{LG} GR^{1/3})^2 - (0.05 + R_{LG} + R_{LG} GR^{1/3})^2 \right] + 0.005\pi^2 (0.055 + R_{LG} + R_{LG} GR^{1/3})^2 \quad [\text{Eq. 6.11}]$$

$$C_H = g_{steel} W_H \quad [\text{Eq. 6.12}]$$

6.2.1.7 Turbine Shaft

There must be a shaft that connects the turbine blades to the gearbox and the gearbox to the generator. These shafts will be approximated as a function on the radius of the small gear in the gearbox. This shaft is modeled as in Equation 6.13. The cost of the shaft is shown in Equation 6.14.

$$W_{shaft} = 0.25 \rho_{steel} R_{LG} G_R^{1/3} \quad [\text{Eq. 6.13}]$$

$$C_{shaft} = g_{steel} W_{shaft} \quad [\text{Eq. 6.14}]$$

6.2.1.8 Payback Period

The final form for the mathematical objective for the power generation system is shown below in Equation 6.15. This function is the nominal “payback period” of the power generation system that will be referred to throughout this paper. The additional \$2000 added to the total cost is an assumed fixed

installation cost related to a full turbine system plus manufacturing cost related to the power generation system.

$$f_{gen}(P_{max}, G_R, R_{LG}, t_G) = \frac{C_{gear} + C_{elecgen} + C_{shaft} + C_H + 2000}{\Omega} \quad [\text{Eq. 6.15}]$$

6.2.2 Constraints

The maximum power of the generator will be constrained to practical limitations. As a first approximation, the maximum power must be greater than zero and must not be greater than the highest known wind turbine power output, which is about 6 MW. To narrow the high end of this range, the linearity of the scaling of the generator maps should be limited to a reasonable limit, which could be placed at approximately 3 times the original power, or 225 kW. A single family home, however, would not need that much power, so a high limit of 100 kW was settled upon. The lower limit for the generator power is nominally zero, but because this will cause problems in the calculation steps, the lower limit was set to 0.1 kW.

The gear ratio will be limited by practicality as well. The gear ratio cannot be less than 1:1 (the generator gear must spin at least as fast as the turbine gear), but it also should not be more than 100:1 for a single gear. This is a rule-of thumb constraint, because gear sets with a ratio greater than 100:1 are not commonly seen because of their difficulty to manufacture (Hau, 2006). Because there are 3 gear reductions, the total gear ratio can be higher than this. For ease of implementation, the highest end of the total gear ratio was set to 10000:1 and the low end of the total gear ratio was set to 1:1. This corresponds to single gear ratios of 21.54:1 on the high end and 1:1 on the low end.

The radius of the large gear must be greater than zero and should be less than the height of the tower (for obvious practical reasons). Intuitively, the radius of the large gear should be able to fit into the top of the turbine tower, so the radius should be less than about 1 m. The lower bound constraint was set to 0.025 m so that the radius of the small gear cannot get below 1 mm.

The constraints on the thickness of the gear are more complicated. As an upper limit, the thickness of the gear should be less than 0.5 m, so that the gear sets can reasonably fit into the top of the turbine tower. The lower limit on the thickness of the gear is nominally zero, but a small positive thickness (0.005 m) is applied as a lower limit to avoid calculation problems. The lower limit, however, must also be based on

the maximum torque input from the blades. The gears must not fail under the most extreme loading that the gears will see. The smaller gears in the gearbox will fail first, and will fail according to Equation 6.16.

$$\sigma_{max-steel} = \frac{32T_{max}}{t_G R_{LG} G_R^{1/3}} \quad \text{[Eq. 6.16]}$$

This equation is based on assumptions regarding the size of the tooth in relation to the radius of the small gear. In this equation, a single gear tooth is failing in bending while holding all of the torque that the wind is providing. This equation should then be an overestimate of the stress on the tooth. The maximum torque that the gearbox will see is a function of the generator size as well. As the generator increases in size, the efficiency map will be larger, so a larger torque will be seen by the gearbox before the lock is applied to stop the spinning of the turbine. Comparing equation 6.16 to the strength of steel will give us a nonlinear constraint.

The constraints will be scaled when implemented into MATLAB: The maximum power generation is scaled down by 100; the gear ratio remains un-scaled; the radius of the large gear remains un-scaled; the thickness of the gears is scaled up by 2; the nonlinear constraint is scaled by the yield strength of steel. After this scaling, the range of each of the constraints is from about 0 to 1, so they will each have to same weighting when implemented into MATLAB.

6.2.3 Design Variables and Parameters

The parameters for this optimization problem are numerous. In order to calculate the generator power output, one must assume that the design of the blades is known. The blade design parameters that directly affect the generation system design are the chord (or thickness) of the blades, the span (or radius) of the blades, the tip-speed ratio of the blades, and the coefficient of lift of the blades. Other parameters include the cost and density of the aluminum and steel alloys chosen, the number of gear reductions (3), and the Weibull shape factor ($k = 2$). Finally, an approximate cost of manufacturing the gearbox, housing, and shaft (\$1000) and an approximate cost for installation (\$1000) were included as parameters.

The most important parameter in this optimization problem is the height ($h = 10\text{m}$). The scaling factor for the wind speed Weibull distribution (a parameter - $\lambda_s = 6.0676$) is dependent on the average speed of the air at the height of the tower. The torque delivered by the blades to the input of the gearbox is dependent

on the design of the wind turbine blades, which will be dependent on wind speed distribution, and thus dependent on the height.

The variables of this optimization are given in the objective function. The maximum power of the generator (P_{max}) determines the size and cost of that component, and is sized simultaneously with the gear ratio in order to obtain the most efficient energy transfer from the blades to the grid. The gear ratio (G_R) is an important variable because it determines the weight of the generation system and is greatly variable for maximum power transfer efficiency. The radius of the large gear (R_{LG}) determines the weight and cost of the gearbox, and partially determines the needed thickness for the gears to avoid failure. The thickness of the gears (t_G) is the final variable, and it also determines the weight and cost of the gearbox should always be minimized while allowing for the survival of the gears under the loading from the blades. The lower constraint on the thickness of the gear should always be active. Thus, this optimization should have three degrees of freedom.

6.2.4 Summary Model

Minimize:

$$f_{gen}(P_{max}, G_R, R_{LG}, t_G) = \frac{C_{gear} + C_{elecgen} + C_{shaft} + C_H + 2000}{\Omega} \quad [\text{Eq. 6.17}]$$

$$C_{gear} = 3g_{steel} \rho_{steel} \left[R_{LG}^2 + (R_{LG} G_R^{1/3})^2 \right] \pi t_G \quad [\text{Eq. 6.18}]$$

$$C_{elecgen} = 13200 \frac{P_{max}}{75} \quad [\text{Eq. 6.19}]$$

$$C_H = \pi^2 \rho_{al} g_{al} (3t_G + 1.25) \left[(0.055 + R_{LG} + R_{LG} G_R^{1/3})^2 - (0.05 + R_{LG} + R_{LG} G_R^{1/3})^2 \right] + 0.005 \pi^2 (0.055 + R_{LG} + R_{LG} G_R^{1/3})^2 \quad [\text{Eq. 6.20}]$$

$$C_{shaft} = 0.25 \rho_{steel} g_{steel} R_{LG} G_R^{1/3} \quad [\text{Eq. 6.21}]$$

Subject to:

$$\Omega = 0.1043 * E_{tot} \quad [\text{Eq. 6.22}]$$

$$g_{gen1} = 0.001 - P_{max} \leq 0 \quad [\text{Eq. 6.23}]$$

$$g_{gen2} = P_{max} - 100 \leq 0 \quad [\text{Eq. 6.24}]$$

$$g_{gen3} = 0.0001 - G_R \leq 0 \quad [\text{Eq. 6.25}]$$

$$g_{gen4} = G_R - 0.01 \leq 0 \quad [\text{Eq. 6.26}]$$

$$g_{gen5} = 0.025 - R_{LG} \leq 0 \quad [\text{Eq. 6.27}]$$

$$g_{gen6} = R_{LG} - 1 \leq 0 \quad [\text{Eq. 6.28}]$$

$$g_{gen7} = 0.005 - t_G \leq 0 \quad [\text{Eq. 6.29}]$$

$$g_{gen8} = t_G - 0.5 \leq 0 \quad [\text{Eq. 6.30}]$$

$$g_{gen9} = \frac{32T_{max}}{t_G R_{LG} G_R^{1/3}} - \sigma_{max-steel} \leq 0 \quad [\text{Eq. 6.31}]$$

6.2.5 MATLAB Implementation

The MATLAB implementation of the previously described optimization model begins with a main script to initialize, run, and report the results of an optimization calculation. This script is shown in Appendix D. This script first defines a start point and scales the variables. These scaled variables are what the optimizer (fmincon) manipulates to enter into the objective script and nonlinear constraint script, and what it compared to the upper and lower bound constraints. The lower and upper bound constraints are then set according to the previous section in their scaled forms. The options for convergence criteria are then set: The maximum number of function evaluations is set to 10000; the maximum number of iterations is set to 5000; the constraint tolerance is set to 1×10^{-12} ; the function tolerance is set to 1×10^{-12} ; the variable tolerance is set to 1×10^{-12} . All other options remain at the default values. The optimization is then performed (calling PowerGenCalculation.m and PowerGenConstraints.m) and the results are sent to the MATLAB command window. In addition, as shown in Appendix D, there is a script for running the derivative-free algorithm DIRECT. Because the generator data has many local minima, the objective function can be noisy, so the global optima may not be possible to achieve using fmincon.

Appendix E shows the PowerGenCalculation.m script, which contains the objective calculation steps as described in section 6.1. Appendix F shows the PowerGenConstraints.m script, which contains only the calculation for the ninth constraint (which is nonlinear) as described in section 6.2. Appendix G shows a script file called PowerGenStructuralOutput.m that shows the calculations for the weight of the system that will be used later to output into the structural design during the full system optimization.

6.3 Model Analysis

6.3.1 Monotonicity Analysis

By analyzing the monotonicity of the objective function and the constraints, one can examine whether the problem is well-bounded, and which (if any) constraints will be active. Equations 6.17 to 6.22 on the

previous page show the objective function and the corresponding component costs and cost recuperation rate. The linear constraints are shown in Equations 6.23 to 6.30, and the nonlinear constraint is shown in Equation 6.31. Table 6.1 summarizes the monotonicity analysis as presented in this section.

Table 6.1: Monotonicity analysis table

	P_{max}	$G_R > 0$	$R_{LG} > 0$	$t_G > 0$
C_{gear}	N/A	Monotonic (+)	Monotonic (+)	Monotonic (+)
$C_{elecgen}$	Monotonic (+)	N/A	N/A	N/A
C_H	N/A	N/A	Monotonic (+)	Monotonic (+)
C_{shaft}	N/A	Monotonic (+)	Monotonic (+)	N/A
Ω	Non-Monotonic	Non-Monotonic	N/A	N/A
f_{gen}	Non-Monotonic	Non-Monotonic	Monotonic (+)	Monotonic (+)
g_{gen1}	Monotonic (-)	N/A	N/A	N/A
g_{gen2}	Monotonic (+)	N/A	N/A	N/A
g_{gen3}	N/A	Monotonic (-)	N/A	N/A
g_{gen4}	N/A	Monotonic (+)	N/A	N/A
g_{gen5}	N/A	N/A	Monotonic (-)	N/A
g_{gen6}	N/A	N/A	Monotonic (+)	N/A
g_{gen7}	N/A	N/A	N/A	Monotonic (-)
g_{gen8}	N/A	N/A	N/A	Monotonic (+)
g_{gen9}	Monotonic (+)	Monotonic (-)	Monotonic (-)	Monotonic (-)

The costs of the various components are monotonic in the positive sense for all variables included in those equations. The linear constraints (Equations 6.23 to 6.30) are each upper and lower bounds for each variable, and thus are monotonic in the positive or negative sense to each variable.

The cost recuperation rate (Ω) requires a more complicated analysis of its behavior. By examining Figure 6.1, one can show the non-monotonicity of the gear ratio on the cost recuperation rate. When the gear ratio is smallest, the characteristics of the power entering into the generator will follow a linear line at a very shallow slope (will follow a line to high speeds at relatively low torques). If the slope is very shallow, the power output of the generator will be very low, as the efficiency of the generator is low in those spots. When the gear ratio is largest, the characteristics of the power entering into the generator will

follow a linear line at a very steep slope (will follow a line to high torques at relatively low speeds). If the slope is very steep, the power output of the generator will also be very low, as the efficiency of the generator is also low in those spots. Therefore there should be an optimal trajectory that the gear ratio should find to maximize the cost recuperation rate and therefore minimize the payback period. However, this data is experimental and also includes multiple local minima, so the final optimization may be noisy.

The maximum power of the generator is also non-monotonic with the cost recuperation rate. If P_{max} is too high, the wind speeds that have the highest probability will provide power to the generator near the origin, which is at low efficiency. If P_{max} is too low, however, the wind speeds that have the highest probability will provide power to the generator outside of the range of the generator's map, so therefore the system will be locked in place during those times, not producing any power. Therefore there should be an optimal P_{max} that places the highest probability wind speeds in the 90% efficiency range, thus maximizing the cost recuperation rate and minimizing the payback period.

The nonlinear constraint (Equation 6.28) is has a more complicated monotonicity then the other constraints. The stress on the gear is monotonic in the negative sense for the thickness of the gear, the radius of the large gear, and the gear ratio. The stress on the gear is monotonic in the positive sense for P_{max} . As P_{max} increases, larger wind speeds, and thus larger torques (T_{max}), will be introduced into the system due to the locking mechanism. As P_{max} increases, the efficiency map will get larger, so the wind turbine will accept larger torques before the locking mechanism is applied.

In summary, according to table 6.1, the following can be observed. First, because the objective function is non-monotonic in relation to both the maximum power of the generator and the gear ratio, I do not expect constraints 1 through 4 to be active in any case. Second, because both the maximum power of the generator and the gear ratio are non-monotonic, and they both are bounded above and below, the problem is well-bounded in these variables. Third, because the radius of the large gear and the thickness of the gears are monotonically increasing in the objective function, I do not expect constraints 6 and 8 to be active, but I can expect a combination of constraints 5, 7, and 9 to be active. In most cases, I expect the ninth constraint to be active for both the thickness of the gear and the large gear radius. Because of the noisiness of the generator map, these constraints may not be met using a derivative-based algorithm. Finally, because the radius of the large gear and the thickness of the gears are monotonically increasing in the objective function, and they are monotonically decreasing in constraints 5, 7, and 9, they are both well-bounded variables. This optimization problem, therefore, is well bounded in all variables according to monotonicity principles 1 and 2.

6.3.2 Redundancy of Constraints

In a few cases, some of the constraints are redundant or unnecessary. The sixth and eighth constraints are unnecessary because the objective function is monotonically increasing with respect to the radius of the large gear and the thickness of the gears and these constraints also monotonically increase with respect to these variables. These constraints are included in the optimization formulation in order to have a full set of upper and lower bounds as inputs into the `fmincon` argument. The second, third, fifth, and seventh constraints can be redundant with respect to the ninth constraints in some cases, but are not fully redundant, so are included in the optimization formulation.

6.4 Optimization Study

The optimization problem presented in section 6.2.4 and analyzed in section 6.3 was performed a number of times to find the most optimal generator system design to minimize the payback period of that system. This section introduces the results of these optimization studies.

The optimization of the wind turbine generation system was performed using a constant height parameter of 10 m (near the high end of the allowed range). The optimization is formulated as in section 5 and implemented using the MATLAB code shown in the appendices. Because of the possibility of multiple local minima, the optimization was started at a variety of start points. There were five start points used in an attempt to examine the entire range of start points. This is not a full factorial design of experiments, but it is sufficient for the needs of this study. The start points are listed in Table 6.2 below.

Table 6.2: Start points of the optimization

Start Point	P_{max}	G_R	R_{LG}	t_G
1	50	0.5	0.5	0.2475
2	100	1	1	0.5
3	0.001	0.0001	0.025	0.005
4	100	1	0.025	0.005
5	0.001	0.0001	1	0.5

In only two cases were any constraints active. This confirms the hypothesis that the generator data has multiple local minima, so the objective function is noisy. At the end of each optimization, the message from `fmincon` was:

“Local minimum possible. Constraints satisfied. Fmincon stopped because the predicted change in the objective function is less than the selected value of the function tolerance and constraints were satisfied to within the selected value of the constraint tolerance.”

Table 6.3 below shows the local minima found from each start point, as well as the number of iterations and objective function value (Payback Period, in years).

Table 6.3: Optimization results for each start point

Start Point	P_{max}	G_R	R_{LG}	t_G	Iterations	Payback Period
1	4.54	0.040	0.025	0.472	41	3.16
2	6.70	0.025	0.025	0.362	33	2.54
3	1.16	0.025	0.879	0.005	26	18.39
4	3.21	0.031	0.086	0.102	25	4.29
5	99.90	0.587	0.025	0.009	6	84.86

From Table 6.3, it can be seen that the objective function is too noisy to use a derivative-based algorithm alone. In order to account for this, DIRECT was used (at 1000 function evaluations) to find a derivative-free optimal point. By using DIRECT, the solution is a result of searching the entire design space. After using DIRECT, fmincon was used at the derivative-free optimum to find a new optimum, which is assumed to be the global constrained optimum at a height of 10 m. The results of this study are shown in Table 6.4.

Table 6.4: Optimization results after applying DIRECT

Solver	P_{max}	G_R	R_{LG}	t_G	Iterations	Payback Period
fmincon	12.18	0.030	0.025	0.005	79	2.22
DIRECT	12.18	0.030	0.025	0.005	17	2.21

6.5 Parametric Study

One important interaction between the generator system and the structural and blade designs is the tower height parameter. All three of these designs are assumed to be a function of the height of the tower, as the characteristic wind speeds change as a function of height. This section aims to examine the effect of tower

height on the optimal generator system design. For tower heights of 4 m, 6 m, 8 m, and 10 m, the DIRECT-to-fmincon optimization procedure like that presented in section 6.4.1 was implemented to find a minimum for the payback period as a function for the generator system variables. Figure 6.3 shows the resultant minimum of the objective function as a function of the height of the tower. Figures 6.4 through 6.7 show the resultant values for the minimum for the maximum generator power, gear ratio, large gear radius, and gear thickness, respectively. Figure 6.3 show that, in general, increasing the tower height will decrease the payback period for the generator system. Table 6.5 shows these results in a table format.

Figure 6.3: Effect of tower height on Payback Period

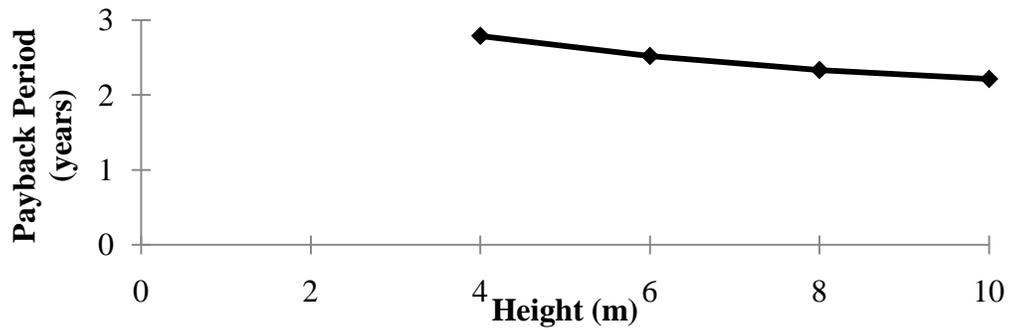


Figure 6.4: Effect of tower height on Optimal Generator Maximum Power

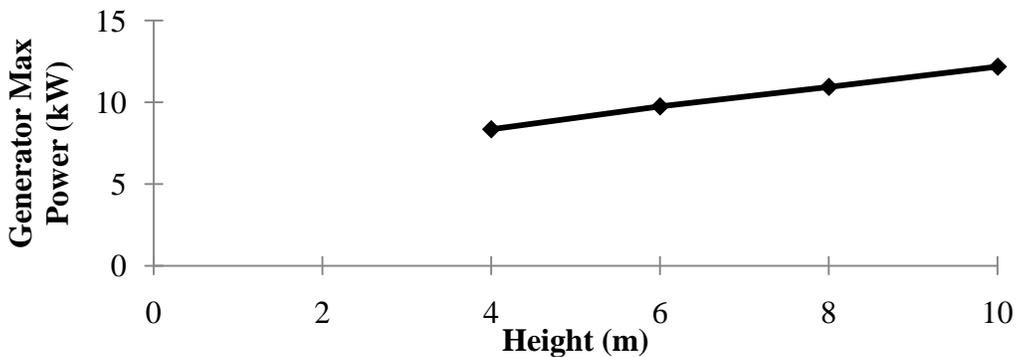


Figure 6.5: Effect of tower height on Optimal Gear Ratio

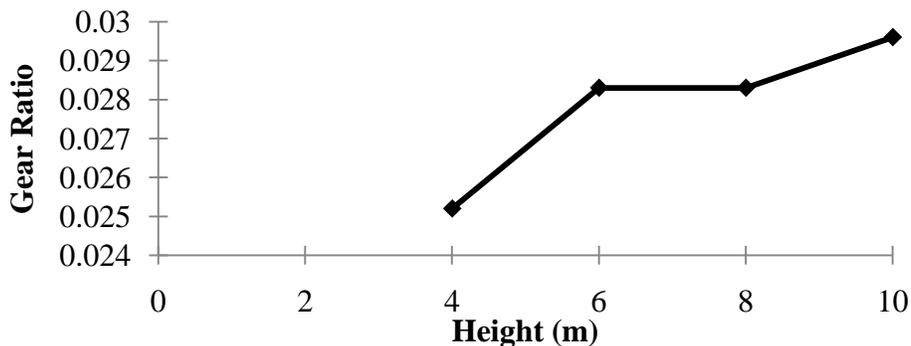


Figure 6.6: Effect of tower height on Optimal Large Gear Radius

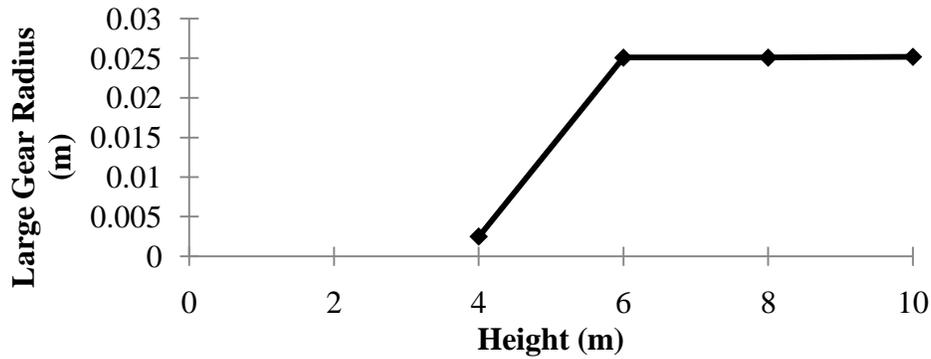


Figure 6.7: Effect of tower height on Optimal Gear Thickness

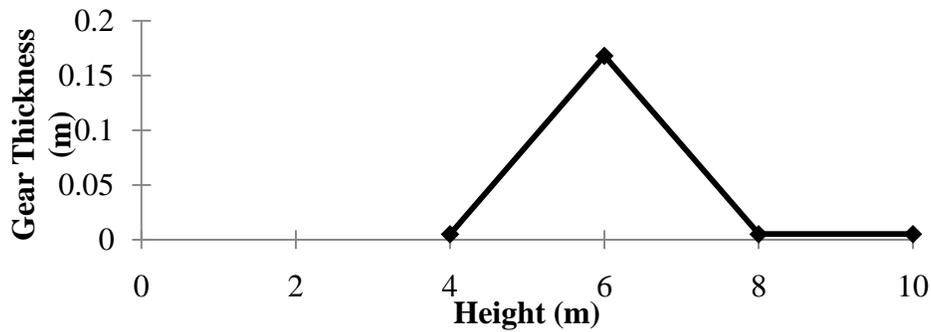


Table 6.5: Optimization results for the parametric study

Tower Height	Payback Period	P_{max}	G_R	R_{LG}	t_G
4	2.79	8.35	0.025	0.025	0.005
6	2.52	9.75	0.028	0.025	0.168
8	2.33	10.94	0.028	0.025	0.005
10	2.21	12.18	0.030	0.025	0.005

6.6 Discussion of Results

The results of the optimization study show that a payback period of 2.2 years can be achieved for a generation specific payback period. The max generator power is about 12 kW, the gear ratio is 33.8:1, the radius of the large gear is 2.5 cm, and the thickness of the gear is 5 mm. The lower constraints on the large gear radius and the gear thickness are satisfied. The activity of these two constraints is consistent with the monotonicity analysis for this subsystem. This model is nonlinear and has multiple local minima, so I do not expect the full system analysis to have active constraints in the constraint set of this sub problem.

The payback period in this section neglected the cost of the structure and of the blades of the turbine. Each of these designs will include tower height (equivalent to wind speed) as a parameter, but the overall optimization study will have tower height as a variable. There is an interesting design tradeoff in the full optimization problem due to the addition of height as a variable. A higher tower size will be desirable for the power generation system, in order to maximize the cost regeneration rate. A higher tower height will also be optimal for the blade design, because a higher height means higher wind speeds, which result in a higher cost recuperation rate that will minimize the payback period. In contrast, it is clear that the structural design will want to minimize the height of the tower in order to reduce materials cost (higher heights should result in a larger tower)

From the parametric study, it is clear that this system will attempt to maximize the height of the full wind turbine system. If the generator cost is dominant over the structural or blade costs, this result should be achieved. In addition, the cost recuperation rate is an important factor in the overall optimization, and will be maximized with higher heights. Because this is so important to the end system objective, I expect that the full system optimization will maximize the height of the system.

7 STRUCTURAL SUPPORT TOWER DESIGN – Kevin Schaefer

For the optimization of the structural subsystem of the wind turbine, a free standing tower design was considered. The major design tradeoff considered during wind turbine structure design is that with increasing height comes instability and cost, but also access to higher quality wind energy, and thus efficiency. In turn, the optimization objective for this subsystem was to minimize the volume of material used, and thus the cost, while still providing a structurally sound tower. The specific tower design that was considered was a hollow cylinder, having unique inner and outer radii at both the top and bottom of the structure. The radii were constrained to vary only linearly with respect to the height of the tower.

7.1 Nomenclature

A	=	Projected two-dimensional area of tower, m^2
C_{d-t}	=	Drag coefficient of tower
d_t	=	Average outside diameter of tower, m
E	=	Modulus of elasticity of tower material, Pa
F_{cr}	=	Critical buckling force of tower, N

F_d	=	Maximum design resultant drag force of wind on tower, N
f_K	=	Shedding frequency of Kármán vortices, Hz
f_r	=	Principle resonant frequency of tower, Hz
F_s	=	Factor of safety
g	=	Gravitational constant, 9.81 m/s ²
g_{al}	=	Cost of 6061-T4 Aluminum, 1.57 \$/kg
h	=	Height of tower, m
I_t	=	Area moment of inertia of tower, m ⁴
K	=	Column effective length factor for buckling
L_{rotor}	=	Length of rotors, m
m_{rotor}	=	Mass of rotor subsystem, kg
m_{gen}	=	Mass of power generation subsystem, kg
P	=	Poisson's Ratio of tower material
R_1	=	Inner radius of tower base, meters
R_2	=	Outer radius of tower base, meters
R_3	=	Outer radius of tower top, meters
R_4	=	Inner radius of tower top, meters
Re	=	Reynolds Number
R_{max}	=	Maximum allowable tower radius, m
R_{min}	=	Minimum allowable tower radius, m
T_{rotor}	=	Maximum design thrust from rotor, N
t_{min}	=	Minimum wall thickness of tower, mm
u_{avg}	=	Average rotor tip tangential velocity, m/s
V	=	Wind velocity, m/s
V_{avg}	=	Average wind velocity, m/s
V_{max}	=	Maximum design wind velocity, m/s
λ	=	Tip-speed Ratio
ρ_{tower}	=	Density of tower material, kg/m ³
ρ_{air}	=	Density of air, 1.225 kg/m ³
σ_{Yield}	=	Yield strength of material, Pa
ν	=	Kinematic viscosity of air, 2.076 x 10 ⁻⁷ m ² /s
ω_{avg}	=	Angular velocity of average rotor operation, rad/s

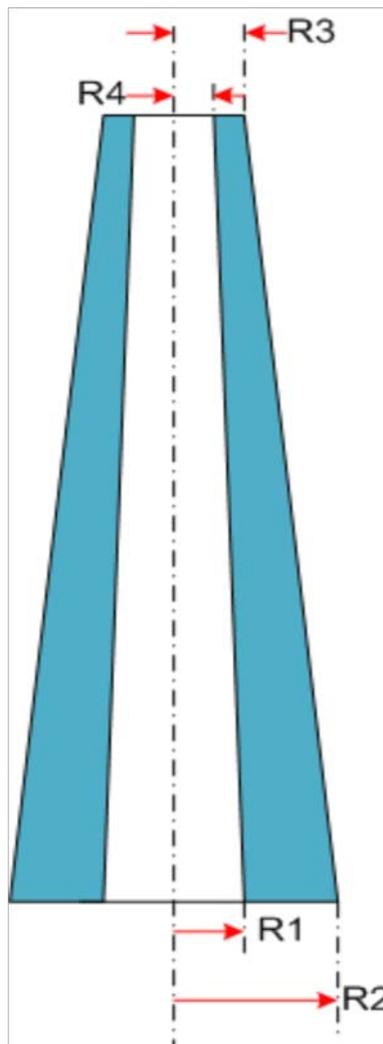
7.2 Mathematical Model

7.2.1 Objective Function

The objective function used in the optimization of the subsystem was the material volume of the tower. The material volume of the tower can be evaluated by subtracting the volume of a conic section based on the inner radii of the tower, from the volume of the conic section based on the outer radii of the tower. The volume of a conic section is determined by the major and minor radii, and the height of the section, and is shown below in Equation 7.1.

$$\text{Volume of Conic} = \frac{\pi h}{3} (R_{\text{major}}^2 + R_{\text{major}} R_{\text{minor}} + R_{\text{minor}}^2) \quad [\text{Eq. 7.1}]$$

Figure 7.1: Conceptual Diagram of Tower Subsystem



The volume of the structural subsystem was then determined by subtracting the volume of a conic section calculated using the inner radii, R_1 and R_4 from that which is calculated using the outer radii, R_2 and R_3 , and is shown below in Equation 7.2.

$$\text{Tower Volume} = \frac{\pi h}{3} (R_2^2 + R_3^2 - R_1^2 - R_4^2 + R_2 R_3 - R_1 R_4) \quad [\text{Eq. 7.2}]$$

During the higher level system optimization, the volume of the tower will be directly correlated to a raw material cost, and incorporated into the calculation of total system cost and potential payback period.

7.2.2 Constraints

Constraints on the design of the structural subsystem included the allowable stress in the tower, the allowable buckling load, and natural frequency constraints with respect to both the frequency of the rotor blades and with respect to the shedding frequency of Karman vortices behind the tower. Geometric constraints including minimum and maximum radii, minimum wall thickness, and tower shape were also considered.

7.2.2.1 Maximum Stress, g_{tl}

One of the largest loads that any wind turbine tower is subjected to, is the thrust generated by its own rotor. This force acts in the direction of the wind velocity along the axis of rotation of the rotors, creating a large bending moment and leading to high stresses, specifically at the base of the tower.

The second major force taken into consideration during the design of the structural subsystem was the weight of the power generation system, including both generator and gear set, and the weight of the turbine blades. The magnitude of the weight of these components can vary greatly based upon their own designs loads, and thus play an important role in the design of the structural support subsystem. During the optimization of the tower, the loads were assumed to be distributed evenly on the upper surface of the tower, and centered on its vertical axis. Values for the weight of other components integral to any wind turbine design, such as the nacelle and yaw system, were not made as the mass of these components were considered negligible compared to those of the turbine blades and generation system. The mass of both the power generation system and turbine blade system will be an input variable to the structural subsystem when the higher order system is optimized, however, nominal values were assumed for the structural subsystem optimization.

Another loading condition considered during was the drag force resulting from the air flow around the tower. To calculate this force the tower was approximated as a regularly shaped cylinder. The drag force is given by Equation 7.3.

$$F_d = \frac{1}{2} \rho_{\text{air}} V_{\text{max}}^2 A C_{d-t} \quad [\text{Eq. 7.3}]$$

Where: $A = h(R_2 + R_3)$ [Eq. 7.3.1]

And: $C_{d-t} = 0.2$ for a cylinder [Eq. 7.3.2]

The determination of the resulting stress in the tower was carried out via finite element analysis in ANSYS, a three dimensional modeling and simulation software package. During the simulations, the tower was considered to be rigidly fixed at its base, as the footing support system or ground conditions were not included in the scope of the optimization. The resultant stresses of all combined loads determined via the simulation, were required to be less than the yield strength of the selected material divided by a safety factor.

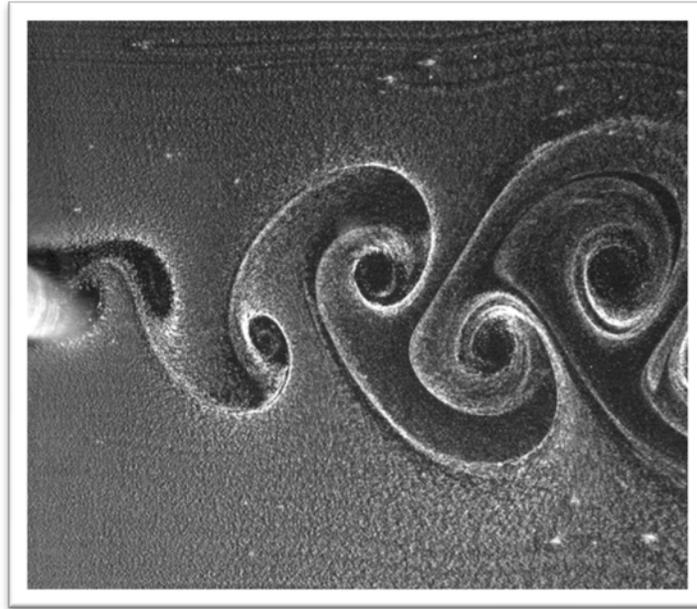
7.2.2.2 Rotor Resonance, g_{12}

The angular velocity of the wind turbine rotor causes cyclically alternating forces felt throughout the entire wind turbine. To reduce the possibility of resonance on the structural subsystem, the first natural frequency of the tower was required to be higher than the excitation frequency of the turbine blades, (rotational frequency divided by the number of blades). The natural frequencies of the tower were also determined in ANSYS, via modal analysis.

7.2.2.3 Vortex Resonance, g_{13}

Similar to the rotor resonance constraint, the vortex resonance constraint required that the primary natural frequency of the tower subsystem be higher than the frequency of the alternating wake vortices, also known as Karman vortices. Karman vortices result from fluid flow around a body in the fluid stream, and create alternating low pressure zones on either side of the tower. Should the primary natural frequency of the around which the fluid is flowing coincide with that of the vortices, resonance could occur and lead to extremely large stress and deformations. Thus, the primary natural frequency of the tower was required to be greater than the vortex frequency created by the maximum wind velocity, and thus highest frequency of the Karman vortices.

Figure 7.2: Karman Vortices behind a Cylindrical Bluff Body



(<http://www.aps.org/units/dfd/pressroom/gallery/kumar.cfm>)

Credit: Sanjay Kumar and George Laughlin; Department of Engineering, University of Texas, Brownsville.

To calculate the frequency of the wake vortices, the tower was approximated as a regular cylinder, using its average diameter. The frequency of the wake vortices is given by Equation 7.4 (Wikipedia, 2010).

$$F_K = \frac{V_{\max}}{d_t} 0.198 \left(1 - \frac{19.7}{Re} \right) \quad [\text{Eq. 7.4}]$$

Where: $Re = \frac{V_{\max} d_t}{\nu}$ [Eq. 7.4.1]

And: (4.2) $d_t = \frac{(2R_2 + 2R_3)}{2}$ [Eq. 7.4.2]

7.2.2.4 Column Buckling, g_{14}

The weight of the turbine and power generation subsystems creates an axial load on the tower subsystem. As the length to diameter ratio of the tower was expected to be greater than 50, design considerations were made to account for column, or Euler, buckling. The axial load resulting from the power generation and turbine subsystem weights was constrained to be less than the critical load defined in Equation 7.5 (Efunda, 2010). The moment of inertia was approximated by using the average radii at the top and bottom of the tower.

$$F_{cr} = \frac{\pi^2 EI_t}{(KL)^2} \quad [\text{Eq. 7.5}]$$

Where: $I_t = \frac{\pi}{4} \left[\left(\frac{R_2 + R_3}{2} \right)^4 + \left(\frac{R_1 + R_4}{2} \right)^4 \right]$ [Eq. 7.5.1]

And: $K = 2.0$ for a column fixed at one end [Eq. 7.5.2]

7.2.2.5 Minimum Radius at Tower Bottom, g_{15}

To ensure that the tower is a hollow cylinder, a minimum radius constraint was imposed on the inner radius at the bottom of the tower.

7.2.2.6 Minimum Wall Thickness at Tower Bottom, g_{16}

To ensure that the wall thickness of the tower did not decrease below a nominal value, a minimum wall thickness constraint was imposed on the inner and outer radii at the bottom of the tower. If the stress, frequency, or buckling constraints had been inactive with respect to the inner radius at the bottom of the tower, R_1 , the problem would still have been properly bounded below by the minimum radius constraint and above by R_2 via the minimum wall thickness constraint.

7.2.2.7 Maximum Radius, g_{17}

The outer radius of the tower was bound by imposing a maximum radius constraint at the bottom of the tower. Should no other constraints have been active with respect to the outer radius at the bottom of the tower, R_2 , this variable would have been bounded below by the minimum wall thickness constraint and above by the maximum radius constraint.

7.2.2.8 Minimum Radius at Tower Top, g_{18}

Similar to g_{15} , a minimum radius constraint was imposed on the inner radius at the top of the tower.

7.2.2.9 Minimum Wall Thickness at Tower Top, g_{19}

Similar to g_{16} , a minimum wall thickness constraint was imposed on the inner and outer radii at the top of the tower. Specifying separate constraints for the thickness at either end of the tower resulted in the

potential for the wall thickness to change as a function of the tower height. Should no other constraints have been active with respect to the inner radius at the top of the tower, R_4 , the radius would have been bounded below by the minimum radius constraint and above by R_3 via the minimum wall thickness constraint.

7.2.2.10 Cylinder/Taper, g_{10}

The cylinder/taper constraint limited the outer radius at the top of the tower from being larger than the outer radius at the bottom of the tower. This constraint created a proper bound above R_3 , which was bound below by g_{16} .

7.2.3 Design Variables and Parameters

7.2.3.1 Parameter Selection

Parameters included the tower material (this dictated material properties such as yield strength, Poisson's Ratio, etc.), safety factor, maximum and average wind velocities, rotor length, tip-speed ratio and efficiency of the turbine blade design, loading conditions due to the weight of turbine and power generation systems on the top of the tower, and aerodynamic and bending loads from the thrust of the turbine.

The material of construction was chosen to be aluminum. Aluminum was considered because of the high strength to weight ratio and relatively low cost, which is congruent to the higher level system objective of minimizing the payback period of the entire wind turbine system. Although typical safety factors employed in wind turbine structural design range from 1.10 to 1.35 (Hau, 2006), the safety factor used for the model was 2. A higher than nominal safety factor was used to compensate for the relatively simplistic load case employed, and a lack of attention given to fatigue loading and design. The safety factor and nominal values of wrought 6061-T4 aluminum material properties are listed below (Granta, 2009).

Safety Factor, F_s	=	2
Young's Modulus, E	=	6.8×10^{10} Pa
Poisson's Ratio, P	=	0.330
Density, ρ_{tower}	=	2700 kg/m^3
Yield Stress, σ_{yeild}	=	1.03×10^8 Pa

For the design of the structural subsystem, the choice of the tower height was of the utmost importance. From the selection of the height, the wind characteristics, and thus aerodynamic loading conditions were derived. During the integration of the three subsystems, height will be a variable, however, an initial tower height was assumed. The most common wind turbine height restriction in residential zoning areas is 35 feet (Forsythe, 2007), so a height of 10 meters will be used.

Wind data for the area of interest was obtained from the Michigan Department of Energy, Labor, and Economic Growth (michigan.gov/dleg, 2010). Using a Weibull wind-speed probability distribution with a shape factor of two and a scaling factor of 7.75 (Hau, 2006), the 99% percentile wind speed was calculated as 13.021 m/s. This wind velocity was used as V_{\max} for the optimization of the structural subsystem.

Similarly, the average wind velocity, V_{avg} , at the assumed height of the tower was calculated as 6.0676 m/s. The calculated average wind velocity was correlated to an average rotor tip tangential velocity, u_{avg} , via the tip-speed ratio, λ , as shown below in Equation 7.6 (Hau, 2006).

$$\lambda = \frac{u_{\text{avg}}}{V_{\text{avg}}} \quad \text{[Eq. 7.6]}$$

High efficiency three bladed-rotor designs typically have a tip-speed ratio of 7 (Hau, 2006). With this information, the average angular velocity of the rotor was calculated with Equation 7.7.

$$\omega_{\text{avg}} = \frac{u_{\text{avg}}}{L_{\text{rotor}}} \quad \text{[Eq. 7.7]}$$

Using an assumed rotor length, L_{rotor} , of one-half the tower height, the following parameters were defined:

Maximum velocity, V_{\max}	=	13.021 m/s
Average wind velocity, V_{avg}	=	6.0676 m/s
Tip-speed ratio, λ	=	7
Average tip velocity, u_{avg}	=	42.4732 m/s
Rotor Length, L_{rotor}	=	5 m
Average angular velocity, ω_{avg}	=	8.49464 rad/s

For the overall system optimization, the thrust generated will be an output of the turbine blade subsystem. However, during the optimization of the structural subsystem, the thrust was estimated by the Rankine-Froude Actuator Disc Theory (Eggleston, 1987). Assuming that the efficiency of the turbine blades is the ideal 59.3% derived by Betz (Hau, 2006), the thrust on the rotor was found via Equation 7.8.

$$T_{\text{rotor}} = \left(\frac{4}{9}\right) \rho_{\text{air}} V_{\text{max}}^2 A_{\text{swept}} \text{ where } A_{\text{swept}} = \pi L_{\text{rotor}}^2 \quad [\text{Eq. 7.8}]$$

$$T_{\text{rotor}} = 7249.902 \text{ N}$$

The remaining parameters for the completion of the structural subsystem model were the mass of the rotor and power generation subsystems, the minimum and maximum radii, and the minimum wall thickness. The mass of the rotor and power generations subsystems will be outputs of their respective optimization programs, but were selected as 900 and 500 kilograms, respectively. The minimum and maximum radii were not desired, nor expected, to be active constraints, and thus were consequently given values outside of the expected design range. The minimum wall thickness was selected to be the thickness of 8 gauge sheet metal.

m_{rotor}	=	900 kg
m_{gen}	=	500 kg
R_{max}	=	0.5 m
R_{min}	=	0.002 m
t_{min}	=	3.2639 mm

7.2.3.2 Design Variables

The formal design variables were chosen as the inner and outer radii at both the top and bottom of the support tower.

R_1	=	Inner radius, bottom of tower
R_2	=	Outer radius, bottom of tower
R_3	=	Outer radius, top of tower
R_4	=	Inner radius, top of tower

7.2.4 Summary Model

Minimize:

$$\text{Volume}(R_1, R_2, R_3, R_4,) = \frac{\pi h}{3} (R_2^2 + R_3^2 - R_1^2 - R_4^2 + R_2 R_3 - R_1 R_4) \quad [\text{Eq. 7.9}]$$

Subject to:

$$g_{t1} = \sigma_{\text{act}}(h, R_1, R_2, R_3, R_4, E, P, m_{\text{rotor}}, m_{\text{gen}}, T_{\text{rotor}}, F_D) - \frac{\sigma_{\text{yeild}}}{F_s} \leq 0 \quad [\text{Eq. 7.10}]$$

$$g_{t2} = \frac{3\omega_{ss}}{2\pi} - f_r(h, R_1, R_2, R_3, R_4, E, P, \rho_{\text{tower}}) \leq 0 \quad [\text{Eq. 7.11}]$$

$$g_{t3} = \frac{0.198V_{\text{max}}}{d_t} \left(1 - \frac{19.7}{Re}\right) - f_r(h, R_1, R_2, R_3, R_4, E, P, \rho_{\text{tower}}) \leq 0 \quad [\text{Eq. 7.12}]$$

$$g_{t4} = (m_{\text{rotor}} + m_{\text{gen}})g - \frac{\pi^2 E I_t}{(Kh)^2} \leq 0 \quad [\text{Eq. 7.13}]$$

$$g_{t5} = R_{\text{min}} - R_1 \leq 0 \quad [\text{Eq. 7.14}]$$

$$g_{t6} = R_1 + t_{\text{min}} - R_2 \leq 0 \quad [\text{Eq. 7.15}]$$

$$g_{t7} = R_2 - R_{\text{max}} \leq 0 \quad [\text{Eq. 7.16}]$$

$$g_{t8} = R_{\text{min}} - R_4 \leq 0 \quad [\text{Eq. 7.17}]$$

$$g_{t9} = R_4 + t_{\text{min}} - R_3 \leq 0 \quad [\text{Eq. 7.18}]$$

$$g_{t10} = R_3 - R_2 \leq 0 \quad [\text{Eq. 7.19}]$$

7.3 Model Analysis

7.3.1 Monotonicity Analysis

The results of a monotonicity analysis of the objective and constraint functions with respect to each design variable are shown below in Table 7.1.

The monotonicity of constraint functions g_{t2} and g_{t3} (modal frequency constraints) was not known, as they relied on simulation results as opposed to analytical expressions. Similarly, constraint function g_{t1} (stress constraint) also relied on simulation results, however, its monotonicity was estimated based on the premise that adding volume to the tower will decrease stress, and vice versa.

Table 7.1: Monotonicity Analysis

Function	Design Variables			
	R1	R2	R3	R4
Objective	-	+	+	-
g_{t1} (estimated)	+	-	-	+
g_{t2}	?	?	?	?
g_{t3}	?	?	?	?
g_{t4}	-	-	-	-
g_{t5}	-	n/a	n/a	n/a
g_{t6}	+	-	n/a	n/a
g_{t7}	n/a	+	n/a	n/a
g_{t8}	n/a	n/a	n/a	-
g_{t9}	n/a	n/a	n/a	+
g_{t10}	n/a	-	+	n/a

7.3.2 Constraint Activity

As shown in the monotonicity analysis table above, each design variable had at a minimum, one constraint with opposite monotonicity of the objective function. Even if the estimated monotonic behavior in constraint function g_{t1} had not been considered, the design variables would have been properly bounded by the geometric constraint functions (g_{t4} through g_{t10}). Including the assumed monotonicity of constraint function g_{t1} , each design variable had more than one constraint function with opposite monotonicity of the objective function. Consequently, none of the constraints were critical to any design variable, and none of the constraint functions could be “optimized out” with any degree of confidence.

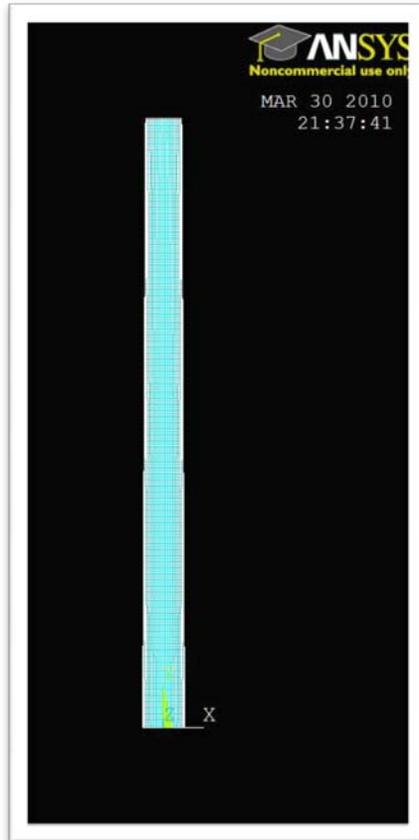
7.4 Optimization Study

7.4.1 Finite Element Analysis Model

ANSYS, a computer based modeling and simulation software package, was used to generate a three dimensional finite element model of the wind turbine support tower. The Solid45 element was used to mesh the solid model, which allowed for analysis of both mechanical stress and modal frequencies. The

model was generated with Mechanical APDL (ANSYS Parametric Design Language), thus allowing the model to be executed in a batch run mode, effectively reducing computation time. The code is viewable in Appendix H. An example of the meshed solid model is shown in Figure 7.3.

Figure 7.3: ANSYS Solid Model



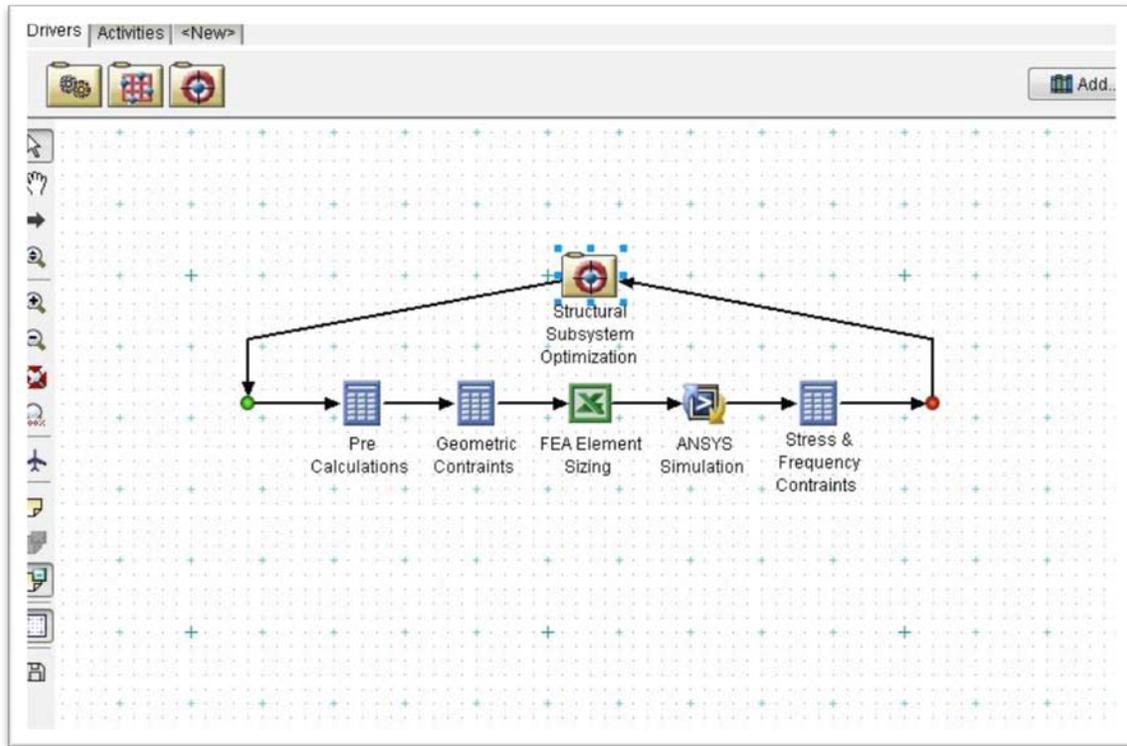
7.4.2 ISight Model

ISight was used to create an optimization program for the structural subsystem. The optimization protocol that ISight was designed to follow is roughly outlined below:

- Start with an initial design point (input by the user) for the tower and calculate the geometric constraints
- Parse design information into the APDL input file and run the simulation in batch mode
- Retrieve stress and frequency results from the simulation output file
- Calculate remaining constraint functions and evaluate results for feasibility, descent, convergence, etc.

NLPQL, a local gradient based sequential quadratic programming algorithm within ISight, was used as the optimization algorithm to iteratively choose the subsequent values of the design variables. The ISight Design Gateway for the structural subsystem is shown below in Figure 7.4.

Figure 7.4: ISight Optimization Design Space



During initial runs of the ISight optimization program, ANSYS computation time was observed to have an extreme sensitivity to changes in the design variables. This was traced to a disproportionate increase in the number of finite elements upon a change in any design variable that caused an increase in the volume of the tower. To combat the sensitivity of the model to changes in geometry, the size specification of the finite element within ANSYS was included in the model as a dependant variable. A simple “if/else” loop was created in Excel to cap the increase in the number of elements in the solid model. If the greatest tower wall thickness of any design iteration was higher than a user defined value, the Excel file output an element size that created only a nominal amount of element divisions in the tower wall.

7.4.3 Results and Initial Point Sensitivity

The optimum volume of the turbine support tower for the defined conditions was 31857 cubic centimeters, leading to a mass of approximately 86 kilograms. The ISight output for this run is available in Appendix I. The optimum volume was achieved with design variable values as follows:

R_1	=	29.51 cm
R_2	=	29.84 cm
R_3	=	1.64 cm
R_4	=	1.31cm

During the optimization run, the constraints that were observed to be active were the maximum stress constraint, **g1**, and the Karman frequency constraint, **g3**. This hints to some degree of viability for the model and to the results of the optimization, as a tapered/conic design is more stable and less prone to vibration, and offers the most support at the base, where the bending moment from the rotor thrust would create the highest stress levels.

It was also observed that the minimum thickness constraint possessed some degree of activity. This makes the choice of the minimum thickness a very important decision. However, smaller values of wall thickness were not considered, as no criteria was included in the constraint equations for dent and puncture resistance. Intuitively, the optimal tower design would have an increasingly larger radius at its base with a decreasing minimum thickness value, thus creating an undesirable footprint for the tower.

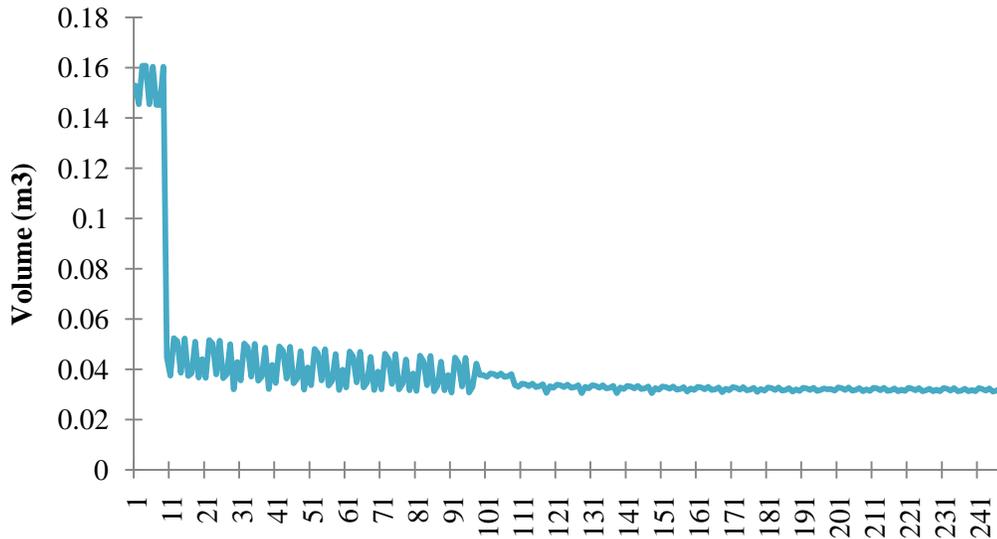
Four starting design points were input into ISight to initiate the optimization procedure from distinct locations within the feasible design space. The starting design points and the results of their simulations are located below in Table 7.2. Trial runs #5 and #6 were required do to the high number of iterations needed for convergence, and are simply continuations of the fourth initial design point (i.e. their respective starting points were the ending points of the previous iterations).

Although different algorithmic step sizes and convergence criterion were tested, the optimization algorithm needed a large number of design iterations to successfully converge at a solution. This is evident in the termination reason for four out of the six trial runs. Interestingly, trial #3 resulted in a local minimum, with the geometry of a regular cylinder. Figure 7.5, below, shows the design volume versus iteration for runs 4, 5, and 6.

Table 7.2: ISight Optimization Results Summary

Trial #	Starting Point (centimeters)				Termination Reason	Ending Point (centimeters)				Objective Value (cm ³)
	R1	R2	R3	R4		R1	R2	R3	R4	
1	0.20	0.53	0.53	0.20	Max # of Iterations	25.80	26.91	1.38	1.06	72062
2	49.50	50.00	50.00	49.50	Max # of Iterations	28.25	28.68	10.46	10.14	46612
3	0.20	50.00	50.00	0.20	Optimality Satisfied	31.98	32.31	32.31	31.98	65702
4	24.00	25.00	25.00	24.00	Max # of Iterations	29.50	29.82	7.40	7.08	37710
5	29.50	29.82	7.40	7.08	Max # of Iterations	29.59	29.91	1.83	1.50	32082
6	29.59	29.91	1.83	1.50	Optimality Satisfied	29.51	29.84	1.64	1.31	31857

Figure 7.5: Volume vs. Design Iteration



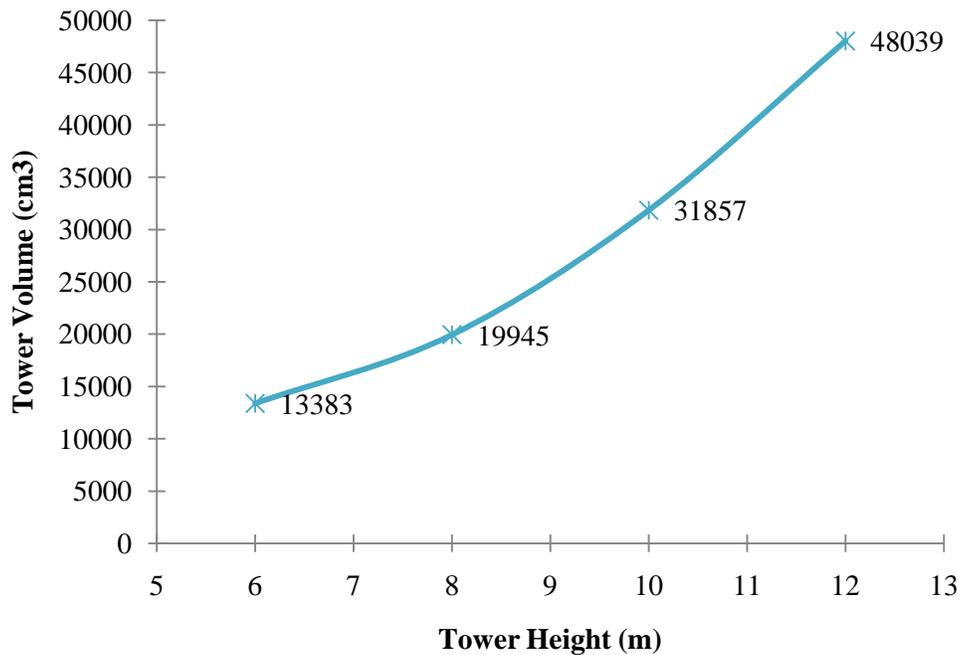
7.5 Parametric Study

A parametric study was performed by varying the height of the wind turbine structural support tower. Three additional tower heights were considered: 6 meters, 8 meters, and 12 meters. The converged results of the parametric study are shown below in Table 7.3. As expected, an increase in tower height resulted in an increase in volume; however, the volume did not vary linearly with the height as shown in Figure 7.6.

Table 7.3: **Parametric Study Results**

Height	R1 (cm)	R2 (cm)	R3 (cm)	R4 (cm)	Volume (cm ³)
6	16.59	17.01	1.51	1.17	13383
8	23.09	23.41	1.38	1.06	19945
10	29.51	29.84	1.64	1.31	31857
12	35.78	36.13	1.62	1.29	48039

Figure 7.6: Parametric Study of Tower Height vs. Tower Volume



7.6 Discussion of Results

The optimum values for the design variables were successfully obtained using the optimization algorithm within ISight. However, the values of many of the parameters used during this subsystem optimization were based on assumptions that will change when integrated into the higher level system optimization. For example, component masses from other subsystems will change with each design iteration, as will values such as the angular velocity and thrust created by the rotor. Thus the subsystem optimization is essentially a proof of concept for one third of the higher order system.

The largest tradeoff that will be addressed by the completion of the total system optimization is that with increased tower height, higher wind energy becomes available, but initial cost of the system will increase

due to higher loads and stresses on the tower. The goal of the overall system optimization will be to minimize the payback period for the wind turbine system. This will properly address the interactions between all variables of each subsystem to obtain the best overall design.

8 SYSTEM INTEGRATION STUDY

8.1 Objective

The overall system objective was to minimize the payback period of the entire wind turbine system, which is the total of the individual subsystem costs divided by the cost recuperation rate. Manufacturing costs associated with the blade and structural subsystems were estimated as constants, and are included in the overall system objective function shown below. Manufacturing and installation costs are also included in the generation system cost.

$$PP = \frac{(Volume_{tower} \rho_{tower} g_{al} + 500) + (w_{blades} g_{blade} + 500) + (C_{gen})}{\Omega} \quad [\text{Eq. 9.1}]$$

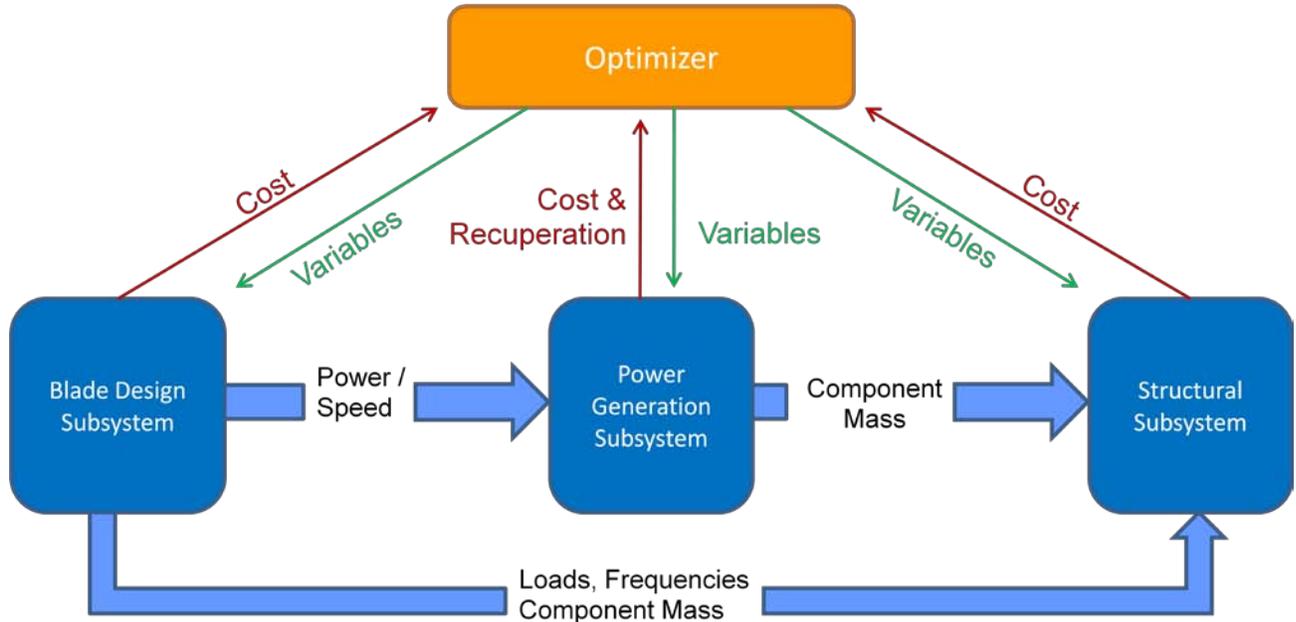
8.2 System Tradeoffs

The major tradeoff of the total system design is that with an increasing height came an increase in the amount of power generated, but also an increase in cost. The interaction of height with payback period was taken into account during the total system optimization because, unlike the subsystem optimizations, height of the wind turbine tower was treated as a variable during the overall system optimization. Before the start of the optimization, it was not clear whether the cost of the tower subsystem (higher at taller heights) would dominate the height optimization or whether the cost recuperation rate would dominate (larger at taller heights).

In addition to this major tradeoff, there is also the issue of subsystem interaction. Figure 9.1 shows the flow of information in the full optimization problem. It can be seen from this figure that the structural subsystem is directly affected by the mass of the power generation subsystem and the thrust load, blade frequencies, and component mass of the blade design subsystem. In addition, the power generation subsystem is affected by the power and speed that the blade subsystem can provide. When all of these

interactions are taken into account, it is expected that the optimizer will have more information to come up with a better result than the individual subsystem optimizations can provide.

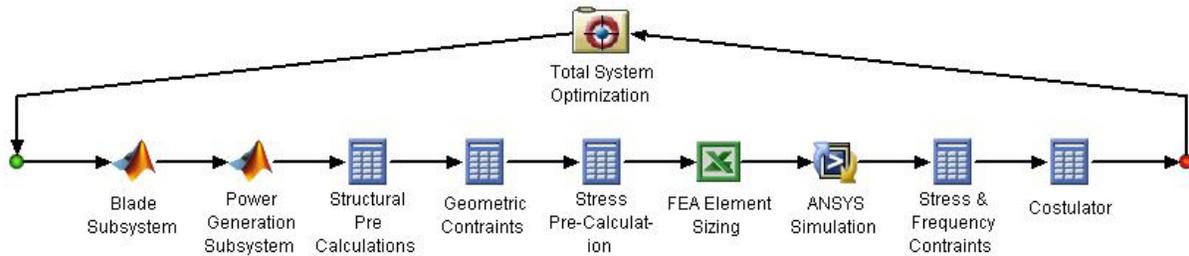
Figure 9.1: Data Flow of Total System Optimization



8.3 Optimization Study

The three systems were integrated using ISight, which allowed for a smooth flow of information from one subsystem to the other. The optimization algorithm used was NLPQL, a sequential quadratic programming algorithm within ISight. The information flow between the subsystems and to and from the ISight optimizer is shown in Figure 9.1. The ISight design space is shown below in Figure 9.2.

Figure 9.2: ISight Design Gateway



8.4 Results

The initial design points used for the total system integration were the optimal designs of the three distinct subsystems. Each subsystem was optimized with reduced payback period in mind, but the parameters in each subsystem that were dependant on the designs of the other subsystem were not all consistent. For example, the chord and span of the optimized blade design was much smaller than that used as parameters in the power generation design. Although the designs were inconsistent, the individual optimization results were used as an initial starting point, which gave an initial payback period of over 163 years.

As stated above, height was included in the optimization as a design variable, and was set to an initial value of ten meters. After 837 design iterations with a termination criterion of 0.01 years, an optimal design was obtained which had a payback period of 6.19 years. The optimal variable values are shown below in Table 9.1 along with the initial start point. The results for a full system optimization were better because the flow of information from one subsystem to the others was not allowed in the individual subsystem optimizations.

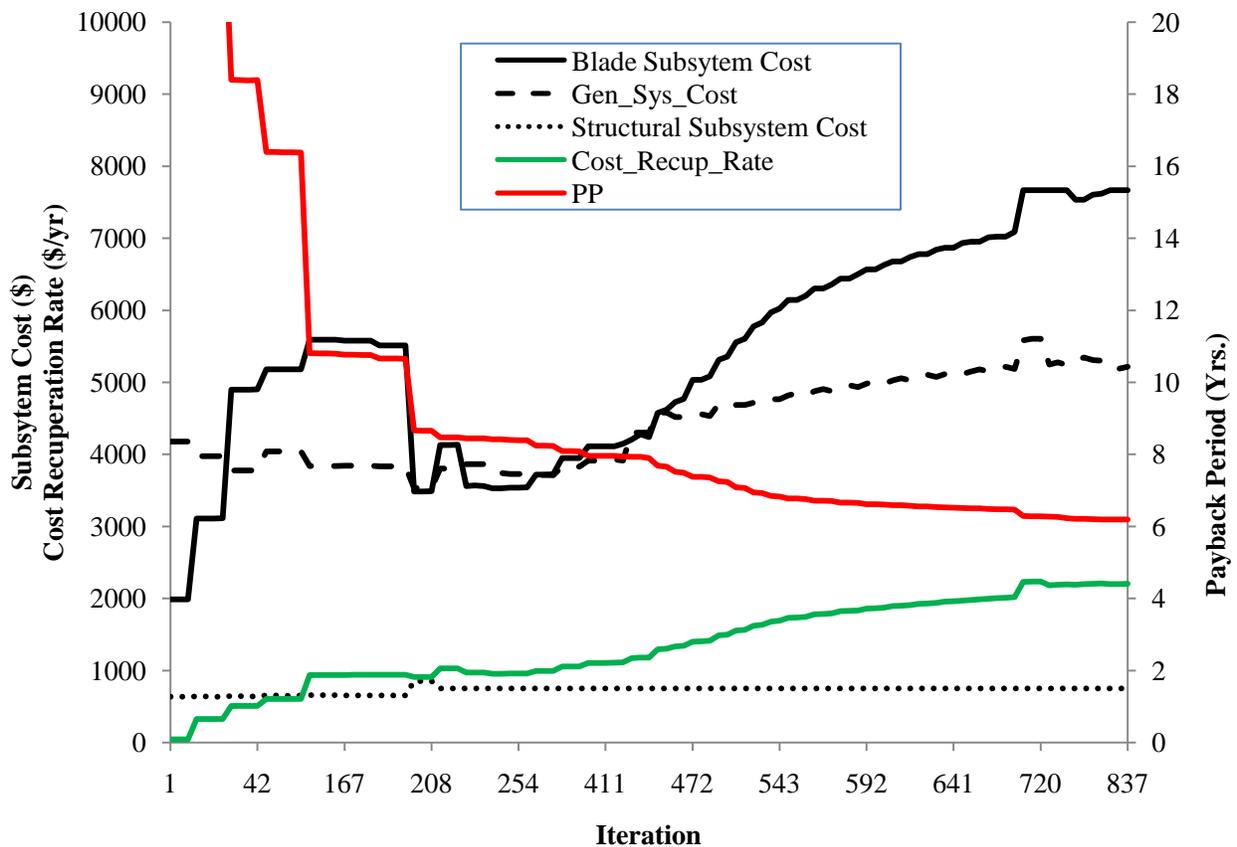
Table 9.1: Total System Optimization Results

Subsystem	Variable	Initial Design Point	Optimal Design
Blade Design	Angle of Attack (°)	5.45	5.45
	Chord (cm)	11.00	10.00
	Span (m)	1.37	8.00
	Tip Speed Ratio	3.58	8.00
Power Generation	Gear Thickness (cm)	1.00	2.00
	Radius, Large Gear (cm)	3.00	1.00
	Gear Ratio	0.03	0.02
	Maximum Generator Power (kW)	12.18	18.17
Structural	Inner Radius, Bottom of Tower (cm)	29.51	27.13
	Outer Radius, Bottom of Tower (cm)	29.84	27.46
	Outer Radius, Top of Tower (cm)	1.64	27.46
	Inner Radius, Top of Tower (cm)	1.31	27.13
Overall	Tower Height (m)	10.00	10.67
Payback Period (years)		163.62	6.19

8.5 Constraint Activity

Interestingly, the optimal design point was observed to be at the upper bound of the height constraint. This was a result of the differences in magnitude of the costs of the individual subsystems. Specifically, the increase in cost associated with making the structural system taller was dominated by the increase in power generated for the same height increase. Figure 9.3 shows the relative costs of the three subsystems during the system optimization.

Figure 9.3: Subsystem Costs, Payback Period, and Cost Recuperation Rate vs. Iteration



Several constraints were observed to be active during the total system optimization. Variables pertaining to the blade design that were located at constraint boundaries for the optimum design point were the chord (at the lower bound), the span (at the upper bound), and the tip-speed ratio (at the upper bound). This combination of variable values gives a long, slender blade design that operates at a relatively high angular velocity - thus reducing volume while maximizing surface area and torque output.

As seen in the optimization of the structural subsystem alone, the minimum wall thickness constraint was active at the total system optimal design. However, unlike the results of the subsystem optimization, the shape of the tower was cylindrical and not tapered. It is possible that this is a result of the effect on payback period associated with changing the tower dimensions being smaller in magnitude than effects associated with changes in other design variables – thus the optimization algorithm searched for improvement in the objective function in variables that had steeper descent directions.

Interestingly, no constraints associated with the power generation subsystem were active. As mentioned in the power generation system subsection, a non-gradient based algorithm was needed to find the constrained global minimum for that system. The ISight implementation of the full system optimization problem was gradient based, so it is not a surprise that the power generation constraints were not active. Because of this fact, the final payback period could not be proved to be a global minimum, because a gradient based solver may not be sufficient to overcome the multimodal nature associated with the power generation subsystem.

9 ACKNOWLEDGEMENTS

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11 APPENDIX A – Blade Design Optimizer Script

```
%fmincon calling file for 555 project
```

```
clc; clear all
```

```
options=optimset('Display','iter',...  
    'LargeScale','off',...  
    'MaxFunEvals',10^100,...  
    'MaxIter',5*10^4,...  
    'TolX',10^(-6),...  
    'TolFun',10^(-6),...  
    'TolCon',10^(-6));
```

```
A=[];b=[];Aeq=[];beq=[];
```

```
lb=[0,1,0,0.1];
```

```
ub=[15,8,8,2];
```

```
x0=[2,6,4,.5];
```

```
[xopt,fval,exitflag,output]=fmincon('objectivef',x0,A,b,Aeq,beq,lb,ub,'constraints',options);
```

12 APPENDIX B – Blade Design Calculation Script

%OBJECTIVE FUNCTION FOR 555 PROJECT

%min f = Weight of blades - Aerodynamic efficiency

function obj=objectivef(x)

global alfa lambda L c;

global rho_mat gr W w V_avg Vt N M T omega P P_max efficiency V0

global L_822 L_823 CL_822 CL_823 CD_822 CD_823 FL_822 FL_823 FL_avg FD_avg

alfa=x(1);lambda=x(2);L=x(3);c=x(4);

rho_air=1.225;

rho_mat=500;

gr=9.81;

tower_height=10;

V0=12;

V_avg=4.222*tower_height^0.1575;

%L_822 must be > L_823

L_822=.55*L;

L_823=.45*L;

%CL_822=-3.0619+(1.5929*alfa)-(0.2575*alfa^2)+(0.0209*alfa^3)-(0.0008*alfa^4); %Co-efficient of lift for S822

%CD_822=0.8561-(0.3005*alfa)+(0.0415*alfa^2)-(0.0028*alfa^3)+(0.0001*alfa^4); %Co-efficient of drag for S822

CL_822=.2525+.2081*alfa-.0341*alfa^2+.0036*alfa^3-.0002*alfa^4+.8;

CD_822=.0192-.0107*alfa+.0060*alfa^2-.0007*alfa^3;

FL_822=0.5*CL_822*rho_air*(c*L_822)*V0^2; %Lift force for S822

FD_822=0.5*CD_822*rho_air*(c*L_822)*V0^2; %Drag force for S822

%CL_823=-1.2034+(0.8249*alfa)-(0.1177*alfa^2)+(0.0085*alfa^3)-(0.0003*alfa^4); %Co-efficient of lift for S823

```

%CD_823=0.5564-(0.2634*alfa)+(0.0474*alfa^2)-(0.0040*alfa^3)+(0.0002*alfa^4);    %Co-efficient
of drag for S823
CL_823=.3196+.1171*alfa+.0041*alfa^2-.0012*alfa^3+.0001*alfa^4+.9;
CD_823=.0171-.0084*alfa+.0026*alfa^2-.0003*alfa^3;
FL_823=0.5*CL_823*rho_air*(c*L_823)*V0^2;    %Lift force for S823
FD_823=0.5*CD_823*rho_air*(c*L_823)*V0^2;    %Drag force for S823

FL_avg=(FL_822+FL_823);
FD_avg=(FD_822+FD_823);

% CD_avg=.5*(CD_822+CD_823);
% CL_avg=.5*(CL_822+CL_823);
% FL_avg=.5*CL_avg*rho_air*(c*L)*V0^2;
% FD_avg=.5*CD_avg*rho_air*(c*L)*V0^2;

%%
%omega=V0*lambda/L;    %angular velocity of blades
%N=60*omega/(2*pi);    %Blade RPM
Vt=V0*lambda;
N=(60*Vt)/(2*pi*L);
omega=(2*pi*N/60);
M=FL_avg*L;    %Torque produced by the blades
T=(4/9)*rho_air*(pi*L^2)*V_avg^2;    %Thrust force on the blades

%p1=0.0; p2=1.0; %Pareto weights
w=3*(1.9913*c^2*L)*rho_mat;    %actual weight
W=w/300;    %Normalized weight
%P=3*M*omega;
P=3*FL_avg*Vt;
P_max=(1.0*(0.5*rho_air*(pi*L^2)*V0^3));
efficiency=P/P_max;    %Efficiency
obj=W-efficiency;

```

```

%%
fprintf(1,'w = %g\n',w);
fprintf(1,'W = %g\n',W);
fprintf(1,'P = %g\n',P);
fprintf(1,'P_max = %g\n',P_max);
fprintf(1,'efficiency = %g\n',efficiency);
fprintf(1,'Vt = %g\n',Vt);
fprintf(1,'omega = %g\n',omega);
fprintf(1,'CL_822 = %g, CL_823 = %g\n',CL_822,CL_823);
fprintf(1,'CD_822 = %g, CD_823 = %g\n',CD_822,CD_823);
fprintf(1,'FL_822 = %g, FL_823 = %g\n',FL_822,FL_823);
fprintf(1,'FL_avg = %g\n',FL_avg);
fprintf(1,'FD_avg = %g\n',FD_avg);
fprintf(1,'N = %g\n',N);
fprintf(1,'M = %g\n',M);
fprintf(1,'T = %g\n',T);
fprintf(1,'\n');

%pareto1=p1*W;pareto2=p2*efficiency;
%fprintf(1,'pareto for W = %g\n',pareto1);fprintf(1,'pareto for eff = %g\n',pareto2);

```

13 APPENDIX C – Blade Design Constraint Script

%Constraint function for 555 project

```
function [g,h]=constraints(x)
```

```
global alfa lambda L c;
```

```
global rho_mat gr W w V_avg Vt N M T omega P P_max efficiency V0
```

```
global L_822 L_823 CL_822 CL_823 CD_822 CD_823 FL_822 FL_823 FL_avg FD_avg
```

```
alfa=x(1);lambda=x(2);L=x(3);c=x(4);
```

```
Plb=1000;
```

%Constraints

```
g=[-W;
```

```
W-1;... %Normalized weight
```

```
-w;
```

```
w-300;... %Actual weight
```

```
-CL_822;
```

```
-CL_823;
```

```
-CD_822;
```

```
-CD_823;
```

```
%Plb/(3*V0*1)-FL_avg;... %1=lambda_min
```

```
FL_avg-P_max/(3*8*V0);... %8=lambda_max
```

```
-FL_avg;...
```

```
%-FL_822;...
```

```
%-FL_823;...
```

```
-efficiency;...
```

```
efficiency-1;...
```

%Positivity constraints for all others

```
-V_avg;...
```

```
-Vt;...
```

```
-N;...
```

```
N-300;...
```

```
-M;...
```

```
-T;...
```

```
-omega;...  
Plb-P;...  
-L_822;...  
-L_823;...  
-FD_avg;...  
];  
h=[];
```

14 APPENDIX D – Power Generation Optimizer Script

```
% ME555 Final Project - Power Generation Optimization
% Author: Kurt Cunningham

clc
clear all

%% Starting Point Variables

Gen_Max_Pwr = 11.2228; %Maximum Rated Power of the Generator - kw
GR = .0211; %Gear Ratio
R_LG = .0251; %Large Gear Radius - m
t_G = 0.06; %Thickness of the gears - m

% Scaling
sc1 = 100;
sc2 = 1;
sc3 = 1;
sc4 = 0.5;

x0 = [Gen_Max_Pwr/sc1, GR/sc2, R_LG/sc3, t_G/sc4];

% Note: 3 Gear step downs within the single gear ratio. GR = (Small Gear /
% Large Gear Radius)^3

% Constraints
ub = [100/sc1, 1/sc2, 1/sc3, 0.5/sc4];
lb = [0.1/sc1, 0.0001/sc2, 0.025/sc3, 0.005/sc4];

%% fmincon Optimization

% Optimization
options=optimset('Algorithm','active-set','MaxFunEvals',100000,'MaxIter',5000,'TolCon',1e-
12,'TolFun',1e-12,'TolX',1e-12,'Display','iter');
```

```

[x,fval,exitflag,output] =
fmincon(@PowerGenCalculation,x0,[],[],[],[],lb,ub,@PowerGenConstraints,options);

End_Message = output.message;
Iterations = output.iterations
Optimal_State = [x(1)*sc1; x(2)*sc2; x(3)*sc3; x(4)*sc4]

% Final Constraint Values
[Gear_Str_Con] = PowerGenConstraints(x);
Gear_Str_Con = Gear_Str_Con*1.03E9
Upper_Constraints = [(x(1)-ub(1))*sc1; (x(2)-ub(2))*sc2; (x(3)-ub(3))*sc3; (x(4)-ub(4))*sc4];
Lower_Constraints = [(lb(1)-x(1))*sc1; (lb(2)-x(2))*sc2; (lb(3)-x(3))*sc3; (lb(4)-x(4))*sc4];

% For Output to Structural Subsystem
[Gen_Sys_Mass] = PowerGenStructuralOutput(x)

% Final Results
Payback_Period = fval
Gen_Max_Pwr = Optimal_State(1);
Single_GR = (1/Optimal_State(2))^(1/3);
R_LG = Optimal_State(3);
t_G = Optimal_State(4);

%% Direct Optimization
% GLOBAL.MaxIter=0;% set max. iterations to zero
% GLOBAL.MaxEval=1000;% DIRECT will terminate after 1000 function evals
% xopt = gclSolve('PowerGenCalculation', 'PowerGenConstraints', lb, ub, [], [], [], [], [], [], GLOBAL);
%
% % Final Results
% x = xopt.x_k;
% Optimal_State = [x(1)*sc1; x(2)*sc2; x(3)*sc3; x(4)*sc4]
% [Gen_Sys_Mass] = PowerGenStructuralOutput(x)

```

15 APPENDIX E – Power Generation Calculation Script

% ME555 Final Project - Power Generation Calculation

% Author: Kurt Cunningham

function [Payback_Period] = PowerGenCalculation(x)

%De-Scaling

sc1 = 100;

sc2 = 1;

sc3 = 1;

sc4 = 0.5;

Gen_Max_Pwr = x(1)*sc1;

GR = x(2)*sc2;

R_LG = x(3)*sc3;

t_G = x(4)*sc4;

%% Constants and Parameters

% Constants

rho_a = 1.2; %kg/m³

% Unchanging Parameters

n_G = 3; %Number of gear reductions

k = 2; %weibull shape parameter

% Steel data using CES Edupack for AISI 6150 Low allow Steel (Tempered at

% 540C and oil quenched)

rho_steel = 7805; %kg/m³

g_steel = 1.12; %\$/kg

% Aluminum data using CES Edupack for 6061-T6 wrought Aluminum

rho_al = 2700; %kg/m³

g_al = 1.57; %\$/kg

h = 4; %height - m

L_bl = 3; %Length of the blade - m (part of blade design)

```

d_bl = 1; % Width of the blade - m (part of blade design)
TSR = 4; % Tip speed ratio (part of blade design)
% Manufacturing Cost
Manf_Cost = 1000;
% Installation Cost
Inst_Cost = 1000;

%% Parameters from Blade design
Cl_1 = 1.0; % Coeff of lift for base blade (blade design input)
Cl_2 = 1.0; % Coeff of lift for tip blade (blade design input)

% Weibull distribution of wind velocity
V_avg = 4.222*(h^.1575); % average wind velocity - m/s - Weibull scale parameter
V = [0:0.01:49.99]; % m/s
for j = 1:5000
    Prob(j) = (k/(V_avg^k))*V(j)*exp(-(V(j)/V_avg)^k);
end

A_pl_1 = d_bl*0.45*L_bl; % Planform area - m^2
A_pl_2 = d_bl*0.55*L_bl; % Planform area - m^2
Fl_1 = Cl_1*.5*rho_a*A_pl_1*V.*V; % Lift force on each blade - N
Fl_2 = Cl_2*.5*rho_a*A_pl_2*V.*V; % Lift force on each blade - N
T_bl_1 = Fl_1*0.45*L_bl/2;
T_bl_2 = Fl_2*L_bl*(1-0.55/2);
T_bl = 3*(T_bl_1 + T_bl_2); % Total torque of blades - Nm (Assuming constant blade distribution)
w_bl = V*TSR/L_bl; % Radial velocity of the blades - rad/sec

%% Generator Data
% Continuous Power = 32kW, Peak Power = 75kW
% Data provided by Unique Mobility, Inc.
% 425 Corporate Circle
% Golden, CO 80401
% (303)278-2002
% This data is taken from a data sheet for a 75Kw Unique

```

```

% Mobility SR218H brushless permanent magnet motor coupled
% with its companion CA40-300L inverter.
% Similar to: http://www.uqm.com/propulsion\_specs.php (PowerPhase 75)
% Proprietary : Public

gc.init.inertia      = 0.0544;          % kg-m^2
gc.init.time_response = 0.05;
gc.init.t_max_trq    = 120; % Time the generator can remain at max torque
gc.init.spd_opt      = 3600*2*pi/60;
gc.init.ratio        = 1;
gc.init.gear_eff      = 1;
gc.init.generator_mass = 50;
gc.init.controller_mass = 10;
gc.init.system_cost   = 13200; % Found this using search for Unique Mobility SR218N cost on google
gc.init.spd_base      = 2200*2*pi/60; % rad/s
gc.init.curr_max      = 300; % (A), maximum current allowed by the controller and motor
gc.init.cont_to_peak_ratio = 75/32;

gc.init.spd_cont_index =
[0,1000,1500,2200,2400,3000,3600,3900,4200,4800,5400,6000,6600,7200,7500,7600,7700]*2*pi/60;%
rad/s
gc.init.trq_cont_map   =
[150.0,150.0,150.0,150.0,137.6,110.1,91.7,84.7,78.6,68.8,61.2,55.0,50.0,45.9,44.0,0,0];% N-m

gc.init.spd_max_index  = gc.init.spd_cont_index; % rad/s
gc.init.trq_max_map    = gc.init.cont_to_peak_ratio * gc.init.trq_cont_map; %% N-m from INEL
Prius generator file
gc.init.trq_max        = max (gc.init.trq_max_map);

gc.init.spd_min_index  = gc.init.spd_cont_index; % rad/s
gc.init.trq_min_map    = -gc.init.trq_cont_map;

gc.init.spd_eff_index  = gc.init.spd_cont_index;% rad/s

```

```
gc.init.trq_eff_index =
[0,3,13,23,35,45,55,66,77,87,100,108,118,129,141,150]/150*gc.init.trq_max;%AyR - ensures that the
maximum torque is included in the efficiency map
```

```
% Efficiency data was provided; (REINTERPOLATE BY HAND?)
```

```
gc.init.eff_trq_map = ...
[ .0, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00;...
.0, .83, .87, .88, .88, .87, .87, .86, .86, .87, .86, .82, .80, .80, .80;...
.0, .84, .88, .89, .89, .89, .88, .88, .87, .87, .86, .84, .82, .81, .80, .80;...
.0, .84, .88, .89, .89, .90, .90, .89, .88, .87, .86, .84, .83, .84, .82, .81;...
.0, .86, .90, .91, .91, .91, .91, .91, .91, .90, .89, .88, .87, .86, .85, .83;...
.0, .89, .92, .92, .93, .93, .92, .92, .92, .91, .91, .90, .89, .89, .89, .89;...
.0, .89, .94, .94, .93, .94, .93, .93, .93, .93, .92, .92, .92, .92, .92, .92;...
.0, .91, .94, .94, .93, .94, .93, .93, .92, .92, .92, .92, .92, .92, .92, .92;...
.0, .90, .93, .93, .93, .93, .92, .92, .92, .92, .92, .92, .92, .92, .92, .92;...
.0, .82, .89, .91, .91, .92, .91, .91, .91, .91, .91, .91, .91, .91, .91;...
.0, .80, .90, .91, .91, .92, .92, .92, .92, .92, .92, .92, .92, .92, .92;...
.0, .80, .89, .91, .91, .92, .92, .92, .92, .92, .92, .92, .92, .92, .92;...
.0, .79, .87, .90, .91, .91, .91, .91, .91, .91, .91, .91, .91, .91, .91;...
.0, .79, .86, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90;...
.0, .79, .86, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90;...
.0, .79, .86, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90;...
.0, .79, .88, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90 ];
```

```
%% Generator Scaling
```

```
% Scale the generator based on power
```

```
gc.calc.pwr_mech_max = 75; %kW
gc.calc.eff_max = max(gc.init.eff_trq_map); %kW
gc.scale.specific_cost = gc.calc.pwr_mech_max./gc.init.system_cost;
gc.scale.pwr_max_des = Gen_Max_Pwr;
gc.scale.sr = gc.scale.pwr_max_des/gc.calc.pwr_mech_max; %calculate the scaling ratio
gc.init.trq_cont_map = gc.scale.sr*gc.init.trq_cont_map;
gc.init.trq_max_map = gc.scale.sr*gc.init.trq_max_map;
```

```

gc.init.trq_eff_index    = gc.scale.sr*gc.init.trq_eff_index;
gc.init.inertia         = gc.scale.sr*gc.init.inertia;
gc.init.curr_max        = gc.scale.sr*gc.init.curr_max;
gc.init.system_cost     = gc.scale.pwr_max_des./gc.scale.specific_cost;

```

%% Objective Function Calculations

% Generator Cost/Mass

```

Generator_Mass = gc.init.controller_mass + gc.init.generator_mass; %kg
Generator_Cost = gc.init.system_cost;

```

% Gear Cost/Mass

```

Gear_Mass = n_G*rho_steel*((R_LG^2)+(R_LG*(GR^(1/n_G)))^2)*pi*t_G; %kg
Gear_Cost = n_G*rho_steel*g_steel*((R_LG^2)+(R_LG*(GR^(1/n_G)))^2)*pi*t_G;

```

% Input Shaft Cost/Mass

```

W_Shaft = rho_steel*0.5*(R_LG*(GR^(1/n_G)))*0.5;
Shaft_Cost = W_Shaft*g_steel;

```

% Gear Housing (Hollow Cylinder) Cost/Mass

```

W_H = rho_al*[((pi*(0.055+R_LG+R_LG*GR^(1/3))^2)-
(pi*(0.05+R_LG+R_LG*GR^(1/3))^2))*(t_G*3+0.25+1) +
0.0025*2*(pi*(0.055+R_LG+R_LG*GR^(1/3))^2)];
H_Cost = W_H*g_al;

```

% Payback Rate Calculation

```

Act_Eff = interp2(gc.init.spd_eff_index,gc.init.trq_eff_index,gc.init.eff_trq_map',w_bl/GR,T_bl*GR);

```

```

for i = 1:length(Act_Eff)

```

```

    if isnan(Act_Eff(i))

```

```

        T_bl(i) = 0;

```

```

        w_bl(i) = 0;

```

```

        Act_Eff(i) = 0;

```

```

    end

```

```

end
Act_Eff = Act_Eff*0.95; %Add an inverter for selling to grid
Generated_Power = w_bl.*T_bl.*Act_Eff/1000; %kW
for j = 1:length(Generated_Power)
    if Generated_Power(j) > Gen_Max_Pwr
        Generated_Power(j) = 0;
        T_bl(j) = 0;
        w_bl(j) = 0;
    end
end
Generated_Energy = Generated_Power.*Prob*8766; %kWh
E_tot = sum(Generated_Energy*0.01);
omega = E_tot*(10.43/100); %10.43 cents per kWh
                        %8766 hr per yr and $1 per 100 cents

% Objective
Gen_Sys_Cost = Gear_Cost+Generator_Cost+H_Cost+Shaft_Cost+Manf_Cost+Inst_Cost;
Payback_Period = (Gen_Sys_Cost)/omega;

%% END

```

16 APPENDIX F – Power Generation Constraint Script

% ME555 Final Project - Power Generation Nonlinear Constraint

% Author: Kurt Cunningham

function [Gen_Sys_Constraints, Eq_Con] = PowerGenConstraints(x)

%De-Scaling

sc1 = 100;

sc2 = 1;

sc3 = 1;

sc4 = 0.5;

Gen_Max_Pwr = x(1)*sc1;

GR = x(2)*sc2;

R_LG = x(3)*sc3;

t_G = x(4)*sc4;

%% Constants and Parameters

% Constants

rho_a = 1.2; %kg/m³

% Unchanging Parameters

n_G = 3; %Number of gear reductions

k = 2; %weibull shape parameter

% Steel data using CES Edupack for AISI 6150 Low allow Steel (Tempered at

% 540C and oil quenched)

rho_steel = 7805; %kg/m³

g_steel = 1.12; %\$/kg

% Aluminum data using CES Edupack for 6061-T6 wrought Aluminum

rho_al = 2700; %kg/m³

g_al = 1.57; %\$/kg

h = 4; %height - m

L_bl = 3; %Length of the blade - m (part of blade design)

```

d_bl = 1; % Width of the blade - m (part of blade design)
TSR = 4; % Tip speed ratio (part of blade design)
% Manufacturing Cost
Manf_Cost = 1000;
% Installation Cost
Inst_Cost = 1000;

%% Parameters from Blade design
Cl_1 = 1.0; % Coeff of lift for base blade (blade design input)
Cl_2 = 1.0; % Coeff of lift for tip blade (blade design input)

% Weibull distribution of wind velocity
V_avg = 4.222*(h^.1575); % average wind velocity - m/s - Weibull scale parameter
V = [0:0.01:49.99]; % m/s
for j = 1:5000
    Prob(j) = (k/(V_avg^k))*V(j)*exp(-(V(j)/V_avg)^k);
end

A_pl_1 = d_bl*0.45*L_bl; % Planform area - m^2
A_pl_2 = d_bl*0.55*L_bl; % Planform area - m^2
Fl_1 = Cl_1*.5*rho_a*A_pl_1*V.*V; % Lift force on each blade - N
Fl_2 = Cl_2*.5*rho_a*A_pl_2*V.*V; % Lift force on each blade - N
T_bl_1 = Fl_1*0.45*L_bl/2;
T_bl_2 = Fl_2*L_bl*(1-0.55/2);
T_bl = 3*(T_bl_1 + T_bl_2); % Total torque of blades - Nm (Assuming constant blade distribution)
w_bl = V*TSR/L_bl; % Radial velocity of the blades - rad/sec

%% Generator Data
% Continuous Power = 32kW, Peak Power = 75kW
% Data provided by Unique Mobility, Inc.
% 425 Corporate Circle
% Golden, CO 80401
% (303)278-2002
% This data is taken from a data sheet for a 75Kw Unique

```

```

% Mobility SR218H brushless permanent magnet motor coupled
% with its companion CA40-300L inverter.
% Similar to: http://www.uqm.com/propulsion\_specs.php (PowerPhase 75)
% Proprietary : Public

gc.init.inertia      = 0.0544;          % kg-m^2
gc.init.time_response = 0.05;
gc.init.t_max_trq    = 120; % Time the generator can remain at max torque
gc.init.spd_opt      = 3600*2*pi/60;
gc.init.ratio        = 1;
gc.init.gear_eff     = 1;
gc.init.generator_mass = 50;
gc.init.controller_mass = 10;
gc.init.system_cost  = 13200; % Found this using search for Unique Mobility SR218N cost on google
gc.init.spd_base     = 2200*2*pi/60; % rad/s
gc.init.curr_max     = 300; % (A), maximum current allowed by the controller and motor
gc.init.cont_to_peak_ratio = 75/32;

gc.init.spd_cont_index =
[0,1000,1500,2200,2400,3000,3600,3900,4200,4800,5400,6000,6600,7200,7500,7600,7700]*2*pi/60;%
rad/s
gc.init.trq_cont_map  =
[150.0,150.0,150.0,150.0,137.6,110.1,91.7,84.7,78.6,68.8,61.2,55.0,50.0,45.9,44.0,0,0];% N-m

gc.init.spd_max_index = gc.init.spd_cont_index; % rad/s
gc.init.trq_max_map   = gc.init.cont_to_peak_ratio * gc.init.trq_cont_map; %% N-m from INEL
Prius generator file
gc.init.trq_max       = max (gc.init.trq_max_map);

gc.init.spd_min_index = gc.init.spd_cont_index; % rad/s
gc.init.trq_min_map   = -gc.init.trq_cont_map;

gc.init.spd_eff_index = gc.init.spd_cont_index;% rad/s

```

```
gc.init.trq_eff_index =
[0,3,13,23,35,45,55,66,77,87,100,108,118,129,141,150]/150*gc.init.trq_max;%AyR - ensures that the
maximum torque is included in the efficiency map
```

```
% Efficiency data was provided; (REINTERPOLATE BY HAND?)
```

```
gc.init.eff_trq_map = ...
[ .0, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00, .00;...
.0, .83, .87, .88, .88, .87, .87, .86, .86, .87, .86, .82, .80, .80, .80;...
.0, .84, .88, .89, .89, .89, .88, .88, .87, .87, .86, .84, .82, .81, .80, .80;...
.0, .84, .88, .89, .89, .90, .90, .89, .88, .87, .86, .84, .83, .84, .82, .81;...
.0, .86, .90, .91, .91, .91, .91, .91, .91, .90, .89, .88, .87, .86, .85, .83;...
.0, .89, .92, .92, .93, .93, .92, .92, .92, .91, .91, .90, .89, .89, .89, .89;...
.0, .89, .94, .94, .93, .94, .93, .93, .93, .93, .92, .92, .92, .92, .92, .92;...
.0, .91, .94, .94, .93, .94, .93, .93, .92, .92, .92, .92, .92, .92, .92, .92;...
.0, .90, .93, .93, .93, .93, .92, .92, .92, .92, .92, .92, .92, .92, .92, .92;...
.0, .82, .89, .91, .91, .92, .91, .91, .91, .91, .91, .91, .91, .91, .91;...
.0, .80, .90, .91, .91, .92, .92, .92, .92, .92, .92, .92, .92, .92, .92;...
.0, .80, .89, .91, .91, .92, .92, .92, .92, .92, .92, .92, .92, .92, .92;...
.0, .79, .87, .90, .91, .91, .91, .91, .91, .91, .91, .91, .91, .91, .91;...
.0, .79, .86, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90;...
.0, .79, .86, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90;...
.0, .79, .86, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90;...
.0, .79, .88, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90, .90 ];
```

```
%% Generator Scaling
```

```
% Scale the generator based on power
```

```
gc.calc.pwr_mech_max = 75; %kW
gc.calc.eff_max = max(gc.init.eff_trq_map); %kW
gc.scale.specific_cost = gc.calc.pwr_mech_max./gc.init.system_cost;
gc.scale.pwr_max_des = Gen_Max_Pwr;
gc.scale.sr = gc.scale.pwr_max_des/gc.calc.pwr_mech_max; %calculate the scaling ratio
gc.init.trq_cont_map = gc.scale.sr*gc.init.trq_cont_map;
gc.init.trq_max_map = gc.scale.sr*gc.init.trq_max_map;
```

```

gc.init.trq_eff_index    = gc.scale.sr*gc.init.trq_eff_index;
gc.init.inertia         = gc.scale.sr*gc.init.inertia;
gc.init.curr_max        = gc.scale.sr*gc.init.curr_max;
gc.init.system_cost     = gc.scale.pwr_max_des./gc.scale.specific_cost;

```

%% Objective Function Calculations

% Generator Cost/Mass

```

Generator_Mass = gc.init.controller_mass + gc.init.generator_mass; %kg
Generator_Cost = gc.init.system_cost;

```

% Gear Cost/Mass

```

Gear_Mass = n_G*rho_steel*((R_LG^2)+(R_LG*(GR^(1/n_G)))^2)*pi*t_G; %kg
Gear_Cost = n_G*rho_steel*g_steel*((R_LG^2)+(R_LG*(GR^(1/n_G)))^2)*pi*t_G;

```

% Input Shaft Cost/Mass

```

W_Shaft = rho_steel*0.5*(R_LG*(GR^(1/n_G)))*0.5;
Shaft_Cost = W_Shaft*g_steel;

```

% Gear Housing (Hollow Cylinder) Cost/Mass

```

W_H = rho_al*[((pi*(0.055+R_LG+R_LG*GR^(1/3))^2)-
(pi*(0.05+R_LG+R_LG*GR^(1/3))^2))*(t_G*3+0.25+1) +
0.0025*2*(pi*(0.055+R_LG+R_LG*GR^(1/3))^2)];
H_Cost = W_H*g_al;

```

% Payback Rate Calculation

```

Act_Eff = interp2(gc.init.spd_eff_index,gc.init.trq_eff_index,gc.init.eff_trq_map',w_bl/GR,T_bl*GR);

```

```

for i = 1:length(Act_Eff)

```

```

    if isnan(Act_Eff(i))

```

```

        T_bl(i) = 0;

```

```

        w_bl(i) = 0;

```

```

        Act_Eff(i) = 0;

```

```

    end

```

```

end
Act_Eff = Act_Eff*0.95; %Add an inverter for selling to grid
Generated_Power = w_bl.*T_bl.*Act_Eff/1000; %kW
for j = 1:length(Generated_Power)
    if Generated_Power(j) > Gen_Max_Pwr
        Generated_Power(j) = 0;
        T_bl(j) = 0;
        w_bl(j) = 0;
    end
end
Generated_Energy = Generated_Power.*Prob*8766; %kWh
E_tot = sum(Generated_Energy*0.01);
omega = E_tot*(10.43/100); %10.43 cents per kWh
                    %8766 hr per yr and $1 per 100 cents

% Objective
Gen_Sys_Cost = Gear_Cost+Generator_Cost+H_Cost+Shaft_Cost+Manf_Cost+Inst_Cost;
Payback_Period = (Gen_Sys_Cost)/omega;

%% Constraint (negative null form)

Gen_Sys_Constraints = [(max(T_bl)*32/(t_G*R_LG*(GR^(1/n_G)))) - 1.03E9]/1.03E9;
Eq_Con = [0];

%% END

```

17 APPENDIX G – Power Generation Structural Output Script

% ME555 Final Project - Power Generation Structural Output

% Author: Kurt Cunningham

function [Gen_Sys_Mass] = PowerGenStructuralOutput(x)

% De-Scaling

sc1 = 100;

sc2 = 1;

sc3 = 1;

sc4 = 0.5;

Gen_Max_Pwr = x(1)*sc1;

GR = x(2)*sc2;

R_LG = x(3)*sc3;

t_G = x(4)*sc4;

%% Constants and Parameters

% Constants

rho_a = 1.2; %kg/m³

% Unchanging Parameters

n_G = 3; %Number of gear reductions

k = 2; %weibull shape parameter

% Steel data using CES Edupack for AISI 6150 Low allow Steel (Tempered at

% 540C and oil quenched)

rho_steel = 7805; %kg/m³

g_steel = 1.12; %\$/kg

% Aluminum data is same as Kevin's values

rho_al = 2700; %kg/m³

g_al = 1.57; %\$/kg

%Manufacturing Cost

```

Manf_Cost = 1000;

% Installation Cost
Inst_Cost = 1000;

%% Generator Data
% Continuous Power = 32kW, Peak Power = 75kW
% Data provided by Unique Mobility, Inc.
% 425 Corporate Circle
% Golden, CO 80401
% (303)278-2002
% This data is taken from a data sheet for a 75Kw Unique
% Mobility SR218H brushless permanent magnet motor coupled
% with its companion CA40-300L inverter.
% Similar to: http://www.uqm.com/propulsion\_specs.php (PowerPhase 75)
% Proprietary : Public

gc.init.generator_mass = 50;
gc.init.controller_mass = 10;

%% Generator Scaling
% Scale the generator based on power

gc.calc.pwr_mech_max = 75; %kW
gc.scale.specific_power_controller = gc.calc.pwr_mech_max./gc.init.controller_mass;
gc.scale.specific_power_generator = gc.calc.pwr_mech_max./gc.init.generator_mass;
gc.scale.pwr_max_des = Gen_Max_Pwr;
gc.init.controller_mass = gc.scale.pwr_max_des./gc.scale.specific_power_controller;
gc.init.generator_mass = gc.scale.pwr_max_des./gc.scale.specific_power_generator;

%% Output Values for Structural Design

```

```

W_H = rho_al*[((pi*(0.055+R_LG+R_LG*GR^(1/3))^2)-
(pi*(0.05+R_LG+R_LG*GR^(1/3))^2))*(t_G*3+0.25+1) +
0.0025*2*(pi*(0.055+R_LG+R_LG*GR^(1/3))^2)];
W_Shaft = rho_steel*0.5*(R_LG*(GR^(1/n_G)))*0.5;

Gear_Mass = n_G*rho_steel*((R_LG^2)+(R_LG*(GR^(1/n_G)))^2)*pi*t_G; %kg
Generator_Mass = gc.init.controller_mass + gc.init.generator_mass; %kg
Gen_Sys_Mass = Gear_Mass + Generator_Mass + W_H + W_Shaft;

%% END

```

18 APPENDIX H – ANSYS Mechanical APDL Code

```
/BATCH
!/COM,ANSYS RELEASE 12.0.1 UP20090415 12:52:22 03/13/2010
/input,start120,ans,'S:\CAEN\ANSYS12\v120\ANSYS\apdl',,,,,,,,,,,,,,1
/units,mks
/prep7
*SET,R1 ,0.3
*SET,R2 ,0.35
*SET,R3 ,0.3
*SET,R4 ,0.25
*SET,h , 12
*SET,W , 13734
*SET,T , 7678
*SET,DF , 10
!*
ET,1,SOLID45
!*
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,6.89e10
MPDATA,PRXY,1,,0.330
MPDATA,DENS,1,,2.7e3
k,1,r1,0,0
k,2,r2,0,0
k,3,r3,h,0
k,4,r4,h,0
k,5,0,0,0
k,6,0,50,0
l,1,2
l,2,3
l,3,4
l,4,1
al,1,2,3,4
```

```

vrotat,1,,,,,5,6,360,4
ESIZE,0.05,0,
MSHAPE,0,3D
MSHKEY,1
CM,_Y,VOLU
VSEL,all,,,,,,
CM,_Y1,VOLU
CHKMSH,'VOLU'
CMSEL,S,_Y
!*
VMESH,_Y1
!*
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
!*
DA,2,ALL,0
DA,7,ALL,0
DA,12,ALL,0
DA,17,ALL,0
!*
SFA,4,1,PRES,W/(3.14159265*((R3*R3)-(R4*R4)))
SFA,9,1,PRES,W/(3.14159265*((R3*R3)-(R4*R4)))
SFA,14,1,PRES,W/(3.14159265*((R3*R3)-(R4*R4)))
SFA,19,1,PRES,W/(3.14159265*((R3*R3)-(R4*R4)))
!*
SFL,2,PRES,DF/h,
!/PNUM,KP,0
!/PNUM,LINE,1
!/PNUM,AREA,0
!/PNUM,VOLU,0
!/PNUM,NODE,0
!/PNUM,TABN,0
!/PNUM,SVAL,0

```

```

! /NUMBER,0
!*
! /PNUM,ELEM,0
! /REPLOT
!*
SFL,11,PRES,t,
SFL,31,PRES,t,
!
FINISH
/SOL
! /STATUS,SOLU
SOLVE
FINISH
/POST1
AVPRIN,0,0,
ETABLE,EVOLUME,VOLU,
!*
SSUM
!*
*GET,VOLUME,SSUM, ,ITEM,EVOLUME
AVPRIN,0,0,
ETABLE,ESTRESS,S,EQV
!*
ESORT,ETAB,ESTRESS,0,0, ,
!*
*GET,STRESS,SORT,,MAX
!*
/SOL
!*
ANTYPE,2
!*
MODOPT,LANB,1,0,100,,
EQSLV,SPAR
MXPAND,0, , ,0

```

```
LUMPM,0
PSTRES,0
!*
! /STATUS,SOLU
SOLVE
!*
*GET,frequency,MODE,1,FREQ
PARSAV,SCALAR,'results','dat','H:\Private\Ansys Model\
/OUTPUT, output test, txt
```

19 APPENDIX I – Sample ISight Results Output

Optimization Results

Started on Thu Apr 01 16:49:04 EDT 2010

Optimization Technique: NLPQL

Failed Run Objective Value = 1.0E30

Failed Run Penalty Value = 1.0E30

Max Failed Runs = 20

Max Iterations = 10

Min Abs Step Size = 1.0E-4

Rel Step Size = 1.0E-4

Save Technique Log = false

Termination Accuracy = 0.0010

Use Central Differences = true

Starting design point:

R1 = 0.295881822 [0.0 < x < 0.5]

R2 = 0.299145722 [0.0 < x < 0.51]

R3 = 0.018285442 [0.0 < x < 0.51]

R4 = 0.015 [0.0 < x < 0.5]

Completed on Thu Apr 01 16:59:03 EDT 2010

Total design evaluations: 50

Number of feasible designs: 18

NLPQL termination reason: OPTIMALITY CONDITIONS SATISFIED

Optimum design point:

Run # = 202
 Objective = 0.0318569028
 Penalty = 0.0
 ObjectiveAndPenalty = 0.0318569028
 R1 = 0.29510937984579566
 R2 = 0.29837327984579565
 R3 = 0.016390622112347847
 R4 = 0.013112588406147844
 g1 = -1.7476873400000006E7
 g10 = -0.2819826577334478
 g2 = -4.172243342641587
 g3 = -0.037375657221312863
 g4 = -51410.4145435982
 g5 = -0.29310937984579566
 g6 = 0.0
 g7 = -0.20162672015420435
 g8 = -0.011112588406147844
 g9 = -1.4133706200000762E-5
 Volume = 0.0318569028

Calculated constraint values at the optimum:

(constraint value is a difference between the bound or target and the output parameter value, scaled and weighted)

g1 (Upper Bound Constraint) = -1.7476873400000006E7 (satisfied)
 g10 (Upper Bound Constraint) = -0.2819826577334478 (satisfied)
 g2 (Upper Bound Constraint) = -4.172243342641587 (satisfied)
 g3 (Upper Bound Constraint) = -0.037375657221312863 (satisfied)

g4 (Upper Bound Constraint) = -51410.4145435982 (satisfied)
g5 (Upper Bound Constraint) = -0.29310937984579566 (satisfied)
g6 (Upper Bound Constraint) = 0.0 (within tolerance)
g7 (Upper Bound Constraint) = -0.20162672015420435 (satisfied)
g8 (Upper Bound Constraint) = -0.011112588406147844 (satisfied)
g9 (Upper Bound Constraint) = -1.4133706200000762E-5 (satisfied)