THE UNIVERSITY OF MICHIGAN

# OPTIMAL DESIGN OF A PLUG-IN HYBRID ELECTRIC VEHICLE

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## ABSTRACT

Rising fuel prices and increased discussion of global warming mitigation have propelled the Plug-in Hybrid Electric Vehicle into the center of the automotive industry and national transportation policy discussions. PHEVs are considered most promising for their ability to reduce U.S. dependence on foreign oil and perhaps lower greenhouse gas emissions. At this point, several questions remain unanswered:

To what extent can electricity replace fossil-hydrocarbons as transportation energy?

How does electric replacement reduce greenhouse gas emissions, specifically $CO_2$?

Finally, how much can $CO_2$ emissions be reduced, and at what cost?

The answers to these questions require a broad understanding of hybrid-electric vehicles, consumer preferences, manufacturing costs, and electrical grid infrastructure and control. This study offers an initial outline of these answers for a single PHEV architecture applied to a very simple, but broad system containing all the above elements. Mathematical models and optimization techniques revealed clear trends and constraint activity regarding the nature of PHEVs and their societal impact.

# Contents

# INTRODUCTION

PHEVs are viewed as a logical step towards cleaner transportation. PHEVs have the ability to operate in either gasoline-fueled hybrid mode or in all-electric mode with grid supplied energy. The duality allows for long range use and easy refueling similar to that of the conventional vehicle, but with lower tailpipe emissions and reduced petroleum consumption. It is commonly quoted that 70% of the energy needed for the adoption of PHEVs could be supported using the current electricity generating and transmission capacity [1].

Current Estimates of PHEV cost indicate that they will be expensive due to the cost of battery technology. It is known that electricity is a cheaper source of transportation compared to gasoline and will hence offset the high initial investment by the consumer. This study will optimize the vehicle configuration from design to ownership. The three different perspectives are developed as subsystems, and when joined create a link between the automobile and electric sector markets. The vehicle subsystem concentrates on maximizing fuel economy with performance constraints. The second subsystem considers the producer who sets engineering characteristics and price to maximize profit from the PHEV based on consumer data. The third considers the vehicle owner who aims to maximize profits from selling energy back to the electric grid. The different subsystems will then be combined to evaluate the possible benefits for reducing greenhouse gas emissions under different policy scenarios.

The idea is that we can obtain an optimal PHEV vehicle configuration which would have the desired features that help its sales as well as reduce the combined GHG emissions from the transportation and electricity infrastructures by acting as an efficient and convenient storage of energy. Hence our problem is to understand the consumer preference so that the PHEV satisfies demands such as low fuel consumption as well as improves the electric grid by leveling the electric energy consumption over the period of the day, thereby reducing emissions.

# DESIGN PROBLEM STATEMENT

This design problem attempts to minimize a multi-objective function consisting of total emitted greenhouse gas emissions ($CO_2$) and manufacturer profits. Greenhouse gas emissions produced through the PHEV life-cycle are not considered, and external market forces such as government incentives are also not considered. The problem contains vehicle, market, and grid models. The PHEV is closely connected to two areas: 1) The vehicle market, where manufacturers and consumers come together, and 2) The electric grid, where it results in a significant demand increase, but also increases storage capacity. The interaction with the grid creates a second market between power generators and consumers. The vehicle model, separately optimized by Michael Woon, models the PHEV's performance, fuel consumption, and tailpipe emissions as a function of several powertrain design variables. The market model, separately optimized by Carol Girata, models the manufacturing costs, consumer demand, consumer utility, market share, and profits associated with the PHEV. Lastly, the electric grid model, separately optimized by Rakesh Patil, models the charging/discharging costs and emissions attributed to PHEVs.

The idea is that we can obtain an optimal PHEV vehicle configuration which would have the desired features in the market, environment, and on the electrical infrastructure. Hence our problem is to understand the consumer preference so that the PHEV satisfies demands such as low fuel consumption as well as improves the electric grid by leveling the electric energy consumption over the period of the day, thereby reducing emissions.

## NOMENCLATURE

| | |
|---|---|
| $\alpha$ | Multi-objective weights used in PHEV subsystem |
| β | Demand model coefficient parameters |
| $c^B$ | Base Manufacturing cost per vehicle (without engine) |
| $c^I$ | Investment cost |
| $c^E{}_j$ | Engine cost for design *j* |
| $c^V{}_j$ | Variable manufacturing cost per vehicle for design *j* |
| $c^R{}_k$ | Regulation cost for producer *k* |
| $c_c(t)$ | Retail Cost of charging the battery from the grid (¢/kWh) |
| $c_d(t)$ | Retail Cost of discharging the battery to the grid (¢/kWh) |
| $c_{elec,avg}$ | average cost of electricity (¢/kWh) |
| $c_g$ | cost of gasoline per mile (¢/mi) |
| $c_k$ | Total cost for producer *k* |
| $Cycle$ | Drive cycle, or velocity schedule |
| $\delta$ | Cost model coefficient parameters |
| $E_B$ | Battery energy capacity (kWh) |
| $E_{BC}$ | Amount of Electric Energy used from the grid to charge the battery (kWh) |
| $E_{BD}$ | Amount of Electric Energy put into the grid by the battery (kWh) |
| $E_C$ | Amount of Electric Energy generated by Coal (MWh) |
| $E_{demand}$ | Energy demanded from the Sources (MWh) |

| | |
|---|---|
| $E_N$ | Amount of Electric Energy generated by Nuclear (MWh) |
| $E_{NG}$ | Amount of Electric Energy generated by Natural Gas (MWh) |
| $E_W$ | Amount of Electric Energy generated by Wind (MWh) |
| $F_{0-2}$ | Vehicle road load coefficients |
| $f_{elec}$ | Objective function, battery electrical consumption in Wh/mile |
| $f_{fuel}$ | Objective function, fuel consumption in Wh/mile |
| $f_{Total}$ | Multi-objective function, combined fuel and electrical consumption in Wh/mile |
| $g$ | Acceleration of gravity |
| Hwy | Standard U.S. drive cycle, moderate, low power but high speeds |
| $\bar{\eta}_{DT}$ | Average drivetrain efficiency, used for gradeability |
| $i$ | Individual index |
| $j$ | Vehicle design index |
| $J$ | Set of all vehicles produced |
| $J_k$ | Set of all vehicles produced by producer k |
| $k$ | Producer index |
| $m_{PHEV}$ | Total mass of the vehicle |
| $N_V$ | Number of PHEVs being used by the consumers |
| $P_C$ | Maximum capacity of shifting reserves from Coal (MW) |
| $P_H$ | Maximum capacity of shifting reserves from Hydro (MW) |
| $P_{NG}$ | Maximum capacity of shifting reserves from Coal Natural Gas (MW) |

| | |
|---|---|
| $p_j$ | Selling price of design j |
| $\Pi_k$ | Total profit for producer k |
| $P_{cycle,max}$ | Maximum road power required by drive cycle, W |
| $P_{E,Batt}$ | Power to energy ratio for battery, kW/kWh or $h^{-1}$ |
| $P_{elec}$ | PHEV electric power consumed in the battery, W |
| $P_{fuel}$ | PHEV fuel power consumed from gas tank, W |
| $P_{PT,max}$ | Maximum powertrain power |
| $p_V$ | market penetration of PHEVs (%) |
| $q_j$ | quantity demanded of design j |
| $r_g$ | Gradability grade |
| $s$: | Size of the automotive market |
| $SOC$ | Battery state of charge, 0 – 1 |
| $SOC_{swing}$ | Battery state of charge practical limits, ex. 0.9-0.2 = 0.7 |
| $t_{0-60}$ | 0-60 mph time |
| $\bar{t}_{0-60}$ | 0-60 mph time required, 12 sec |
| $t_{EC}$ | Time of day when battery ends charging from the grid (hrs, min) |
| $t_{ED}$ | Time of day when battery ends discharging to the grid (hrs, min) |
| $t_{SC}$ | Time of day when battery starts charging from the grid (hrs, min) |
| $t_{SD}$ | Time of day when battery starts discharging to the grid (hrs, min) |
| UDDS | Standard U.S. drive cycle, least aggressive cycle used in this study |

| | |
|---|---|
| US06 | Standard U.S. drive cycle, most aggressive cycle used in this study |
| $v_g$ | Gradability speed |
| $v_{ij}$ | Observable component of utility for individual *i* design *j* |
| $\mathbf{x}$ | Design variable vector |
| $x_1 Batt$ | Battery variable, energy in kWh |
| $x_2 Eng$ | Engine variable, power in kW |
| $x_3 MG$ | Motor/Generator variable, power in kW |
| $x_1$ | engine bore (mm) |
| $x_2$ | battery power |
| $\mathbf{z}$ | Product characteristics vector |
| $z_{j1}$ | gas mileage (mpg) |
| $z_{j2}$ | performance, time from 0-60 (s) |
| $Z$ | Vehicle characteristics in the demand model |

# SUBSYSTEM DESIGN

## PHEV Powertrain – Michael Woon

*Mathematical Model*

Objective Function

$$\min f_{Total} = \alpha_1 f_{fuel}(x) + \alpha_2 f_{elec}(x), \qquad \sum_i \alpha_{1i} + \alpha_{2i} = 1$$

$$f_{fuel} = \left[\frac{\int P_{fuel}}{\int velocity}\right]_{Cycle} \qquad f_{elec} = \left[\frac{\int P_{elec}}{\int velocity}\right]_{Cycle}$$

$$x = [x_1 Batt, x_2 Eng, x_3 MG]^T$$

Subject to Performance Constraints

$$g_1 \qquad \max\left(0.65\left(\frac{m_{PHEV}}{P_{PT,max}}\right), 5.5\right) - t_{0-60} \le 0$$

$$g_2 \qquad v_g\left(r_g m_{PHEV} g_c + F_2 v_g^2 + F_1 v_g + F_0\right) - x_2 Eng \cdot \bar{\eta}_{DT} \le 0$$

$$g_3 \qquad P_{Cycle,max} - P_{PT,max} \le 0$$

This sub-system optimization minimizes two objectives, fuel energy and electrical energy in Wh/mile, which create a Pareto set that is unique to a given drive cycle. These objectives are evaluated from a quasi-static hybrid vehicle model in Matlab/Simulink. Several Matlab subroutines are first executed to determine the mass of the vehicle, the maximum power, performance attributes, control parameters, and constraints before evaluating fuel economy. The quasi-static model allows drive cycle execution even if the powertrain is underpowered for certain events, so the objective function can be calculated in infeasible regions. Other advantages of this model include fast evaluation time and no driver tuning, while it has disadvantages of a simplified power-split architecture and control strategy.

The PHEV Powertrain design is subject to performance constraints of 0-60 mph time in 12 seconds or less, engine-only gradability of at least 65 mph on 5% grade, and adequate power to complete the drive cycle. No accessory loads were included in the constraints. The 0-60 time in $g_1$ is proportional to mass, and inversely proportional to total maximum power. Using component mass and power density parameters, the total vehicle mass and maximum power is found to evaluate the constraints.

The variables are battery size in kWh, engine power in kW, and combined motor power in kW. The vehicle parameters and powertrain configuration are considered fixed. The vehicle parameters represent an efficient midsize sedan, and the powertrain configuration is a power-split gas-electric hybrid, similar to the Toyota Prius. There are two motor/generators (MG) in the power-split transmission, where MG1 is used to control the engine speed, and MG2 directly transfers power to and from the wheels. Typically, MG2 is bigger than MG1 since it takes more torque to move the vehicle than the engine. In the simplified model, there is no significant distinction between MG1 and MG2, but their combined power, MG, represents the maximum electric power.

Since the proposal, three important model changes have taken place that will be mentioned here before an in-depth look in Discussion of Results. First, the objective function changed from fuel consumption in gallons per mile to a multi-objective function of gasoline and electrical energy in Wh/mile. Second, the number of variables and degrees of freedom decreased from four to three by combining MG1 and MG2 into MG power. Third, the 0-60 time constraint, $g_1$ , was modified to limit vehicle 0-60 time to 5.5 s, or an average of 0.5g acceleration.

*Model Analysis*

The objective function does not clearly reveal monotonicity with respect to the design variables, $x_1 Batt$ , $x_2 Eng$, $x_3 MG$, in kWh, kW, and kW, respectively. However, insight into the model operation, especially the control strategy will be discussed in order to analyze monotonicity. First, electrical energy is given priority over gasoline energy. Gasoline energy is used when the drive cycle demand exceeds electric power and when the battery has been depleted to the minimum SOC of 0.2. Increasing battery size increases electric power and electric range which directly reduces gasoline energy but increases electrical energy. Engine size is simply used to satisfy the constraints, all of which are highly responsive to engine size, and it has very little affect on either fuel or electric consumption. Motor size is used to regulate the power from the battery, and in this model it acts as a control parameter governing battery usage. This analysis is summarized in the monotonicity chart below.

| | x1_Batt | x2_Eng | x3_Mot |
|---|---|---|---|
| f_elec | + | + | + |
| f_gas | - | + | - |
| g1_t60 | - | - | - |
| g2_grade | + | - | |
| g3_cycle | - | - | - |
| KEY | + | Strong | |
| | + | Weak | |
| | | In Likely Region | |

Monotonicity Chart for PHEV Subsystem

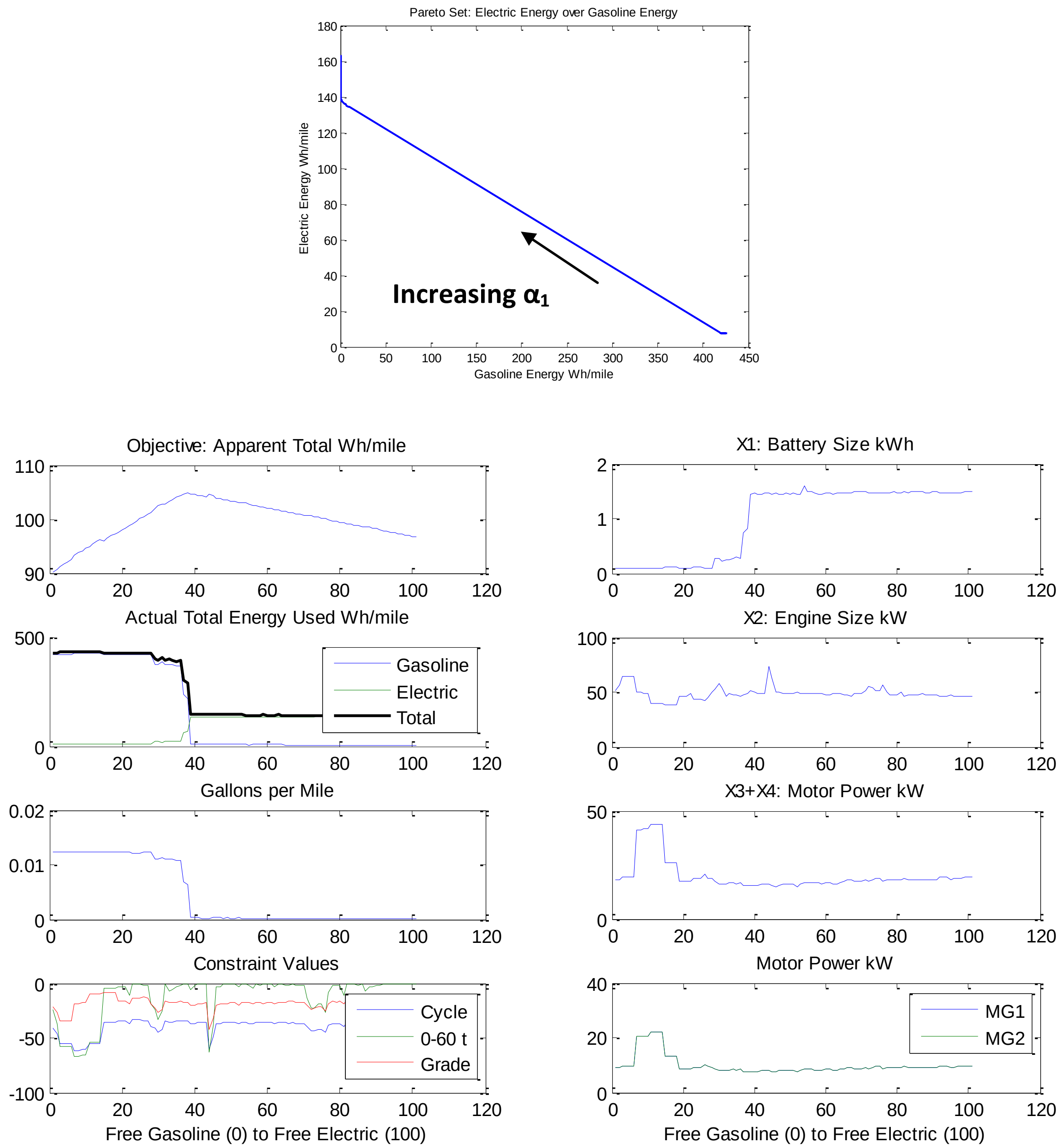
To briefly explain the monotonicity chart, any increase in battery size will increase electric consumption, decrease gasoline consumption, improve 0-60 time, worsen gradability, and improve power required to complete the cycle. An exception to this would be if motor power was much lower than the battery power, such that any increase in battery size would increase only the weight and not the power, thus reducing 0-60 time. However, this exception will most likely result in a poor objective value. The other variables will produce increasing or decreasing values as stated in the chart.

It is expected that when electric consumption is weighted highly, the resulting design will essentially become a conventional gasoline vehicle, with the electric components approaching zero. Lower bounds, which were not stated in the constraints, were placed on all the variables in order to essentially allow the variables to approach zero without blowing up evaluations to infinity. Likewise, when gasoline consumption is weighted highly, the electric components will increase to satisfy all the power and energy demands of the drive cycle.

*Optimization Study*

During the first optimization runs, the SQP algorithm in i-Sight was used with the solver parameters set to the default values. At the time, the objective function was gasoline consumption. The optimization would require many runs, and if it converged to a solution it was usually at the maximum range for battery and motor sizes, with sufficient engine size to satisfy 0-60 time. Once the battery size reached a certain power and energy size it could complete the entire drive cycle without using any gas, minimizing the objective function to 0 Wh/mile. Increasing the battery size beyond this point had no affect on fuel consumption and the objective function remained at 0 Wh/mile. This problem became too simple, since drive cycle analysis quickly indicated the maximum power and energy required to complete the cycle. For example, the drive cycle composed of the Hwy and US06 cycles requires 113 kW and 5.9 kWh over a SOC swing of 0.7 for all electric energy.

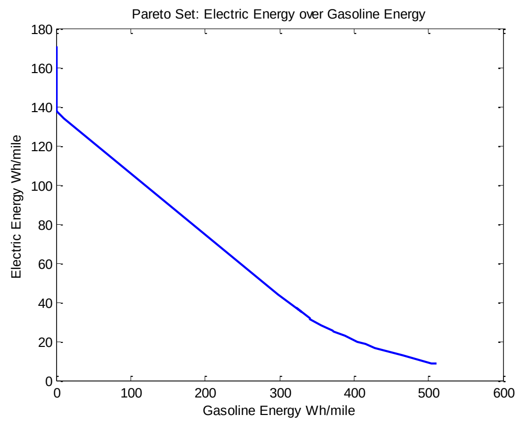
To make the optimization problem more interesting and challenging, the objective function was redefined by two competing objectives, fuel and electricity consumption. First, the solver was chosen based on speed of convergence and range of optimal design values. Downhill-Simplex in i-Sight was chosen because it converged very quickly to values that were interior to the absolute bounds placed on the variables in i-Sight. Optimal values could not be repeated precisely, but it generally converged to the same designs for the same objective function weights. The objective function was very noisy due to the nonlinear control parameters, and as a result local optimization was not successfully achieved.

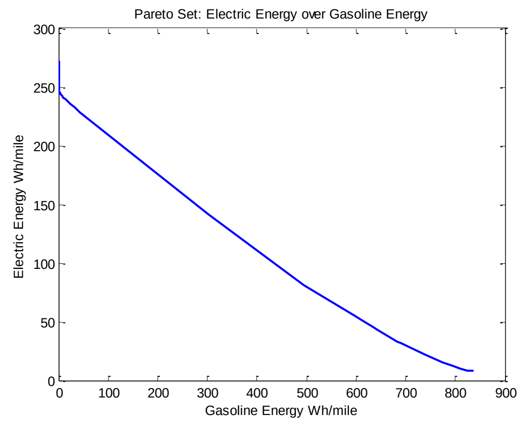



Pareto set and corresponding design variable values and constraint values

*Parametric Study*

One of the most interesting parameters was the drive cycle, since it prescribed how much energy would be consumed at certain power levels. In less aggressive drive cycles, such as the UDDS, initial hybridization was very beneficial to gasoline consumption, while only slightly increasing electrical consumption. This is because mild hybridization improved engine operation and captured braking energy very effectively in less aggressive cycles, but it did not displace large amounts of gasoline energy. In order to displace large amounts of gasoline energy, battery size had to be increased significantly, which increased mass and overall energy consumption. Two Pareto sets were obtained, the first used UDDS and the second used US06, shown below.



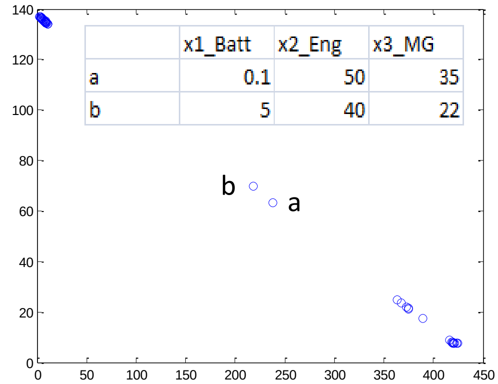

(a) (b)

Pareto sets over the drive cycles UDDS (a) and US06 (b)

In the case of UDDS, small electric components effectively decreased gasoline consumption due to the better engine operation and regenerative braking. In the case of US06, the electric components had to become large in order to effectively reduce gasoline consumption due to the high power demands of the cycle. This clearly indicates the importance of the drive cycle in optimal hybrid vehicle design.

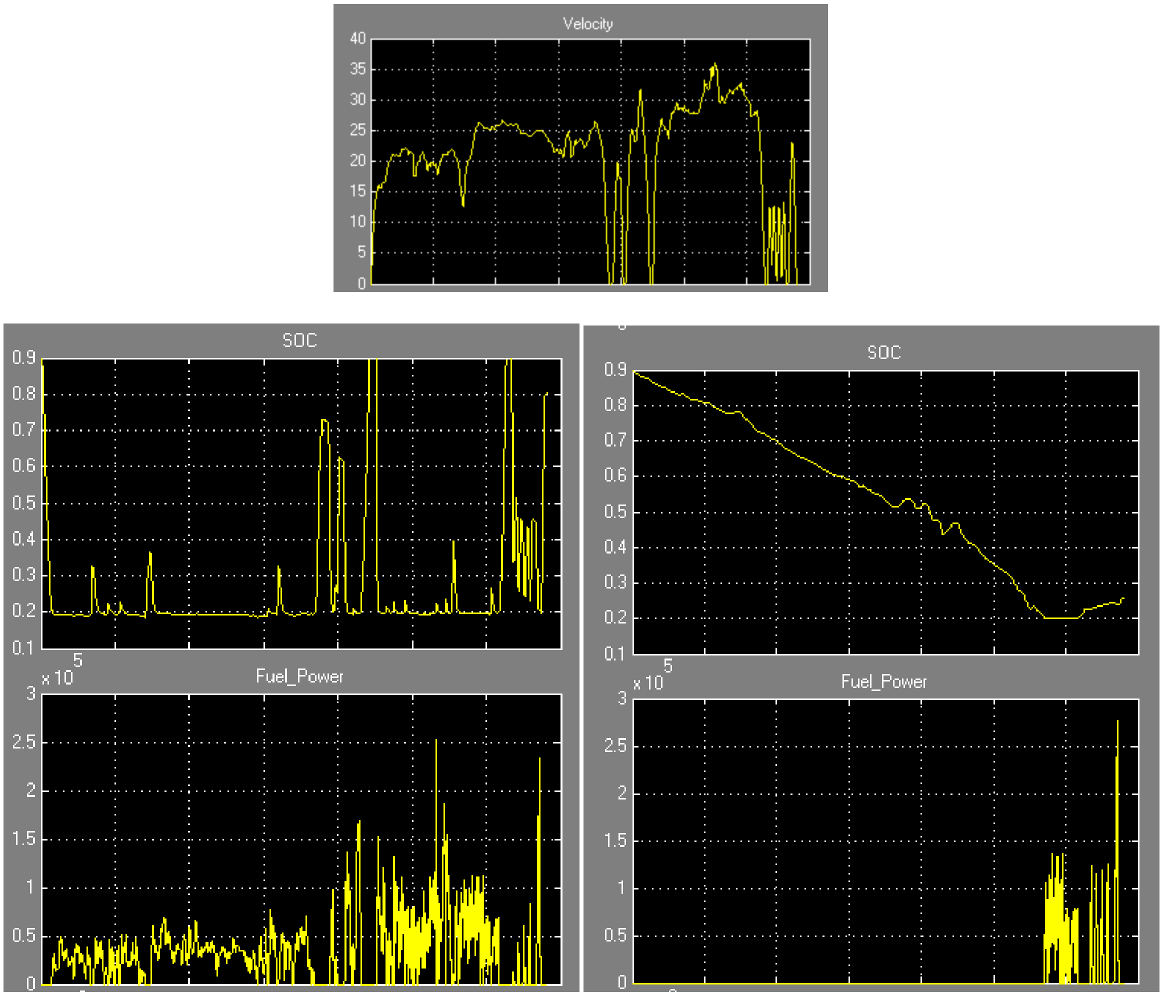
*Discussion of Results*

The most notable behavior observed in this optimization study was the transition from "mild hybrid" to "full hybrid." This distinctly happened as objective weight transitioned from electricity to gasoline. For the US06_Hwy cycle, this happened near $\alpha = [0.236\ 0.764]^T$. For other cycles, the transition occurred very near this weight. The model was observed closely as the optimum design transitioned through this point. The interesting conclusion showed that the control strategy and drive cycle played a large part in the design. A specific example is shown below over the Hwy_US06 cycle.




Finite Pareto set points near $\alpha = [0.236\ 0.764]^T$

All three of the design variables change dramatically from point a to point b in the figure above. This change marks the transition from mild hybrid to full hybrid. Point a is the best mild hybrid in terms of fuel consumption, and point b is the worst full hybrid in terms of fuel consumption. The operation shifted from primarily fuel energy to primarily electric energy, with some fuel use occurring at the end of the drive cycle of the point b design after the battery was depleted. To simply illustrate this, the simulated values for both a and b designs are shown below.




(a) (b)

Simulation of mild hybrid (a) and full hybrid (b) that produce similar objective values

Also, a simple rule was found for quickly designing the minimum electric component sizes to just complete the drive cycle without using any gasoline. The model must first run with very large electric components and use no fuel energy before applying this rule, and the rule can be repeated iteratively for better results. The engine size must meet the strictest performance constraint.

$$x_1 Batt = \max\left[\frac{(Range \times f_{elec})}{SOC_{swing}}, \frac{P_{elec,max}}{P_{E,Batt}}\right]$$

$$x_3 MG = \frac{P_{cycle,max}}{\eta_{DT}}$$

## Automotive Market – Carol Girata

The development of an automobile market model is necessary to analyze how desirable the PHEV will be to a consumer which ultimately affects its ability to penetrate the market. The relationship between producer decisions and consumer preference for the desired vehicle is imperative to evaluate the possible consumer surplus and producer profit associated with the vehicle platform. The goal of this subsystem will be to create a model that would predict design decisions of a firm whose aim is to maximize profit. The producer's decision will depend on the consumer's preference for the vehicle. Consumers will be modeled under utility theory, which assumes that they will select the vehicle that maximizes their utility. The profit will depend on the quantity of vehicles purchased and the cost to manufacture. The market model in this subsystem is based on an existing model used in a study by Michalek, Papalambros and Skerlos (MPS) in 2004 [4].

### *5.1 Problem Statement*

The objective of this sub-system is to maximize profit for a PHEV vehicle configuration. The resulting vehicle configuration determined by the design decisions will affect the cost and the market share of the firm. The optimal design will give the price and design decisions of the optimal vehicle that maximizes the profit of a given firm. The size and performance constraints on the vehicle resemble that of a Toyota Prius.

### *5.2 Mathematical Model*

Objective Function

The objective for each producer is to maximize profit. The profit function $\Pi_k$ for each producer $k$ is calculated as revenue minus cost. Assuming that the producer has perfect knowledge of the market and thus produces only the number of vehicles demanded, the revenue is equal to the price of the vehicle, $p$, times the quantity demanded, $q$.

$$\Pi_k = q_j p_j - c_j$$

The quantity of vehicles sold depends on the utility consumers would receive from the vehicle. A demand model as a function price and vehicle characteristics, z, will calculate this quantity. The vehicle characteristics under consideration will be those that are observed by the consumer therefore able to affect the purchasing decision. The characteristics used will be performance, the time to accelerate from 0 to 60, operating costs, in mpg, and the price of the vehicle.

**Variables**

Each firm is able to vary the price and design characteristics to find the optimal configuration that maximizes profit. The engine power, $x_1$, and battery energy, $x_2$, will be the two varying design characteristics.

**Constraints**

The constraints on vehicle characteristics are detailed in subsystem 1 above.

**Parameters**

In the cost model, the following are considered fixed parameters: the investment cost $c^I$, additional cost to manufacture the vehicle $c^B$ which includes the battery cost., the coefficient δ's, and the base engine size $b_M$. In the demand model, the following are considered fixed parameters: the size of the automobile market $s$ and the coefficient β's used to describe the observed portion of utility.

**Cost Model**

We will use the cost model from the paper Michalek et al [13]. The model divides cost into three components: the investment cost $c^I$, variable cost $c^V$, and the regulation cost, $c^R$. The initial subsystem model views regulation costs as a parameter. The total cost $c^P$ to manufacture $q$ units of a vehicle:

$$c^P(M,\mathbf{x}) = c^I + qc^V(M,\mathbf{x})$$

The investment cost $c^I$ is \$550 million per vehicle design. The variable cost is defined as:

$$c^V = c^B + c^E$$

where $c^B$ is the cost to manufacture the rest of the vehicle including the battery. A simple relationship was used to model the cost of the battery and the associated motor generators.

$$C^B = x_1\gamma_1 + (MG1 + MG2)\gamma_2$$

where $\gamma_1$ = 240 (\$/kWh) and $\gamma_2$ = 1 (\$/kWh). The equation for the cost to manufacture the engine $c^E$ is

$$c_E(M,x) = \{\delta_1 \exp(\delta_2 b_M x_1) if\, M \in SI\}$$

where $\delta_1$ = 670.51, $\delta_2$ = 0.0063, $\delta_3$ = 26.23, and $\delta_4$ = 1642.8. The final equation for the total cost for manufacturer k to produce q vehicles is:

$$c_k = \left( \sum_{j \in J_k} c_j^P \right) + c_k^R$$

Engineering Model: The engineering model created in subsystem 1 provides the mapping function of vehicle characteristics **z** given design variables **x**.

$$\mathbf{z} = f_M(\mathbf{x})$$

**Consumer Demand Model**

A simple logit model developed by Boyd and Mellman [16] was chosen to model consumer demand. In a logit model every consumer in the vehicle market is assumed to get the same utility from a given vehicle. The utility $u_{ij}$ provided to individual $I$ by product $j$ is composed of a deterministic component $v_{ij}$, which can be calculated based on observed characteristics, and a stochastic error component $\varepsilon_{ij}$, which is unobserved so that

$$u_{ij} = v_{ij} + \varepsilon_{ij}$$

The probability $P_{ij}$ of an individual choosing product $j$ from a set of products is

$$\begin{aligned} P_{ij} &= \Pr\left[ u_{ij} > \{u_{ij'}\}_{\forall\, j' \neq j} \right] \\ &= \Pr\left[ v_{ij} + \varepsilon_{ij} > \{v_{ij'} + \varepsilon_{ij'}\}_{\forall\, j' \neq j} \right] \end{aligned}$$

The ε error terms are unobserved random variables that are described by a probability distribution. The logit model makes the assumption that the error terms are independently and identically distributed following the double exponential distribution:

$$\begin{aligned} F_\varepsilon(\varepsilon_{ij}) &= \exp\left(-e^{-\varepsilon_{ij}}\right) \\ f_\varepsilon(\varepsilon_{ij}) &= e^{-\varepsilon_{ij}} \times \exp\left(-e^{-\varepsilon_{ij}}\right) \end{aligned}$$

this assumption leads to a very simple formula for choice probabilities:

$$P_{ij} = \frac{e^{v_{ij}}}{\sum_{j'} e^{v_{ij'}}}$$

The logit model assumes that every consumer will get the same utility from a given vehicle therefore observable component of utility $v_{ij}$ is assumed to depend only on the characteristics of the product. The term *product characteristic* is used specifically to describe objective measurable aspects of the product that are observed by and relevant to the consumer during the choice process. The value of the product characteristics of product j are written as the real-valued vector $\mathbf{z}_j$, and $v_j$ is a function of $\mathbf{z}_j$ as well as the products price $p_j$, which is not included in $\mathbf{z}_j$.

We will initially use the functional form for the observable component of utility $v$ proposed by researchers Boyd and Mellman [16]. The functional relationship for vehicles includes the price $p_j$, gas mileage $z_{j1}$, and performance measured as time to accelerate from 0-60 mph $z_{j2}$. With the following form:

$$v_j = \beta_0 p_j + \beta_1 \left( \frac{1}{z_{j1}} \right) + \beta_2 \left( \frac{1}{z_{j2}} \right)$$

where $\beta_0$, $\beta_1$, and $\beta_2$ are coefficients.

**Summary Model**

max $\Pi_k = q_j p_j - c_j$

*With respect to:* $\{M_j, \mathbf{x}_j, p_j\}$ for all $j \in J_k$

Subject to:

$g_1$ $\quad \max\left(0.65\left(\frac{m_{PHEV}}{P_{PT,max}}\right), 5.5\right) - t_{0-60} \le 0$

$g_2$ $\quad v_g\left(r_g m_{PHEV} g_c + F_2 v_g^2 + F_1 v_g + F_0\right) - x_2 Eng \cdot \bar{\eta}_{DT} \le 0$

$g_3$ $\quad P_{Cycle,max} - P_{PT,max} \le 0$

$h_1$: $\mathbf{z}_j = f_M(\mathbf{x}_j)$

$h_2$: $q = sP$

$h_3$: $v = \beta_0 p + \beta_1 \left( \frac{1}{z_1} \right) + \beta_2 \left( \frac{1}{z_2} \right)$

$h_3$: $c_j^V = c_B + \{\delta_1 \exp(\delta_2 b_M x_1)\, if\, M \in SI\}$

*Model Analysis*

It is not possible to complete a monotonicity analysis on this model because it is dependent on the results from a simulation. Relationships developed within the model will provide a check simple check for the analysis.

It was necessary to scale the design variables within the range of 1 and 200 due to a large magnitude difference between the price and all other variables. The variable scaling also created a greater need to scale the objective function. Initially on the order of 10^9, a satisfactory decrease of the scale by 10^-5 was able to capture small changes in the design variables.

*Optimization Study*

**Algorithm Selection**

During initial evaluation of the market model, MATLAB was used to execute the optimization routine. The function fmincon in MATLAB is a traditional optimization algorithm designed to find a minimum. Therefore it was necessary to optimize the negative profit. Fmincon varied the design variables in order to maximize the profit. For the starting point, an initial design decision, an equilibrium price was found. This equilibrium price is defined as the point at which changing the price will no longer increase its profit which is affected most directly by the consumer demand for that design. After the price has been found for the starting point, the expected profit is calculated at that point. Thus, the idea is to optimize the price for every design decision that determines the profit and then using those results find the design decision that generates the highest profit.

The first stage of the design process implemented a surrogate model to map design and vehicle characteristics. Fmincon initially determined that it could not calculate the gradient which led to the examination of scaling. The objective was found to be orders of magnitude larger than the design variables and therefore small changes in variables resulted in negligible changes of the optimum. After resolution of this problem, the algorithm was able to find sufficient descent but would consistently take too large a step and not be able to return to the feasible space terminating with no solution.

To improve upon the reliability of the model, the optimization package, iSight, was used to directly integrate the automobile market with the vehicle design model eliminating the need for a less reliable surrogate model. The initial optimization was run using the sequential quadratic programming, SQP, algorithm. After evaluating the optimization from different starting points, it was discovered that the function is very noisy. The vehicle model has discontinuities. Each start point found a different optimal value. The simulation is not very computationally expensive therefore an acceptable remedy for this situation was to use a gradient free algorithm to search the entire feasible design space to attempt to locate the global optimum.

**Multi-Start Analysis**

A design of experiments study was performed on the decision variables to determine a good set of starting values for the downhill simplex algorithm. Table 1 shows the results from the multi-point analysis and Table 2 lists the design variables at the optimal. The optimal value showed the max profit of 557 million dollars.

Table 1. Multiple-Start Analysis with Scaled Values

| | Starting Values | | | | | | Optimal Design Values | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Run # | x1_batt | x2_eng | x3_mg1 | x4_mg2 | x5_price | f_profit | x1_opt | x2_opt | x3_opt | x4_opt | x5_opt |
| 1 | 1 | 120.4 | 1 | 120.4 | 120.4 | -4429.8 | 55.6 | 89 | 2 | 87 | 105.7 |
| 2 | 40.8 | 200 | 40.8 | 1 | 1 | -3055.6 | 64.3 | 107 | 98 | 95 | 149.8 |
| 3 | 80.6 | 40.8 | 160.2 | 200 | 40.8 | -4303 | 57.9 | 56 | 108 | 108 | 103.3 |
| 4 | 120.4 | 160.2 | 120.4 | 160.2 | 160.2 | -4383 | 55.0 | 86 | 56 | 63 | 108.0 |
| 5 | 160.2 | 80.6 | 80.6 | 40.8 | 80.6 | -4132 | 58.7 | 155 | 146 | 146 | 108.2 |
| 6 | 200 | 1 | 200 | 80.6 | 200 | -4269.2 | 64.8 | 37 | 121 | 46 | 107.8 |

Table 2. Design Variables at Optimal Multi-Start Point

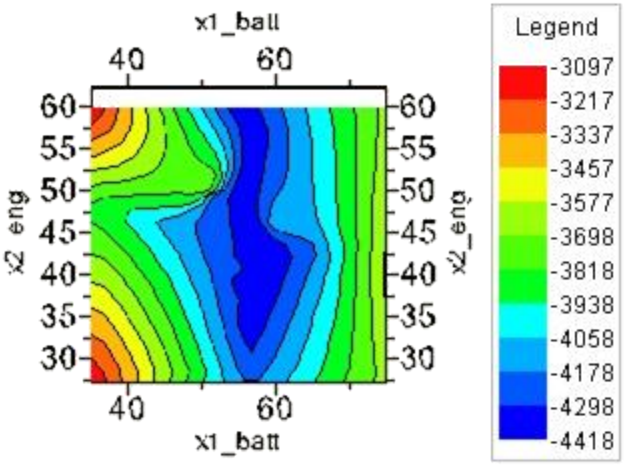
| Design Variables | | |
|---|---|---|
| Battery Energy (kWh) | Engine Power (kW) | Price ($) |
| 5.56 | 89 | 31700 |

It was not expected that the engine power would be so large. To further investigate the design and analyze the local behavior, the range of the design variables were set to within 10% of the optimal value to evaluate local convergence behavior. Local convergence was evaluated with the sequential quadratic programming algorithm as well as the non gradient based method, downhill simplex. The difference in the design variable optimal values is negligible as shown below in Table 3.

Table 3. Local Convergence Analysis

| Starting Values | | | | | | Optimal Design Values | | | |
|---|---|---|---|---|---|---|---|---|---|
| x1_batt | x2_eng | x3_mg1 | x4_mg2 | x5_price | f_profit | x1_opt | x2_opt | x3_opt | x4_opt | x5_opt |
| 55.8 | 36.5 | 79 | 79 | 106 | -4431 | 55.9 | 39 | 81 | 70 | 106 |
| 55.8 | 36.5 | 79 | 79 | 106 | -4436 | 55.9 | 36 | 74 | 71 | 107 |

The size of the engine was reduced significantly.  To understand the behavior between the Figure 1 shows a comparison of the battery capacity and engine size of the PHEV as a function of the profit.  A notable result is that the optimal battery size has been determined while a range of engine sizes could produce the same objective results.  From the design perspective, the engine size is more accurately represented as discrete values which could be realized and selected in the ranges provided.




*Constraint Activity*

During the optimization exercise, constraint activity was recoginized.  As the model attempted to maximize profit, it would consistently try to reduce the size of the engine in order to lower the manufacturing cost.  However, when this would occur the vehicle would have difficulty meeting the gradeability requirement and would subsequently have to increase the size of the engine to avoid an infeasible design.  The gradeability constraint is active at the solution.

*Parametric Study*

Utility Coefficients

The most significant parameters currently in the model are the coefficients used in the utility function. By changing the fuel economy coefficient of the utility function in the demand model, we would expect to see a change in the design of the vehicle to conform to the new desires of the consumers. Subsequently when the consumer has a greater preference for fuel economy, the producer will have to manufacture and sell a more expensive vehicle and will subsequently make the firm less profit.  It is currently cheaper to manufacture less fuel efficient vehicles that generate a greater profit. The same set of starting points considered above was used to evaluate the hypothesis. The results are consistent with expectation on the profit. The firms profit was reduced to 338 million dollars.

Table 3. Parametric Study of the Effects of Fuel Economy Utility Coefficient

| | Starting Values | | | | | | Optimal Design Values | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Run # | x1_batt | x2_eng | x3_mg1 | x4_mg2 | x5_price | f_profit | x1_opt | x2_opt | x3_opt | x4_opt | x5_opt |
| 1 | 1 | 120.4 | 1 | 120.4 | 120.4 | -3381.3 | 55.7 | 89 | 18 | 87 | 107.1 |
| 2 | 40.8 | 200 | 40.8 | 1 | 1 | -3359 | 63.5 | 107 | 98 | 95 | 149.8 |
| 3 | 80.6 | 40.8 | 160.2 | 200 | 40.8 | -3333.5 | 57.9 | 54 | 112 | 108 | 102.6 |
| 4 | 120.4 | 160.2 | 120.4 | 160.2 | 160.2 | -3193.1 | 55.0 | 86 | 56 | 63 | 107.0 |
| 5 | 160.2 | 80.6 | 80.6 | 40.8 | 80.6 | -3339.6 | 58.7 | 153 | 149 | 146 | 108.2 |
| 6 | 200 | 1 | 200 | 80.6 | 200 | -1889.4 | 64.8 | 37 | 121 | 46 | 107.8 |

The utility definition did not affect the design of the vehicle. This occurred because the design in the original optimization was already taking advantage of the largest possible fuel economy to maximize profit. It should be noted that significant changes in the utility function would affect the expected performance of the demand model. This occurs because the B&M model aimed to fit actual market data.

*Discussion of Results*

Two forces affect the accumulated profits, the manufacturing cost and the consumer demand. If the manufacturing cost is dominant, very similar vehicle designs will occur regardless of change in consumer preference. Conversely, any change in manufacturing cost will produce very different vehicle designs. On the other hand, if consumer demand dominates manufacturing costs, vehicle designs will change according to consumer demand, and very little according to manufacturing costs. In the first case, the profit margin per vehicle is most important, and in the second case the volume of vehicles sold is most important.

The PHEV has the potential to travel having lower operating costs however the design did not improve when consumer's preference for lower operating costs increased. This is attributed to the fact that the PHEV was already being maximized for fuel economy under the performance constraints. As battery technology continues to improve, the vehicle platform would be more capable of adapting to changes in consumer preferences.

A more accurate representation of the current automotive market could be obtained if the utility function of the consumer preference model was updated to show the new preferences for vehicles that have arisen due to higher fuel prices and climate change concerns. The current model was developed using data and preferences from 1977. The current market climate is very different and therefore this relationship might not be as accurate as desired however it is currently the most understood and accepted model of vehicle utility.

An important note on vehicle cost is the inability of the model to reduce other vehicle component costs as the size of the engine is reduced. If a full vehicle cost model could be determined, the affects of reducing engine size could be analyzed. For major market competitors like GM or Ford, typical business practice has shown a tendency to evolve current vehicle models to fit new designs. While this might be current best practice, there is a need to investigate the possible benefit of an entirely new vehicle design that takes advantage of the current state of the art of all technologies.

## Electric Grid – Rakesh Patil

The Electric Grid is a vital infrastructure that would be a part of the energy cycle if PHEVs make significant market entry, as hoped. Currently, the electricity demand is uneven through the course of the day and through the course of the year depending on the seasons [6], [7]. PHEVs are unique due to their ability to transfer energy to and from the grid and use the on board battery as an electricity storage unit as well as a vehicle driving power source. In addition to displacing petroleum based fuel use in the transportation infrastructure, PHEVs also have the potential to improve the resilience of the grid by making responses to major disruptions (e.g. a blackout or peak demand). However, without proper system-level design, PHEVs could have negative effects if, for example, they are charged at peak hours or if they use fuel energy derived from the IC engine to charge the battery and power the grid.

In our model of the grid, we wish to capture this advantage of the PHEV. Hence our inputs will be Grid power demand profiles, the number of PHEVs and the size of their on-board battery. The variables of interest are the times of day when the PHEV should begin and end charging to and discharging from the grid, and the choice of electricity generating power sources. The objective function would be the amount of money spent by a PHEV owner on electricity. The idea that consumers will earn money by discharging into the grid at the correct times of the day will be vital in this model. These “correct” times are stipulated by the desirable grid power demand/availability profile to be achieved. The Grid Capacity and Vehicle to Grid (V2G) ideas will be modeled based on information in [8], [9].

*Problem Statement*

The electric grid problem looks at optimizing the financial benefit of the consumer for a given electric power demand profile. After considering the daily transportation needs of the consumer (which are assumed to be an average value for all consumers), the remaining electric energy in the battery can be supplied back to the grid for some financial incentive. The grid controller decides when the cars can charge or discharge their power and it is assumed that if people are not using their cars for transportation their cars are available to the grid controller. The cost of electricity varies during the period of the day depending on the demand and the combination of sources used to produce this power. In this problem, we try to find the best combination of capacities of the power sources, the number of PHEVs and their battery size to maximize the monetary benefit for the consumer who buys a PHEV. The problem is looked at from a grid controller point of view, meaning the grid has control over all the power sources and has access to all the PHEVs, thus using them to either charge during low load demand or discharge back during high load demands.

*Mathematical Model*

**Design Variables and Parameters**

The design variables used in the subsystem are PHEV market penetration ( $p_V$ ), which gives the number of PHEVs ( $N_V$ ) assuming a certain number for the total number of vehicles in a region. The battery energy capacity ( $E_B$ ) is also a variable. The base electric load is assumed to be supplied by coal and nuclear, the maximum capacities of shifting electricity sources ( $P_C$ , $P_H$ , $P_{NG}$ ) , coal, hydro and natural gas are also variables. The shifting electricity is the difference between the current power demand and the minimum power demand (trough). All the variables are normalized before being used in the optimization routine. The base values used for normalization are 15% for PHEV market penetration, 25 kWh for the battery energy capacity and 6000 MW for maximum generating capacity for each of the three shifting electricity power sources.

The Grid controller: The grid model controller has the following two functions 1) decide when to let PHEVs charge and discharge from the grid. This ensure that the PHEVs do not charge at peak or discharge when the loads are low. It is assumed that the grid can connect or disconnect the PHEV when it is stationary (which is 23 hours a day, one hour being used for transportation.) 2) The grid has a certain structure of using power sources due to their abundance. The structure is as follows: if shifting power demand < 40% of difference between peak and trough, power source is 75% coal and 25% hydro; if 40% of difference between peak and trough < shifting power demand < 70% of difference between peak and trough, power source is 40% coal, 40% hydro, 20% natural gas; if 70% of difference between peak and trough < shifting power demand < 90% of difference between peak and trough, power source is 50% hydro, 50% natural gas; if 90% of difference between peak and trough < shifting power demand, power source is 100% natural gas. This structure tries to imitate the way the grid is operated currently.

The electric power demand profile is obtained from the website of CAISO (California Independent System Operator [7]). This is the combined electric power demand for over 30 million California residents and it is assumed that this represents approximately 90% of California's population. The data is downloaded for the year 2008.

The price of electricity per kWh and the cost of gasoline can be used as parameters. The optimization studies presented here are for different objectives. This study tries to understand the profitability of owning a PHEV for a consumer considering that the grid peak load and emissions are within certain desirable limits.

**Objective Function**

The subsystem level objective was the retail cost of electricity to the consumer due to the added load of the PHEV considering the savings from using less gasoline for transportation and discharging energy back into the grid. The consumer will be charged for the electricity consumed to charge the battery and will be rewarded for the amount of energy discharged back into the grid. Note that this is the extra cost incurred or extra money gained only because the consumer owns a PHEV, and hence does not include the currently existing electrical energy consumption from appliances etc.

$$f = \sum_{t=t_{SC}}^{t=t_{EC}} c_c(t)E_{BC} + \sum_{t=t_{SD}}^{t=t_{ED}} c_d(t)E_{BD} + (c_g - c_{elec,avg})E_{transport}$$

It is understood that the consumer might earn money depending on when he discharges into the grid, hence $c_d(t)$ can be negative, but $c_c(t)$ is the cost of the electricity consumed and this function will always be greater than zero. $c_g$ is the cost/ miles of using gasoline and it taken as a constant. $c_{elec,avg}$ is the average cost of the electricity used for transportation and this depends on the combination of power sources used to supply the grid.

**Constraints**

<u>*Physical Constraints*</u>: Since all design variables have been normalized they can take values only in [0,1]. The battery size cannot be zero as the grid subsystem would not have any meaning in that case, the minimum battery size was chosen as 4 kWh which is approximately the amount of energy needed for 12 miles of electric driving. So

$$E_B > 4kWh$$

The power demand of the electric grid should be satisfied at all times. So, the sum of base load power supplied by coal and nuclear, variable power from the wind (100% of which is incorporated into the grid) and the shifting power sources should provide the exact amount of power demanded by the grid and in doing so operate at different efficiencies in terms of cost and emissions.

<u>*Practical Constraints*</u>: The sum of the maximum power available from all the grid power sources should be higher than the maximum shifting power demand (difference between peak and base load power)

$$P_C + P_H + P_{NG} > 12960MW$$

It is reasonable to assume emission constraints in the form of kg of $CO_2$ produced per day or per month. In the initial simulations different restrictions were tried.

**Summary model**

$$\min\ f = \sum_{t=t_{SC}}^{t=t_{EC}} c_c(t)E_{BC} + \sum_{t=t_{SD}}^{t=t_{ED}} c_d(t)E_{BD} + (c_g - c_{elec,avg})E_{transport}$$

*with variables:* $p_V$ , $E_B$ and $P_C, P_H, P_{NG}$

*subject to:*

*Physical Constraints:* $E_B \geq 4kWh$

*Practical Constraints:* $P_C + P_H + P_{NG} \geq 12960MW$

*Model Analysis*

As we could not obtain an analytic expression for the objectives and the constraints in terms of the variables, monotonicity analysis could not be performed. The effects of the variables were understood on a more general level, for example, increasing the capacity of coal and natural gas sources would increase the emissions. Several constraints presented in the previous report were consolidated and the ones mentioned above were obtained.

*Optimization Studies - Results and Discussions*

First, the optimization code fmincon was used to optimize the model. It was found that starting at different points gives different results and different constraint activities also. The non-gradient based algorithm DIRECT was then used to give some results presented here. The algorithm performed number of function evaluations equal to 200 times the number of variables considered and then increments of 500 function evaluations were run till there was no appreciable improvement in the objective. This means that if there was no improvement over the last 500 function evaluations the solution was considered an acceptable optimum.

There are four cases studied here, each studying an optimization problem with different objectives, in order to understand the effect of PHEVs on cost savings on gasoline as well as the prospect of vehicle to grid.

**Case 1:** The first problem considered was to find the optimal capacities of the power sources to obtain the least electricity cost for the consumer. Note that this case does not consider any PHEVs. Here it is assumed that the grid has complete control of the capacities of the power sources and a combination of the sources to reduce the monthly bill of the consumer. The algorithm was run for a total of 1100 function evaluations and the objective function improved as shown (Figure 1).

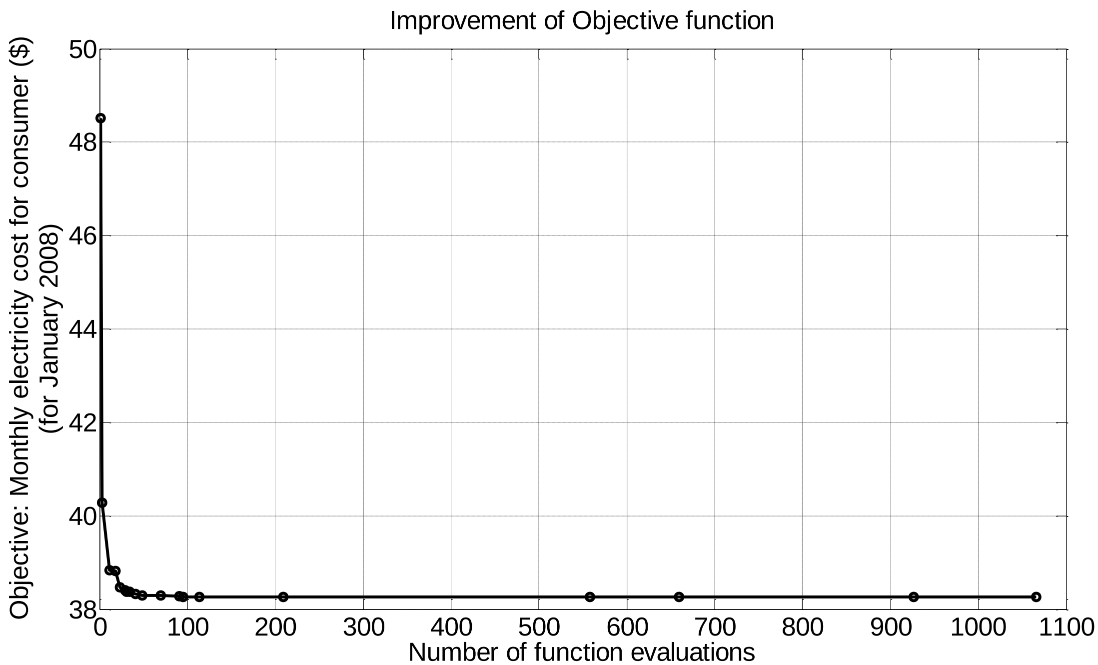



Figure 1 –improvement in objective function with function evaluations for case 1

Following are the optimal variable and objective values:

Maximum Capacity of shifting coal = 5938 MW

Maximum Capacity of shifting natural gas = 2703 MW

Maximum Capacity of shifting hydro = 4407 MW

Objective: Electricity bill per person over one month = $38.26

The results show that it is desirable to have large coal capacities, then hydro and then natural gas. This is due to two reasons: 1) The costs of energy coming from the sources is coal < hydro < natural gas. Since the cheapest electricity comes from coal and the hydro has no emissions, the optimal variables are as above. 2) The grid controller uses the power sources according to the structure explained in the model description section. At lower load demands the grid uses more coal and hydro than natural gas. Hence justifying the need for larger reserves of these sources.

**Case 2:** Here the problem is set up to find the optimal capacities of the power sources, market share of PHEV and battery size in the PHEV to obtain the least electricity cost for the consumer, which is the first term in the expression for objective mentioned above in the summary model section. The algorithm was run for a total of 1000 function evaluations and the objective function improved as shown (Figure 2).

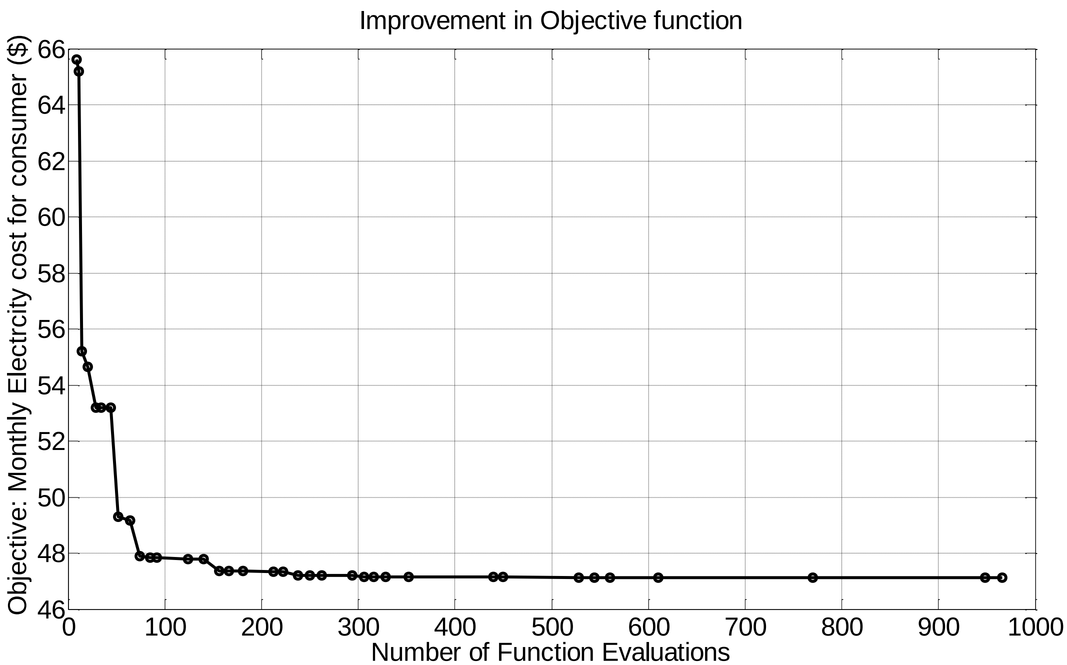



Figure 2 –improvement in objective function with function evaluations for case 2

Following are the optimal variable and objective values:

Market Share of PHEV = 3.429*10^-3 %

Battery Size in the PHEV = 4 kWh

Maximum Capacity of shifting coal = 5781 MW

Maximum Capacity of shifting natural gas = 4004 MW

Maximum Capacity of shifting hydro = 4380 MW

Objective: Electricity bill per person over one month = $47.13

The result shows that if the cost savings of the PHEV for transportation or discharging back into the grid are not considered the smallest battery size is the optimal. We can also see that the capacities of the power sources desired are similar to case 1 for coal and hydro but higher for natural gas. This is due to the structure of power usage in the grid controller which uses more natural gas as the load demands are higher. The load demand is higher due to charging of the batteries.

**Case 3:** Here the problem is set up to find the optimal capacities of the power sources, market share of PHEV and battery size in the PHEV to obtain the least electricity cost for the consumer while considering the savings in transportation, which is the first and third terms in the expression for objective mentioned above in the summary model section. This study compared the cost savings in gasoline consumption of a 25 MPG car, with gas at $2/gallon. The algorithm was run for a total of 1500 function evaluations and the objective function improved as shown (Figure 3).

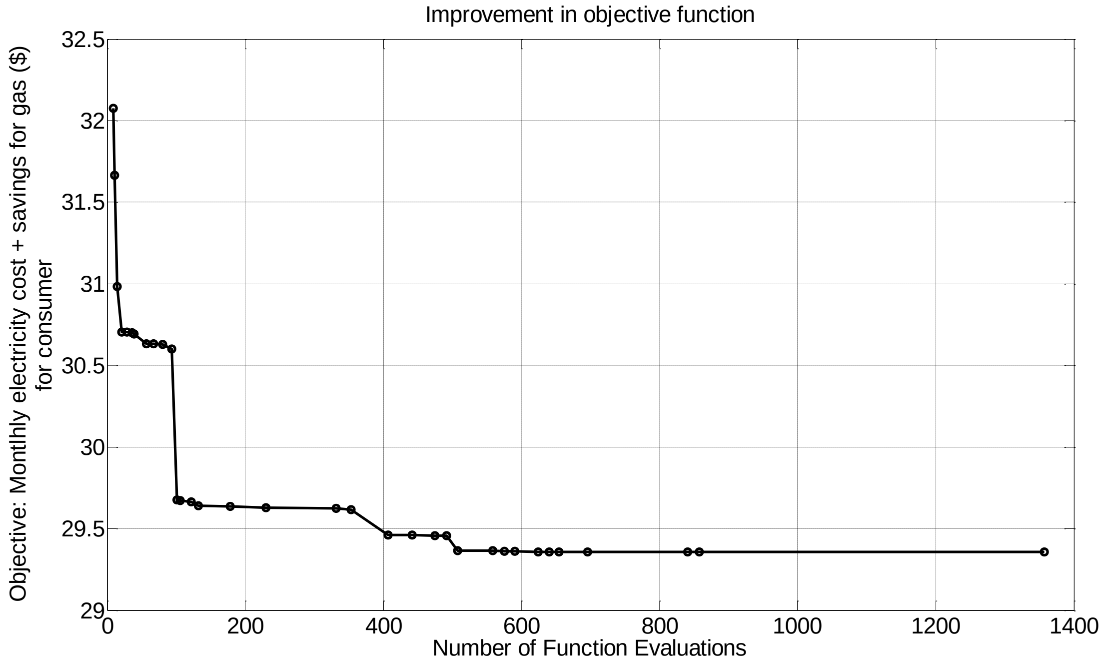



Figure 3 –improvement in objective function with function evaluations for case 3

Following are the optimal variable and objective values:

Market Share of PHEV = 0.55 %

Battery Size in the PHEV = 14.645 kWh

Maximum Capacity of shifting coal = 5946.5 MW

Maximum Capacity of shifting natural gas = 2720.2 MW

Maximum Capacity of shifting hydro = 4300.4 MW

Objective: Electricity bill per person over one month considering savings in gas usage for transportation = $29.36

Comparing the results to case 1 and 2 we see that by considering the cost savings in transportation due to lower usage of gasoline at the optimum the consumer is able to save money over the extra electricity usage. Running the model with the above optimum variable values we see that the electric bill of the consumer is $63.31 and the savings in gas cost is $33.95. As the battery size and number of PHEVs increase there is an increase in the cost/kWh of electricity due to the increase in load on the grid. This increase in cost/kWh as well as the increase in consumption result in higher electric cost than the previous two cases but it is compensated by the savings earned due to lower gas usage. It was assumed that on average people drive 1000 miles a month, which results in $80 of gasoline costs. Thus we can see that this consideration itself justifies the use of PHEVs keeping the emissions constrained to the same level as in cases 1 and 2.

**Case 4:** Here the problem is set up to find the optimal capacities of the power sources, market share of PHEV, battery size in the PHEV to obtain the least electricity cost for the consumer while considering the savings in transportation and the money earned by discharging electricity back into the grid, which is the expression for objective mentioned above in the summary model section. This study compared the cost savings in gasoline consumption of a 25 MPG car, with gas at $2/gallon. It is assumed that money earned by discharging back to the grid is at 0.8 times the cost of electricity at the time of discharging. The algorithm was run for a total of 1500 function evaluations and the objective function improved as shown (Figure 4).

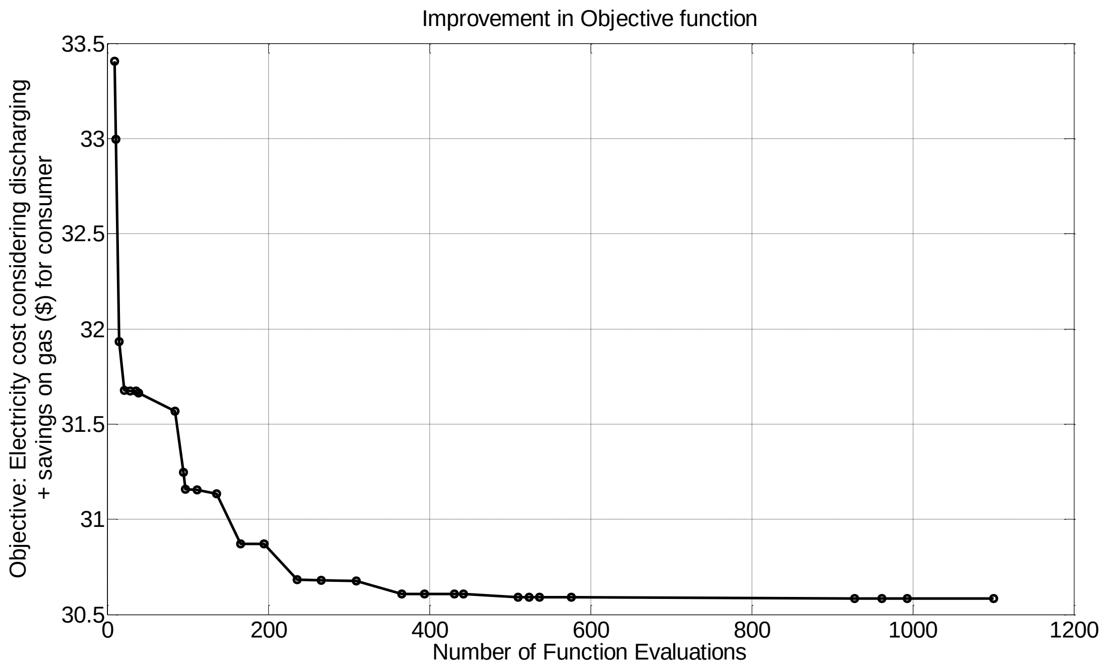



Figure 4 –improvement in objective function with function evaluations for case 4

Following are the optimal variable and objective values:

Market Share of PHEV = 0.36 %

Battery Size in the PHEV = 13.337 kWh

Maximum Capacity of shifting coal = 5672.2 MW

Maximum Capacity of shifting natural gas = 2541.8 MW

Maximum Capacity of shifting hydro = 4923.2 MW

Objective: Electricity bill per person over one month considering savings in gas usage for transportation = $30.58

Running the model with the above optimum variable values we see that the electric bill of the consumer is $58.02 and the savings in gas cost is $27.44. Comparing the results of case 3 to 4 according to the cost structure assumed in the grid and the gas price of $2/gallon, it is actually not profitable to discharge energy back into the grid. As gas prices rise, from a monetary point of view it is financially more advantageous to use the battery for transportation than discharging back into the grid. Another point to note is that the optimal shifting reserves of both coal and natural gas are lower than in case 3, while the hydro reserves are higher. This is due to the reduction in peak demands which is an effect of batteries discharging back into the grid at higher loads. This results in a reduction of $CO_2$ emissions over a period of one month from 58900 kg to 58818 kg. This effect offers an interesting avenue to explore at the system level.

# SYSTEM INTEGRATION

To analyze the entire system, an absolute population was chosen to operate in. The availability of the grid power demand schedule for most of California led to selecting that state's approximate population of 29.7M as the absolute population in the system. This was the number of daily drivers and consumers of electricity. For clarification, no geographic or demographic traits associated with California affected the parameters in the Market or Grid subsystems.

Assumptions regarding the population were that all PHEV owners drove the average mileage for the U.S. at 12500 miles per year (~16 mile commute) at a rate of 0.3 kWh/mile. Each PHEV produced so many tailpipe emissions according to the PHEV Fuel Economy model over a 16 mile drive cycle consisting of the federal city test and supplemental FTP (UDDS + US06). Each non-PHEV, or conventional vehicle, had the average U.S. fuel economy of 20.4 mpg and drove the same annual milage.

Mathematical Model

Objective Function

$$\min f(x) = \alpha_1 f_{CO_2}(x) + \alpha_2 f_{Profit}(x)$$

Subject to

$$g_1 \qquad \max\left(0.65\left(\frac{m_{PHEV}}{P_{PT,max}}\right), 5.5\right) - \bar{t}_{0-60} \leq 0$$

$$g_2 \qquad v_g\left(r_g m_{PHEV}\, g_c + F_2 v_g^2 + F_1 v_g + F_0\right) - x_2 Eng \cdot \bar{\eta}_{DT} \leq 0$$

$$g_3 \qquad P_{Cycle,max} - P_{PT,max} \leq 0$$

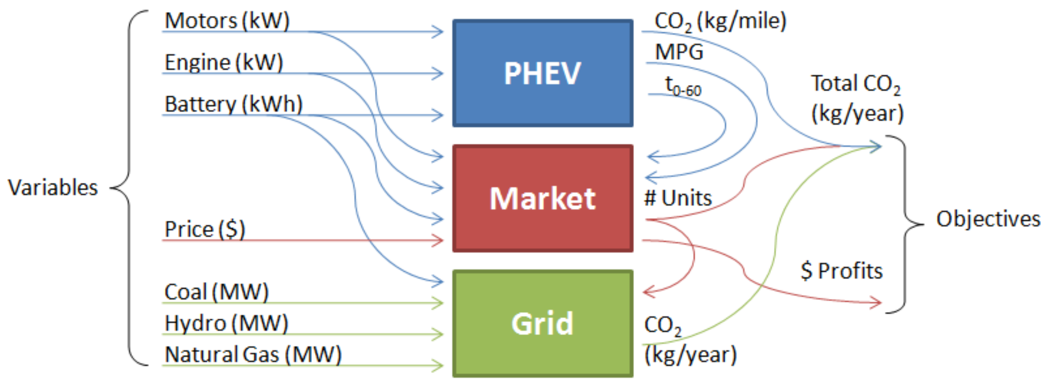
$$g_4 \qquad 28424\ MW - (P_C + P_H + P_{NG}) \leq 0$$

$$g_5 \qquad \varepsilon_{RMS} - 1\mathrm{e} - 3 \leq 0$$

Where

$f_{CO_2}$ represents the percent change in the population's GHG emissions with PHEVs compared to without PHEVs

$f_{Profit}$ is the manufacturer's profit LOSSES in \$M, where losses < 0 are desirable to the manufacturers.



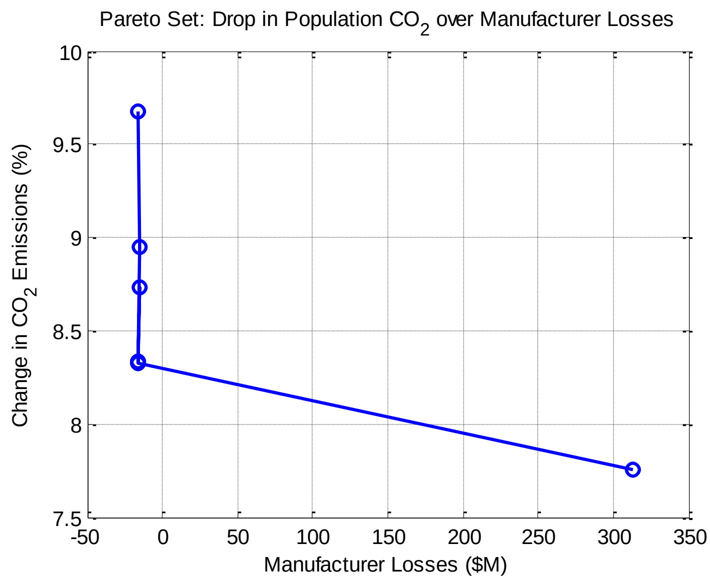

Interconnections between subsystems to make the integrated system

From the PHEV and Market subsystems, the tradeoff between lower gasoline consumption and greater manufacturing costs due to increasing battery size became well understood. The main purpose of the integrated system was to discover what happens to the grid emissions when adding transportation load but also storage capacity. The change in grid emissions includes the grid and the all the cars, both conventional and PHEV. Initial observations showed that the most effective way to reduce GHG emissions was to sell the PHEV at a loss, resulting in the multi-objective function, above. The first three inequality constraints come from the PHEV subsystem, while the last two apply to the Grid. The fifth constraint, $g_5$, allows some error between the power supplied and the power demanded, noting that the error has an order of magnitude 1e 4.

## Discussion of Results

The system results showed that the GHG emissions are highly dependent upon the grid composition. By transferring transportation loads to the grid, the grid emissions increased more so than the tailpipe emissions decreased. Thus, considering the grid variables from Case 1 in the Grid subsystem optimization, which considered no PHEVs, the integrated system produced more emissions. Along the Pareto set, the gradability constraint and both grid constraints were active.

One clear trend that was observed was that in order to significantly reduce emissions by using PHEVs, it comes with a great cost to the manufacturer. The troubling part about the Pareto set, shown below, was the optimal solution could not return to the baseline case, where no PHEVs were present. This is most likely due to the starting point, which assumed a desirable design for manufacturing. Also, although 11 optimization points were chosen, only 5 distinct results appear, suggesting that the SQP easily falls into local optima.




Although much more could be achieved through more advanced models, it appears the existing model can offer more information by experimenting with different solvers, perhaps more exploratory solvers such as DIRECT or NOMAD. The authors wish to further investigate into these areas.

# ACKNOWLEDGMENTS

We would like to thank Prof. Ian Hiskens of EECS and Prof. Steve Skerlos of the Mechanical Engineering Department for their valuable guidance. We would also like to thank research assistants Scott Moura and Brian Adornato for pointing us towards potential sources of data to be used for grid modeling.

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# APPENDIX

(see attached Appendix)