OPTIMAL DESIGN OF AN EXTENDED RANGE ELECTRIC VEHICLE

by

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Abstract

The concept of an extended range electric vehicle (E-REV) is a new one that is being ushered to market through a particularly fast product development cycle. With such a short concept-to-production timeline certain aspects of the design could benefit from a formal optimization study.

This optimization study manipulates the Volt's T-shaped battery pack dimensions as well as the locations of four key vehicle components – the front seats, the rear seats, the vehicle wheelbase and the battery pack- in search of maximum all-electric range (AER), occupant comfort and the best possible vehicle handling characteristics.

An optimum set of these design variables were found by using an SQP algorithm. The optimum design has a battery volume of 0.2m^3 while placing the front and rear seats according to optimality conditions provided by Hamza et al. and the understeer coefficient increased approximately 9% from the optimal design in the handling subsystem, still maintaining good handling characteristics across the range of driving conditions.

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1.) Introduction

In 2010, GM plans to release the Chevrolet Volt as its brand new electric vehicle. With the growing demand for green technologies, and gasoline independence, this release is being welcomed with open arms by both the public as well as the rest of the automotive industry. Current IC engines have gone through many years of designs and redesigns. This has lead to the current vehicle design which has been optimized throughout the long course of its development. The release of this vehicle has already sparked other car makers to begin development of their own electric vehicles, and thus begins a new age in vehicle design. Designers can no longer design cars based off of “accepted norms” because they haven’t been established yet, and thus optimizations are necessary to ensure vehicle quality.

One of the largest tasks the auto companies have is to try to maximize the all electric range of their vehicles, while still maintaining current comfort, safety, and handling norms. General Motors’ research has revealed that most consumers suffer from what they call “range anxiety.” That is, a substantial feeling of angst as the charge on their ell-electric vehicle nears zero. Their best solution to this problem is the Chevy Volt which offers 40 miles of AER and additionally can sustain mobility via a small IC engine. An AER of 40 miles was chosen because their research showed that the majority of drivers travel less than 40 miles on their daily commute. The idea is to dramatically reduce the use of gasoline, a fuel made controversial for its recent price instability and for its heavy ties to foreign countries. Undoubtedly, increasing the AER will allow further reduced dependence on foreign oil and, perhaps more importantly, is necessary if they are to achieve their planned goal of pure-electric vehicles in the near future.

The Chevy volt has decided on a particular design, and it is our goal to try to optimize their design by maximizing the amount of battery space (which would lead to more mileage) while still maintaining safe and responsive handling characteristics. These are the two sub-systems which will be optimized.

The two-subsystems are linked through the location of the c.g. of the battery, and the passenger seats. As the battery volume increases, the rear and forward seats are going to have to move to accommodate the increased growth. All of these will affect the overall c.g. of the vehicle, which will thus affect the handling characteristics. This overlap in design variables creates a large trade off between optimizing the battery volume, and optimizing the handling characteristics. It is anticipated that the optimized battery volume will fill as much of the rear section of the passenger cabin as possible (constrained by mostly comfort constraints). This will push the c.g. of the vehicle towards the rear wheels, reducing the handling characteristics; thus resulting in the sub-system optimizations being in opposition.

2. Nomenclature

2.a) Battery Subsystem

Symbol	Description	Units	Symbol	Description	Units
ArmX*	Battery arm x dimension (length)	[mm]	$L_{firewall}$†	Length from front end to firewall	[mm]
ArmY*	Battery arm y dimension (width)	[mm]	$L_{overall}$†	Length of vehicle	[mm]
ArmZ*	Battery arm z dimension (height)	[mm]	$L_{seat\ bottom}$†	Length of seat bottom (x dim)	[mm]
BaseX*	Battery arm x dimension (length)	[mm]	L_{trunk}‡	Length of trunk (cabin to rear end)	[mm]
BaseY*	Battery arm y dimension (width)	[mm]	$L_{wheelbase}$‡	Length of wheelbase	[mm]
BaseZ†	Battery arm z dimension (height)	[mm]	$t_{seat\ back}$†	Thickness of seat back	[mm]
Leg_f‡	Front leg room	[mm]	$t_{seat\ bottom}$†	Thickness of seat bottom	[mm]
Leg_r‡	Rear leg room	[mm]	$w_{seat\ bottom}$†	Width of seat (y dim)	[mm]
L_{cabin}*	Cabin length (firewall to trunk)	[mm]	$w_{vehicle}$†	Vehicle width (y dim)	[mm]
L_{FrSeat}*	Front seat position (from front of vehicle to front of seat-back.	[mm]	AER‡	All electric range	[miles]
e1†	Volt overall length	[mm]	e2†	Max BaseY length	[mm]
e3†	Prius front leg room	[mm]	e4†	Prius rear leg room	[mm]
e5†	Est.Volt tire outer diameter	[mm]	e6†	Est. Volt length to firewall from front	[mm]
e7†	Typical seat width (y-direction)	[mm]	e8†	Min. required UDDS fuel economy	[mpg]
e9†	Door panel widths (y-direction)	[mm]	e10†	Est. Volt pedal to firewall distance	[mm]
e11†	Volt official vehicle width	[mm]	e12†	ArmZ parameter	[mm]
e13†	Typical thickness of seat-back	[mm]	e14†	Typical seat-bottom length (x-dir)	[mm]
e15†	Max height of BaseZ	[mm]			[mm]

Table 1 - Symbols and descriptions for the battery subsystem

Note: values with an * denote design variables, †denote vehicle parameters, ‡denote calculated quantities.

2.b) Handling Subsystem

Symbol	Description	Units	Symbol	Description	Units
$C_{\alpha f}$†	Combined cornering Stiffness on the front tires.	[N/rad]	M†	Mass of vehicle	[kg]
$C_{\alpha r}$†	Combined cornering Stiffness on the front tires.	[N/rad]	m_B†	Mass Ratio of Battery	[kg]
C_B*	Distance to c.g. of the Battery	[m]	m_E‡	Mass Ratio of Excess	[kg]
C_E‡	Distance to c.g. of the Excess Mass	[m]	m_F†	Mass Ratio of Front Passengers	[kg]
C_F*	Distance to c.g. of the Front Passengers	[m]	m_R†	Mass Ratio of Rear Passengers	[kg]
C_G‡	Distance to c.g. of entire vehicle	[m]	m_P†	Mass Ratio of Powertrain	[kg]
C_P*	Distance to c.g. of the Powertrain	[m]	L_{front}†	Distance to the front axle	[m]
C_R*	Distance to c.g. of the Rear Passengers	[m]	$L_{overall}$†	Length of Vehicle	[m]
δ†	Steering angle	[rad]	$L_{wheelbase}$*	Distance between front and rear axles	[m]
d1†	Constraint on how far forward the front passengers can be	[m]	R†	Radius of curvature	[m]
d2†	Constraint on how far forward the rear seats can be	[m]	R_T†	Radius of tire	[m]
d3†	Minimum constraint for the wheelbase	[m]	v_0†	Vehicle forward velocity	[m/s]
g†	Acceleration due to Gravity	[m/s^2]	W_F‡	Weight on front axle	[N]
K_{US}‡	Understeer Coefficient		W_R‡	Weight on rear axle	[N]

Table 2 - Table of symbols and descriptions for the handling subsystem

Note: all distances measured from the front of the bumper unless otherwise noted, values with an * denote design variables, †denote vehicle parameters, ‡denote calculated quantities.

3) Subsystem Analysis

3.a) Battery Packaging Subsystem

In order to provide sufficient electric-only range, the vehicle must dedicate a significant portion of its volume to the battery. Generally speaking, increasing the size of the battery pack will allow for more energy storage. Higher energy storage affords the vehicle more all-electric drive range. However, increasing the energy storage comes at the expense of additional vehicle mass. Increased mass should yield reduced fuel economy and reduced all-electric drive range. There is a tradeoff worth investigating here.

Extended Range Electric Vehicles (E-REVs), namely those based on the Chevrolet Volt platform, currently carry the battery pack in T-shaped package that runs down the passenger compartment with the top of the 'T' residing beneath the rear passenger seats. Because of its sheer size, the battery packaging significantly affects other interior packaging.

Thus we see the interactions between battery pack dimensions, electric drive range, and interior occupant comfort. Larger pack dimensions will yield more energy storage but will reduce occupant space (assuming fixed overall vehicle dimensions) and thus reduce occupant comfort. This interaction of pack dimensions and occupant space will be the essence of this subsystem's optimization.

The previous work put into battery packaging for an E-REV has principally been done by GM during its product development for the Chevrolet Volt. We choose to build upon their work by basing our optimization model heavily on the Volt. The most qualitative example of this is that we constrain ourselves to a T-shaped battery pack. This follows the assumption that the extensive product development done by GM revealed the general T-shaped pack to be the best. Rather than attempt to find an optimum shape for a battery pack, we focus on finding the dimensions for the T-shaped pack subject to our occupant comfort.

We note now that much of our constraints and design assumptions stem from the idea that we are working with the Volt. In other words, we are optimizing the battery pack dimensions while staying within the existing dimensions of the Volt. As a result, many constraints and assumptions draw on existing dimensional specifications for the Volt or its closest known vehicle segment competitors.

3.a.1) Mathematical Model

Objective function

The design objective is to maximize the all-electric range and front and rear occupant comfort for the vehicle. The objective function evenly weighs each of these. PSAT simulations were used to create a surrogate model relating all-electric range and fuel economy as a function of volume of the battery pack. The occupancy comfort portion of the objective function comes from a set of optimal positions for driver and passenger hip points for a mid-sized passenger vehicle (Hamza et al.)

We now draw attention to the PSAT vehicle simulation model. The PSAT simulation model was created by Rakesh Patil of the Lay Automotive Labs at the University of Michigan as part of his research work. His work pertained to a parametric study aimed towards identifying the effects of increased battery storage on fuel economy, all electric range, emissions, and several other important vehicle performance metrics. His research involving the model will be the focus of an upcoming publication. The model was specifically aimed at modifying a Chevrolet Volt.

A separate model was independently made in the AVL Cruise vehicle simulation environment. It was simulated over a wide range of battery volumes but because of higher experience and credibility behind the Patil PSAT simulation data, the in-house AVL Cruise model was put aside. It is available by request through the author of this subsection.

The exact parameters of the PSAT are included in Appendix B. All credit towards developing the model is given to Rakesh Patil for his excellent work.

Given the general shape of the battery pack to be two rectangular prisms as shown in Figure 1, taken from GM Media Online, we note that the pack volume is the simple equation for a rectangular solid. This equation is simply

$$vol = length * width * height$$

The T-shaped-battery pack volume, is the sum of two such solids.

$$Vol_{batt} = ArmX * ArmY * ArmZ + BaseX * BaseY * BaseZ$$

Because our surrogate models express all-electric range and fuel economy as functions of battery pack volume, we denote them as $AERange = f(Vol_{batt})$, $FuelEcon = f(Vol_{batt})$.

Regarding occupant comfort, Hamza et al. provided a set of optimal locations of driver and rear passenger hip points for maximum comfort. Rather than force these dimensions on our model as constraints, we opt to weigh them into our objective function. Any deviation from these optimal positions would result in a reduced objective function value. We describe the proximity of the design variables for front and rear seat position to the optimal location with rear seat optimality calculated similarly as

$$FrontSeat_{optimality} = \frac{\left|\left(L_{FrSeat} - L_{FrSeat_{OPTIMAL}}\right)\right|}{L_{FrSeat_{OPTIMAL}}}$$

The objective function combines all together as

$$Max\ f = \frac{1}{3}(AER/40) + \frac{1}{3}\left(FrontSeat_{optimality}\right) + \frac{1}{3}\left(RearSeat_{optimality}\right)$$

The decision to normalize all-electric range by 40 stems from GM's extensive research concluding that the vast majority of drivers in the U.S. drive for 40 miles or less per day. Thus the gold standard for all-electric range is 40 miles. Any additional electric range would boost the weighted objective contribution further.

The battery arm height will be fixed in this study as parameter. This was done to avoid having to include other ergonomic aspects into our model. The height of the arm relates to the height of the center console in between the front passengers. To model it appropriately as a design variable would require constraints related to occupant safety during side impact collisions, non-forward visibility limitations, arm rest height comfort ratings, and other complex and relatively ambiguous parameters. For those reasons, as well as to keep the number of design variables to a manageable range, we fix ArmZ as a simple parameter, e_{14}, inspired by the current Volt's standard battery pack's dimension.

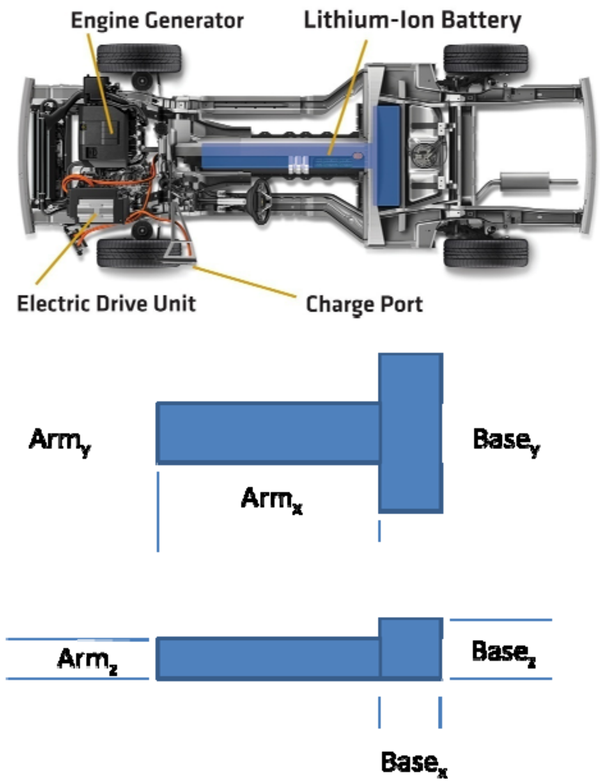

Figure 1 - The Volt battery pack in place, and the various dimensions used to describe the battery pack volume in this optimization problem.

Constraints

Because this project aims to optimize the Volt's design, we must draw on its existing dimensions to provide constraints so that this project's results will be directly applicable to the Volt. The first constraint is the overall vehicle length. Without a constraint on overall length, the battery could grow without bound, providing an unrealistic system to optimize. The overall length is comprised of three sections: the front end, the cabin, and the trunk. The sum of these sections must be equal to the existing Volt's length.

$$g_1:\ L_{firewall} + L_{cabin} + L_{trunk} = e1$$

Focusing attention on the cabin section of the vehicle, we examine the constraints that limit the battery pack dimensions. In keeping with the current Volt design, the overall longitudinal length of the battery pack cannot spill into the trunk space. Therefore we say that the longitudinal components of the pack dimensions must be less than or equal to the length of the cabin.

$$g_2\colon\ ArmX + BaseX \le L_{cabin}$$

The base of the battery, the 'top' of the 'T' that rests beneath the rear seats, carries a dimensional constraint based on the current Volt platform design. As can be seen in Figure 1**,** above**,** there exists a bay within which the base must be seated. This bay will impose an upper bound on the BaseY dimension of the pack.

$$g_3\colon\ BaseY \le e_2$$

A constraint also exists on the height of the battery pack's base. Because the rear seats sit atop the pack's base, an increase in pack base height yields an increase in rear seat height which in turn reduces rear occupant headroom. We wish to maintain satisfactory occupant head room. Although no official interior occupant dimensions have been released by GM, we do know that a relevant competitor would be the Toyota Prius. Although the Prius is not an E-REV like the Volt, we see from the Volt's other dimensions that the Volt designers must have had the Prius in mind. To provide evidence to this claim we briefly look at the dimensions officially released by GM for the Volt and compare them to the Prius's respective dimensions. Table 1 presents the data.

Dimension	**Chevrolet Volt**	**Toyota Prius**
Wheelbase [mm]	2685	2700
Length [mm]	4404	4445
Width [mm]	1798	1725
Height [mm]	1430	1491

Table 3 - Comparison of Volt and Prius dimensions shows them to be in similar markets regarding occupant comfort.

Having established that the Volt and Prius will likely share very similar dimensions and that the Volt designers likely expected to be closely compared to the Prius, we conclude that it is reasonable to establish constraints on the optimization project such that we must meet or exceed

Prius occupant comforts. The following constraints for front and rear leg room stem from this. Figure 2 shows the calculation of legroom for front and rear occupants.

$$Leg_f \geq e_3$$

$$Leg_r \geq e_4$$

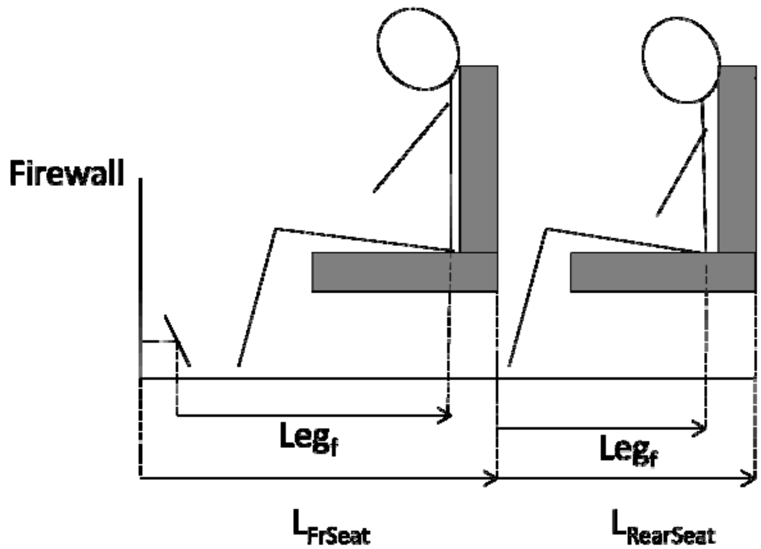

Figure 2 - Leg$_f$ and Leg$_r$ definitions shown visually.

We now define Leg$_f$ and Leg$_r$ mathematically as functions of the other design parameters.

$$Leg_f = L_{frSeat} - e_{15} - e_7 - e_{12}$$

$$Leg_r = L_{cabin} - e_{15} - L_{frSeat}$$

Parameter e7 is the Volt's firewall length, taken to be constant for this optimization problem and e12 is the distance from the firewall to the accelerator pedal and e15 is the seat back thickness. We can now rewrite constraints g4an d g5 as

$$g4\text{: } L_{FrSeat} - e_{15} - (e_7 + e_{12}) \geq -e_3$$

$$g5\text{: } L_{cabin} - e_{15} - (L_{frSeat} - L_{firewall}) \geq e_4$$

The next constraint deals with fuel economy. Because increases in battery pack volume lead to increases in both energy storage capacity and mass, there is a chance that fuel economy will decrease with enough additional battery volume. Because of this we instill a fuel economy minimum constraint. Again we refer to the current highest-selling hybrid, the Prius. The 2010

Prius's preliminary fuel economy numbers have been announced at 50 mpg. Its fuel economy becomes a minimum constraint.

$$g6\colon FuelEcon \geq e_{10}$$

We now spend a moment to explore the constraints on L_{trunk}. The cabin length is variable in this optimization study, and the length forward of the cabin (the $L_{firewall}$) is fixed. This leaves us in need of a constraint for trunk length so that the cabin cannot run to the end of the vehicle. To find this constraint we look to a trait common to all vehicles of moderate ride height and that is that the rear wheels must be placed behind the rear seat. Usually, the rear seats end right where the rear wheels start. The only constant exception to this rule of thumb are vehicles whose bodies ride high enough that the wheels can literally rest beneath the passenger cabin. For example, transit busses and large SUVs commonly have seats above the tires because they vehicles are tall enough to allow this. Finally, we point out that the current Volt design also embodies this typical design cue. From this we attain a constraint that the trunk length (length from the end of the cabin to the rear of the vehicle) must at least be long enough to house the rear wheel / tire length. Note that this will relationally set a constraint on the wheelbase of the vehicle as the wheelbase and L_{trunk} are directly related.

$$g7\colon\ L_{trunk} \geq e_6$$

The $L_{firewall}$ will remain fixed for this study. This follows the assumption that the front end was designed under much scrutiny by GM. The front end length affects things such as crash safety and powertrain cooling and packaging in such a way that we have deemed it too complex to vary as part of this project's analysis. This leads to a set length.

$$L_{firewall} = e_7$$

There exists a rule of thumb recommended by the National Highway Traffic Safety Administration related to driver's seat distance from the steering wheel. NHTSA recommends that for reduced risk of airbag deployment related injury the driver should be seated 10 to 12 inches (254 to 305 mm) from the steering wheel. The steering wheel distance from the heal point forward was measured from the Volt platform image to be 570 mm. We adapt this into a constraint on seat position but acknowledge up front that it is overlapped by the front seat leg room constraint of g4. Because the leg room is defined by SAE J1100 as the distance from the heel point to the front seat's seatback, we can state the NHTSA requirements as a constraint on leg room.

This seemingly relevant constraint would take the form of the equation given below.

$$L_{FrSeat} - e_{15} - e_7 \geq e_8$$

However, this directly competes with constraint g_7. Because constraint g_7 imposes a stricter constraint (e_6 is greater than e_8), we leave g_7 intact and do not use the NHTSA requirement to impose a (weaker) constraint on leg room.

We also have also a constraint for the maximum Arm_y dimension. Because the pack rides between the two front seats (each of width e_9), we must ensure that the Arm_y dimension is bounded to the Volt's vehicle width overall width minus some amount of space dedicated to interior door panels. The door panel widths, e_{11}, will be estimated for future reports but at the moment is left simply as a parameter to be defined at a later time. It is not a function of any other variables, etc. and will be treated as a constant after a suitable value is found (e_{11}).

$$g8\text{: } ArmY + 2 * e_9 \leq \mathrm{W_{veh}} - 2\mathrm{e_{11}}$$

The next constraint for the subsystem involves the battery pack dimension BaseX. We must simply keep the base of the pack from becoming long enough that it intersects with the front seat. Mathematically this is

$$g9\text{:} L_{frSeat} - e_7 + BaseX \leq L_{cabin}$$

where e15 is the parameter for seat thickness.

The final remaining constraints on the subsystem refer to the positioning of the front seats, which is a design variable. Intuitively, we know the front seat has to be in the cabin. We also know that the front seat bolts directly to the floor (while the rear seat sits atop the battery pack). Therefore the battery pack's base cannot intersect the front seat's position. From these requirements we generate two constraints.

$$g10\text{: } L_{frSeat} \geq L_{firewall} + L_{seat\_bottom}$$

$$g11\text{: } BaseX \geq L_{seat\_bottom}$$

Finally we include a few constraints stemming from the simple physical nature that all of these dimensions must be greater than or equal to zero. We also include a constraint on BaseX so that it is at least as long as the seat bottom which must sit atop of it.

$g12: e16 - BaseX \leq 0$

$g13: BaseZ - e_{17} \leq 0$

$g14: -ArmZ \leq 0$

$g15: -ArmY \leq 0$

$g16: -ArmZ \leq 0$

Design Variables and Parameters

The variables to be adjusted in this optimization study are listed below. Below them are the parameters

DESIGN VARIABLES

x_1: ArmX = Longitudinal battery arm x dimension [mm]

x_2: ArmY = Longitudinal battery arm y dimension [mm]

x_3: BaseX = Transverse battery portion x dimension [mm]

x_4: BaseY = Transverse battery portion y dimension [mm]

x_5: BaseZ = Height of transverse battery portion [mm]

x_6: L_{cabin} = Cabin length (from firewall to trunk) [mm]

x_7: L_{FrSeat} = Length from the front of the vehicle to the front seat position [mm]

x_8: L_{trunk} = Trunk length (from end of cabin to end of vehicle)

PARAMETERS

e1 = 4404 mm, the Volt official overall length

e2 = 790 mm, the maximum battery base width

e3 = 1064 mm, the Toyota Prius frontal leg room

e4 = 981 mm, the Toyota Prius rear leg room

e5 = 948 mm, the Toyota Prius rear head room

e6 = 470 mm, the estimated outer diameter of a Volt tire

e7 = 1224 mm, the estimated length from the front of the Volt to its firewall

e8 = 875 mm, the calculated required leg room to meet NHTSA airbag safety guidelines

e9 = 609 mm, typical seat width

e10 = 50 mpg, the minimum fuel economy as measured on the UDDS drive cycle

e11 = To be determined, the estimated door panel widths for the Volt. A value for this will be determined by either examining typical door panel widths for similar vehicles or by statistically examining

e12 = 242 mm, the estimated pedal distance from the firewall

e13 = 1798 mm, Volt official vehicle width

e14: ArmZ= 250 mm, height of battery pack arm

e15: t_{seat_top} = 150 mm, typical seat (vertical portion, or back) thickness

e16: t_{seat_bottom} = 520 mm, typical seat bottom (thickness)

e17: BaseZ maximum height = 150 mm in order to maintain rear seat head room comfort

Degrees of Freedom and Feasible Point

With the eight design variables, and one equality constraint, we have 7 degrees of freedom. In order to ensure a reasonable design space has been created for the optimization problem, it is standard practice to find at least one feasible point for the system.

The table below summarizes the design variables and the resulting objective function value. The AER and fuel economy were taken from PSAT vehicle simulation. Note that all constraints are met.

objective f =	.81684		Constraint	(neg. null form)
AER	40.61	[miles]	**g1**	0
FuelEcon	58.1935	[mpg]	**g2**	-525
BaseX	1435	[mm]	**g3**	-240
BaseY	550	[mm]	**g4**	-948
BaseZ	54	[mm]	**g5**	-112
ArmX	750	[mm]	**g6**	-8.1935
ArmY	100	[mm]	**g7**	0
ArmZ	100	[mm]	**g8**	-180
LfrSeat	1500	[mm]	**g9**	-999
Lcabin	2710	[mm]	**g10**	-2466
Ltrunk	470	[mm]	**g11**	-999
			g12	-915

Table 4 – Table showing a possible starting point with constraint feasibility proof

3.a.2) Summary Model

Max.

$$f = \frac{1}{3}(AERange/40) + \frac{1}{3}\left(FrontSeat_{optimality}\right) + \frac{1}{3}\left(RearSeat_{optimality}\right)$$

Subject to:

g_1: $e_7 + L_{cabin} + L_{trunk} - e_1 = 0$

g_2: $ArmX + BaseX - L_{cabin} \leq 0$

g_3: $BaseY - e_2 \leq 0$

$g4$: $-L_{FrSeat} + e_{15} + (e_7 + e_{12}) + e_3 \leq 0$

$g5$: $e_4 - L_{cabin} + e_{15} + L_{frSeat} - e_7 \leq 0$

$g6$: $e_{10} - FuelEcon \leq 0$

$g7$: $e_6 - L_{trunk} \leq 0$

$g8$: $ArmY + 2e_9 - e_{13} + 2e_{11} \leq 0$

$g9$: $L_{frSeat} + Base_x - e_7 - L_{cabin} \leq 0$

$g10$: $e_7 + e_{16} - L_{frSeat} \leq 0$

$g11$: $BaseX + (L_{FrSeat} - e_7) - L_{cabin} \leq 0$

$g12$: $e16 - BaseX \leq 0$

$g13$: $BaseZ - e_{17} \leq 0$

3.a.3) Model Analysis

We first investigate the monotonicity table for the subsystem. The table will not be very helpful in determining activity. This is because the objective function is U (undefined) across the board due to components from the objective function coming from simulation output. The monotonicity table is at least useful in determining that the problem is well bounded. As can be seen from the table, every design variable has at least one plus and one minus in its respective column. Therefore, all design variables are bounded.

Function	Design Variables							
	x1	x2	x3	x4	x5	x6	x7	x8
-f	U	U	U	U	U	U	U	U
g1						+		+
g2	+		+			-		
g3				+				
g4							-	
g5							+	
g6	U	U	U	U	U	U	U	U
g7								-
g8		+						
g9			+			-	+	
g10							-	
g11			-					
g12								
g13			+					
g14	-							
g15		-						
g16				-				
g17					-			

Table 5 - Monotonicity table for the optimization problem

The italicized constraints g14 through g17 are very simple constraints imposed by reality. They essentially constrain the dimensions of the battery pack so that they must be greater than or equal to zero. This may seem unnecessary but when dealing with computer optimization software, it is always recommended to provide even the most obvious bounds.

Before feeding the model to Optimus, an attempt was made to eliminate redundancies and unneeded constraints as should always be done prior to cranking a computation-intensive numerical program. Because we have one equality constraint, g1, we know that we can eliminate one design variable. Recall that g1 essentially states that the sum of the length to the firewall, the cabin length, and the trunk length must add up to the Volt's overall length. We choose to eliminate variable x_8, the length of the trunk.

The revised monotonicity table, which would have column x_8 removed, does not provide any further insight into the problem and will not tell us any more information about activity. Because it adds nothing to this discussion it is omitted here.

3.a.4) Optimization Study

First, we show the Optimus results based on a Sequential Quadratic algorithm and those based on an NLPQL algorithm. Both algorithms resulted in approximately identical results.

Design Variables		**Constraints**	
ArmX (mm)	1456	**g2**	**~0**
ArmY (mm)	280	**g3**	**0**
ArmZ(const) (mm)	150	**g4**	**0**
BaseX (mm)	1254	g5	-123
BaseY (mm)	790	g6	-25.80
BaseZ (mm)	150	**g7**	**0**
LfrSeat (mm)	2680	**g8**	**0**
Lcabin (mm)	2710	**g9**	**0**
Ltrunk (mm)	470	g10	-936
Objective, f	**2.013**	g11	-734
Fuel Econ (mpg)	**75.80**	**g12**	**0**
BattVol (mm^3)	**2.10e8**		
Elec. Range (mi)	**168**		

Table 6 – Subsystem Results.

These results are presented visually in Figure 3 below. Table 3 shows us quite a few active constraints. Constraints g2,3,4,8,9 and g12 were all active. This provides insight into the underlying tradeoffs occurring within the problem itself. The surrogate model of the vehicle simulation which provides fuel economy and all-electric range is monotonic. This explains why so many constraints resulted as active.

The gradient-based methods all converged on their final point such that Optimus output a message of "SEQUENCE CONVERGED," indicating that the algorithm's convergence criteria were met within the input tolerance of ε = 0.00001. Thus KKT conditions were met within a related tolerance.

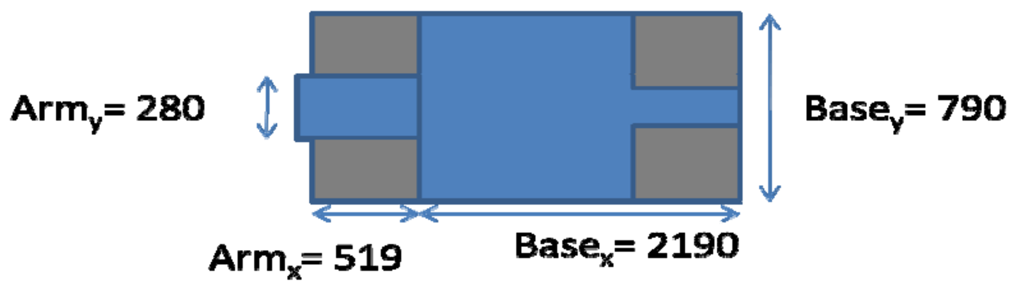

Figure 3 - Graphical representation of the battery dimensions. Note, the rear seats are sitting on top of the battery base, and the battery arm is in between the two front seats.

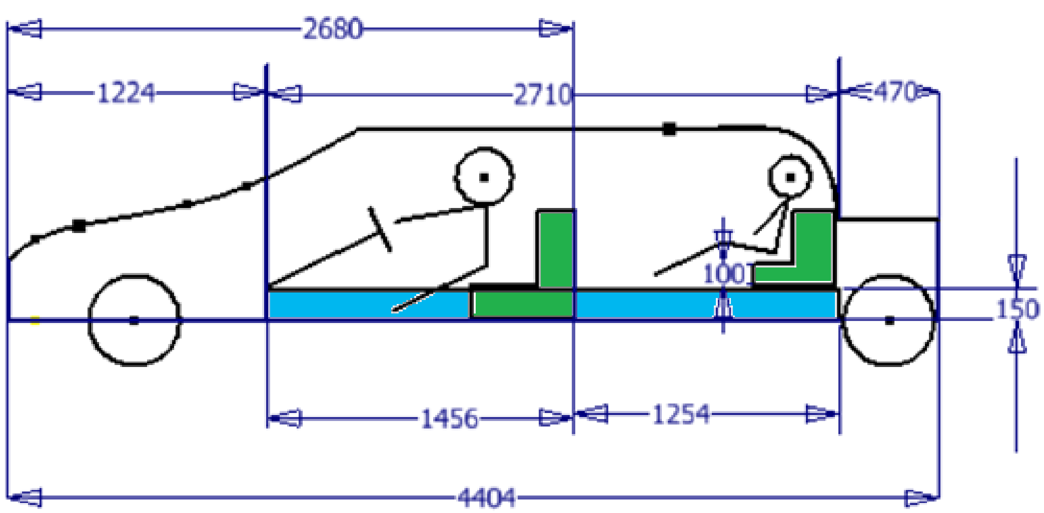

Figure 4 - Graphical representation of the optimal vehicle design based off of the battery and packaging subsystem

The cabin length, Lcabin, was optimized to its upper bound, leaving only enough trunk space length to fit the rear wheel in that space. This is expected if the simulations proved favorably that more battery volume yielded better all-electric range. Maximizing the cabin length allowed for the BaseX and ArmX dimensions to be extended. Similarly, BaseY and ArmY were expanded until they hit their bounds set by overall vehicle dimension constraints.

The optimization also exploited the fact that increasing the dimension BaseX by a unit length results in a greater increase in volume to the battery pack than does an increase to the dimensions ArmX. Hence the algorithm placed the front seats just far enough from the firewall that it met the constraint for frontal leg room while using the rest of the cabin floor to expand the BaseX dimension of the battery pack.

Some investigation was done on varying some of the parameters of the problem as well as exploring objective function weighting modifications. Also, different algorithms were used to attain the same optimum point in order to check for multiple optima. These various trials and variations are discussed in the following section.

Multiple Start Points

Multiple start points were used to ensure that convergence is to a global optimum. Several start points were chose as specified in the table below. The table also shows that regardless of the start point and the algorithm, the same optimum point occurred. A non-gradient method was also used as a second method of to check against multiple optima.

	Start Point 1	**Start Point 2**	**Start Bounds**
ArmX	519	1400	[0 5000]
ArmY	280	280	[0 5000]
BaseX	2190	1000	[0 5000]
BaseY	790	150	[0 5000]
BaseZ	150	150	[0 5000]
LfrSeat	1744	1000	[0 5000]
Lcabin	2710	2000	[0 5000]
Algorithm	SQP	NLPQL	Direct
Optimum f	2.013	2.013	2.013
Converged to same optimum?	Yes	Yes	Yes

Table 7 - Optimization results based off of different starting points, and the use of a non-gradient method, DIRECT

Constraint Activity Discussion

Constraint g2 dictates that both BaseX and ArmX dimensions are combined to take up the entire length of the cabin. We can say, although without certainty, that the optimization schema is seeking is maximizing the battery volume by maximizing the x dimensions of the battery pack. Further investigation must be done to better understand what physical behavior

Constraint g3 is simply an upper bound on the dimension BaseY of the battery pack. Again, we see the optimizing algorithm result in activation of a constraint related to achieving a maximum overall volume of the battery pack. Constraint g4 is also active for similar reasons.

Constraint g7, also found to be active by Optimus, is a lower bound on the length of the trunk. This length has been minimized. Note that because the trunk length competes with the cabin length, and because the battery pack must be contained completely within the cabin, minimizing the trunk length will result in maximum cabin length and thus maximum battery pack x dimensions.

Constraint g8 is also active. This indicates that the battery pack dimension Arm_y has been maximized. Active constraint g10 relates to the position of the front seat. The constraint requires a minimum distance between the front seat and the vehicle firewall. The emerging trend of maximization of battery volume can again explain this active constraint. The idea is that the battery dimension BaseX must be placed entirely behind the front seat. Recall that BaseX and ArmY compete for the total cabin length. So the optimizer must choose which of the two to make longer. If we assume the optimum objective occurs at maximum battery volume, then we can assume the optimum solution occurs when the cabin length is distributed between BaseX and ArmX in such a way that maximizes battery volume.

By causing constraint g9 to be active, BaseX is maximized (versus maximizing ArmX). Because the battery base (top of battery "T") is much wider than the battery arm (vertical part of battery "T"), maximizing the BaseX, as the optimizer does by activating constraint g10, yields the maximum possible battery volume. Thus the continuing trend of the optimum occurring at maximum battery volume continues to hold true from the active constraint g10.

The remaining active constraint is g12 which simply imposes a maximum value for BaseX based on parameter e16. Again, maximum battery volume appears to be the intention.

3.a.5) Parameter Changes

Dimensional

Constraint g7 depends on parameter e6, the estimated diameter of the tire. This constraint single handedly governs the required length of the trunk. While this was taken as true for the work in this particular project, we recognize that in a full blown car design space considerations for the trunk would yield different requirements for trunk space. Because this parameter is the only parameter in active constraint g7, know that a change in parameter e6 would yield a change to the optimum solution. Specifically, it would reduce the maximum possible combined length of $Base_X$ and Arm_X. Accordingly, the maximum battery pack volume would be reduced which would reduce the all electric range and fuel economy.

Objective Weighting

The objective function for the subsystem, repeated below for convenience, is weighted to distribute each of its three components. These weights could also be modified

$$Max\ f = \frac{1}{3}(AER/40) + \frac{1}{3}\left(FrontSeat_{optimality}\right) + \frac{1}{3}\left(RearSeat_{optimality}\right)$$

intelligently in order to yield a more meaningful objective function. For example, modifying *f* so that the weights a, b, and c are set to 1/7, 3/7, and 3/7 could result in an optimal design that better places the front and rear seats. Certainly this would come at the cost of all-electric range as the new weights would favor better front and rear seat position corresponding to the Hamza optimal seat locations. As expected, the AER suffers. Also as expected, the proximity to the ideal front and rear seat locations is improved.

Modification of Parameters

The current model is set up so that the rear seat location is fixed to the end of the cabin. This was a modeling decision made to maintain a manageable degree of simplicity. In actual car design, we would of course have the freedom of pushing the rear seats forward into the cabin if so desired. This could be done to increase trunk space, or to modify center of gravity for vehicle handling, or even to provide better occupant positioning for blind spots, etc.

In the case of our model, this additional design variable of rear seat location (unconstrained from the cabin length) would allow for independent placing of the rear seats to better fit the Hamza criterion for ideal rear seat location. This possible model modification was done for in the fully integrated system design discussed in its respective section in this report.

3.b) Vehicle Handling Subsystem

The handling dynamics of the vehicle are dominated by two main factors: the tire and suspension properties, which will be considered fixed parameters for this subsystem, and the location of the center of gravity. For the majority of consumer grade vehicles, the mass is dominated by four objects: the powertrain, the front passengers, the rear passengers, and the fuel tank. An optimal design for vehicles with internal combustion engines and gas tanks has been created over time to put the powertrain components (engine and transmission) in the front, the passengers in the middle, and the gas tank in the rear to obtain the desired center of gravity location and to satisfy other criteria (like passenger comfort and safety). In lieu of a gas tank, the Chevy Volt has a battery pack, as described above, which presents designers with a new challenge of having a much wider range of placements for the various components. With this freedom it is still important to maintain proper vehicle handling characteristics, and this optimization attempts to do this.

3.b.1) Mathematical Model

Objective Function

The best way to quantify the handling performance is to use a quantity that is a function of the two main factors that influence the handling: the tire parameters and vehicle dimensions (specifically the location of the C_G relative to the front and rear axles). If we assume a scenario of a vehicle driving at a constant velocity along a curve with a fixed radius then we can create a simple two degree of freedom model known as the "bicycle model" (Fig. 5)

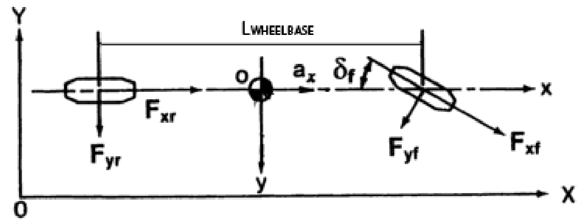

Figure 5 - Bicycle Model - Manipulated from (Wong (2001))

Using this model and knowledge of how the forces on the tires relate to motion in the vehicle we can develop a model for the steering angle required to navigate the curve.

$$\delta = \frac{L_{wheelbase}}{R} + K_{US}\frac{{v_0}^2}{gR}$$

Where K_{US} is known as the understeer coefficient, and is defined below with W_F and W_R being the component of the vehicle weight on each axle, and $C_{\alpha f}$ and $C_{\alpha r}$ being the tire properties on each axle.

$$K_{US} = \left(\frac{W_F}{C_{\alpha f}} - \frac{W_R}{C_{\alpha r}}\right)$$

This equation can also be rewritten using the design variables (in bold) as

$$K_{US} = \frac{M\left(1 + \frac{L_{front}}{\boldsymbol{L_{wheelbase}}}\right)}{C_{\alpha f}} + \frac{M\left(\frac{L_{front}}{\boldsymbol{L_{wheelbase}}}\right)}{C_{\alpha r}} - \frac{\left(\frac{1}{\boldsymbol{L_{wheelbase}}}\right)(m_b\boldsymbol{C_b} + m_eC_e + m_f\boldsymbol{C_f} + m_pC_p + m_r\boldsymbol{C_r})}{C_{\alpha f}} - \frac{\left(\frac{1}{\boldsymbol{L_{wheelbase}}}\right)(m_b\boldsymbol{C_b} + m_eC_e + m_f\boldsymbol{C_f} + m_pC_p + m_r\boldsymbol{C_r})}{C_{\alpha r}}$$

These design variables were chosen because they were directly affected by the battery subsystem. All of the mass and tire properties were fixed for this subsystem analysis as well as the location of the powertrain components and the front axle. Any vehicle components not incorporated in the powertrain, the seats, or the battery, is considered a lumped mass (m_e) with a center of gravity (C_e) at half of the length of the vehicle to emulate an even distribution of mass along the length of the vehicle.

For a first level analysis, an objective function was made to maximize the understeer coefficient.. A monotonicity analysis was performed using the objective developed from the understeer coefficient and the constraints defined in the section below.

	$L_{wheelbase}$	C_F	C_R	C_B
f	-	+	+	+
g1		-		
g2		+	-	
g3		+		-
g4			-	+
g5		-	-	-
g6	-	+	+	+
g7	-			
g8	+			
g9			+	

Table 8 – Monotonicity table for the first level optimization

Analysis of this table showed us that we had activity in all of the design variables, specifically that constraint g8 (again, talked about below) was active with respect to the length of the wheelbase, and that the other design variables were conditionally active. The results of this optimization will be discussed later; but it was determined that this optimization was not sufficient enough to capture the best handling characteristics.

In order to develop a more accurate objective function, some additional information was needed. It has been observed in both research and by industry professionals that the ideal handling behavior is largely dependent on how fast the car is moving. Manipulating the result of the two degree of freedom bicycle models allowed designers to have developed many different constants which represent vehicle handling characteristics for steady turns. One of these values is the yaw rate gain (G).

$$G = \frac{v_0}{L_{WHEELBASE} - K_{US}\frac{{v_0}^2}{g}}$$

This value relates how fast a vehicle can turn with a given steering input; thus, the higher this value the more sensitive the steering will be. At low speeds (also known as parking lot maneuvers) we want the vehicle to be as sensitive as possible. In order to achieve this, the denominator of the yaw rate gain needs to be as high as possible; thus we want to make the K_{US} approach zero and even go negative if possible.

At high speeds designers worry about directional instability of the vehicle; which is how the vehicle responds to some disturbance (like wind or something). If a vehicle is directionally stable, the disturbance won't cause the vehicle to spin out; however, if the vehicle is unstable, then even a small disturbance can cause the driver to lose control. This stability can be determined by solving the eigenvalue problem of a transient vehicle dynamics model. Researchers have shown that if a vehicle's K_{US} value is greater or equal to 0 then that vehicle will always be stable; but if K_{US} is less than 0 then there will be some critical speed where the vehicle becomes unstable. While the actual transient model will not be used, the correlation between the understeer coefficient and stability risk will be used. At higher speeds the importance on steering sensitivity decreases, but the need for good directional stability (more negative eigenvalues) are desired. This corresponds to making K_{US} as large as possible.

Taking into consideration these two criteria, a curve was created the tries to show what the ideal K_{US} value would be as a function of the vehicle forward velocity. This is purely a heuristic function, and only created to visualize a trend. If the mass properties or the tire properties of the vehicle changed, then the curve would be numerically different but still exhibit the same trend.

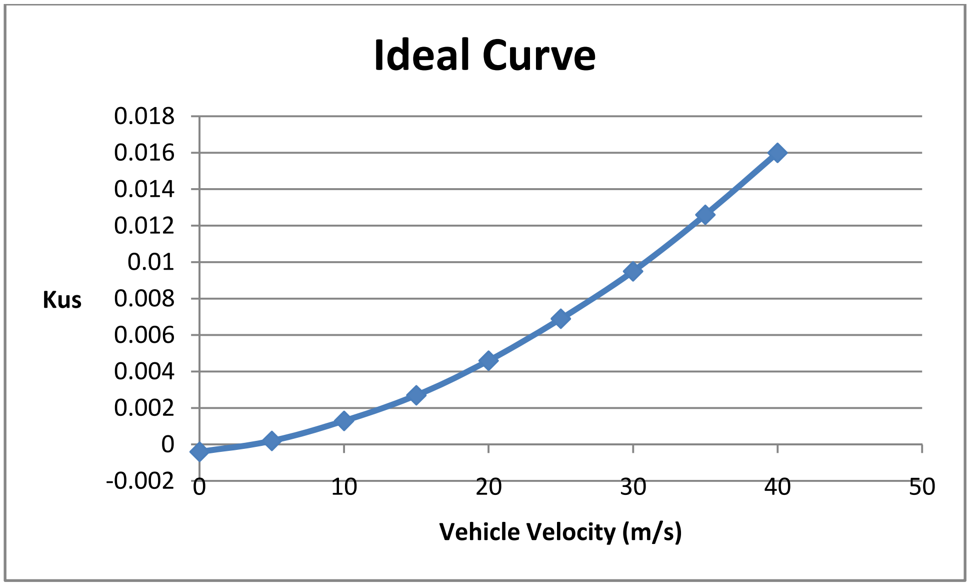

Figure 6 - Ideal understeer coefficient as a function of vehicle forward velocity, v0. (Kus_I = (8e-6)*(v0)^2 + (9e-5)*(v0) - 0.0004)

The objective of this subsystem optimization will be to try to fit a horizontal line (determined by K_{US} = Constant) to this curve, to try to get the best overall characteristics at both high and low velocities. This will be done by doing a least squares summation at various velocities.

$$f = 1 - \sum \sqrt{(K_{US} - K_{US_{Ideal}})^2}$$

This value is known as the residual, R. R exists between 0 and 1, where 1 represents a perfectly fitted curve, and 0 represents no correlation between the data points and the equation for the fitted curve. For this problem it is understood that the residual will never get 1, but we do want to try to maximize the value, as this will give us the best overall performance of the range of velocities.

Constraints

The following constraints are based off of some very rough data, and also based off of another vehicle. While they are accurate enough to describe bounds, if we have activity in any of the constraints than further, deeper, analysis of the parameters would be necessary.

We have a constraint on how far forward the front passengers can be, based off of the front legroom and the distance between the front of the car and the firewall. This legroom constraint was determined based off the legroom of the Toyota Prius due to the reasons give in the section explaining the packaging sub-system, mainly, for it provides a basis for comparison between similar vehicles.

$$g1: C_F \geq d_1$$

There is also a constraint on the legroom the rear passengers can have, and this determines the constraint on how far forward the rear passengers can be, again determined by examining the rear legroom for the rear passengers.

$$g2: C_R \geq C_F + d_2$$

There will also be a constraint on how far forward the c.g. for the battery can be. For this sub-system the specific battery shape will not be considered; rather, it will be assumed the shape will be determined to place the location of the battery c.g. at the optimal position. The assumption will be that the c.g. of the battery cannot be outside of the wheelbase. This assumption is made based off of the idea that due to packaging issues it just isn't feasible to have the c.g. of the battery exist outside an envelope defined by the c.g.'s of the front and rear passengers.

$$g3: C_B \geq C_F$$

$$g4: C_B \leq C_R$$

Additionally we will need constraints on where C_G can be located. In order for the vehicle not to tip into the front or rear bumpers, the C_G must exist within the constraints of the wheel base. The Cg was calculated based off of the design variables and the constraints are given below.

$$g5: Cg \geq L_{FRONT}$$

$$g6: Cg \leq L_{FRONT} + L_{WHEELBASE}$$

Constraints on the wheelbase were also determined by how far forward/backward the wheels can be place on the vehicle. It will be assumed again that lower limit on the wheelbase will be determined by the current Volt specifications (with the assumption that the designers have had some reason for having this be the wheelbase that we don't fully understand). The upper limit will be constrained by how far back we can move the rear wheel. This limit will be assumed that the edge of the wheel cannot extend beyond the trunk which will be determined by knowing the radius of the tire, R_T

$$g7: L_{WHEELBASE} \geq d_3$$

$$g8: L_{WHEELBASE} \leq (L_{OVERALL} - L_{FRONT}) - R_T$$

We also need to specify how far back the rear seat can go, as that will determine how far back the battery will go. This constraint will be the same constraint as the constraint on the wheel base, thus making sure that the rear seat cannot extend past the center of the rear wheel.

$$g9: C_R \leq L_{OVERALL} - R_T$$

The last constraint is a constraint on system stability. Designers tend to design vehicles in order to completely avoid any risk of going unstable, thus they design the car to be under steer, or at the very least, neutral steer.

$$g10: K_{US} \geq 0$$

Design Variables and Constraint Parameter Values

Design Variables	Feasible Value	Parameters	Typical Value
x1: C_F	2.5m	d1	1.8510
x2: C_R	3.5m	d2	0.9800
x3: C_B	3.0m	d3	2.6850
x4: $L_{WHEELBASE}$	2.7m	R_T	0.235

Table 9- Table showing an initial feasible point and nominal parameter values for the handling subsystem

All other parameters are only seen in the .m file (see Appendix D) used in the simulation, and the values are given there.

3.b.2) Summary Model

$$maximize \quad f = 1 - \sum \sqrt{(K_{US} - K_{US_{Ideal}})^2}$$

subject to

$$g1: 1.851 - C_F \leq 0$$

$$g2: C_F + 0.98 - C_R \leq 0$$

$$g3: C_F - C_B \leq 0$$

$$g4: C_B - C_R \leq 0$$

$$g5: 1.05 - C_G \leq 0$$

$$g6: C_G - 1.05 - L_{WHEELBASE} \leq 0$$

$$g7: 2.685 - L_{WHEELBASE} \leq 0$$

$$g8: L_{WHEELBASE} - 4.169 \leq 0$$

$$g9: C_R - 4.169 \leq 0$$

$$g10: K_{US} \geq 0$$

3.b.3) Model Analysis

The results for the first level optimization gave the optimal location for the centers of the front seats, the rear seats, the battery, and the length of the wheel base by trying to maximize the value of the understeer coefficient. While this is not a really good model to try to characterize good vehicle handling, it does bring insight to what the limits of the design variables are (by either looking at the maximum or minimum understeer coefficient). Maximizing the value places the axles as far away from each other as possible, and all of the mass elements shoved as far forward onto the front axle as possible. As predicted, there were four active constraints, g1, g2, g3 and g8. This gave the optimal locations of the vehicles below, and resulted in a K_{US} value of 0.017

Design Variable	Value
$L_{WHEELBASE}$	4.169
C_F	1.851
C_R	2.831
C_B	1.851

Table 10 - Results of the first level optimization

At either the maximum or minimum (solution not shown) we have 4 active constraints (for the 4 design variables) and thus we have one unique solution for each case.

Because this original model was not good enough to capture optimal vehicle handling at all velocities, it was determined to develop another model; however, no monotonicity could be analyzed other than knowing that at maximum/minimum values for K_{US} would result all the variables being defined by constraint activity.

3.b.4) Optimization Study

The results of the second level optimization gave a K_{US} value of 0.0046. This was done doing using both a gradient method of optimization (SQP) and a non-gradient method (DIRECT). The non-gradient method was done with two different design space bounds, but did not yield results that were significantly different than the SQP results and thus weren't used for further analysis, other than verification of model accuracy. This value gives good performance at both low and high speeds, and matches the ideal at around 20m/s (approximately 45mph) which is great for city driving.

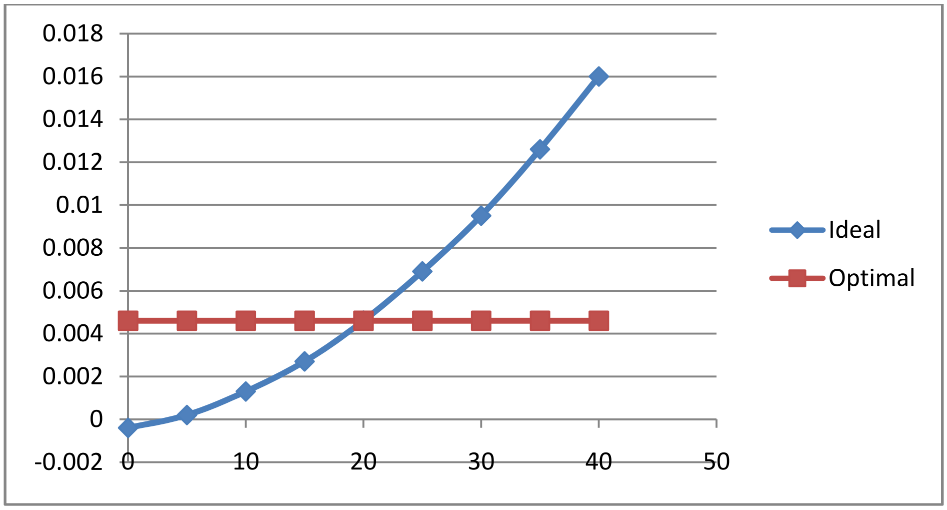

Figure 7 - Optimal Understeer Coefficient in relation to the ideal

Analysis of varying starting points was necessary to make sure that we only had one optimum. Below is a figure showing the optimum from three different starting positions: (1,1,1,1), (2,2,2,2) and (3,3,3,3).

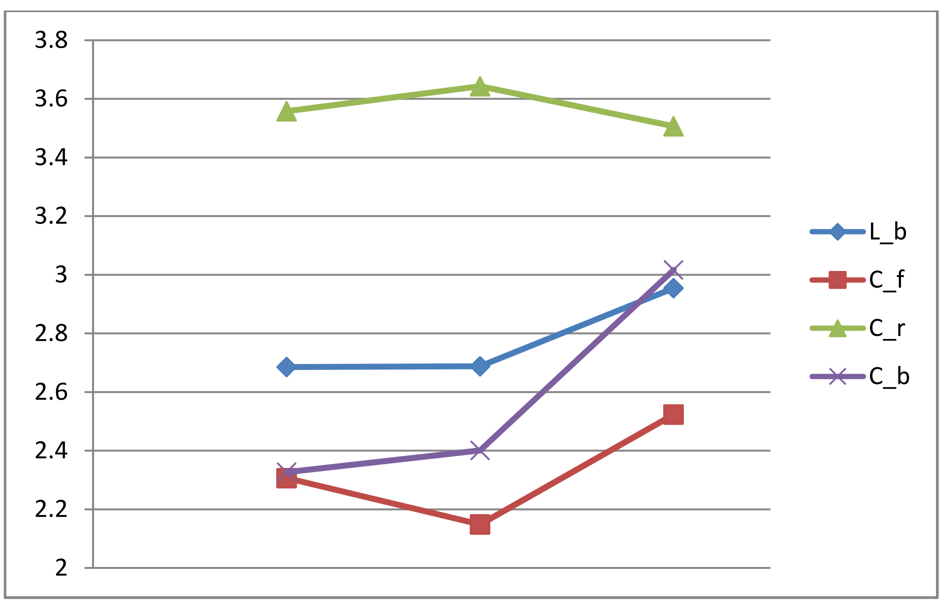

Figure 8 - Results based off different starting locations (L_b = $L_{wheelbase}$)

More data points would be necessary to notice any large trends, but it seems that the C_R and C_F values are inversely proportional. Additional starting points were tested, including a starting point that was heuristically determined to be a starting point that has "good" values that need to be optimized (denoted in **bold**).

Start	$L_{wheelbase}$	C_F	C_R	C_B	K_{US}
(1,1,1,1)	2.685	2.3061	3.55783	2.32649	0.0046
(2,2,2,2)	2.68754	2.1481	3.64303	2.40095	0.0046
(3,3,3,3)	2.95433	2.52304	3.50653	3.0166	0.0046
(4.169,1.851,2.831,1.851)	2.83793	2.54294	3.52323	2.61886	0.0046
(2.685, 3.189, 4.169, 4.169)	3.54593	3.00995	3.99005	3.9707	0.0046
(2.8, 2.5, 3.5, 3)	**2.91681**	**2.4713**	**3.47541**	**2.96597**	**0.0046**
DIRECT(bounds due to activity)	*3.427*	*2.4209*	*4.0947*	*4.0402*	*0.0046*
DIRECT (this gave 3 optima bounds being 0<x<5)	*3.0556*	*2.1296*	*3.7963*	*3.4259*	*0.0046*
	3.0556	*1.9444*	*3.9815*	*3.4259*	*0.0046*
	3.0556	*2.3148*	*3.6111*	*3.4259*	*0.0046*

Table 11 - Results for the Handling subsystem under various starting points, and with both gradient and non-gradient (in italics) methods. The chosen set of design variables is seen in bold

The lack of easily noticeable trend leads to the conclusion that there is some 4-dimensional surface of optimality in the design space, which any point on the surface leads to a K_{US} value of 0.0046. While the objective remains the same regardless of the actual optimal point, it should be noted that some designs are better than others based off additional criteria. The most important to this design is to not have any constraint activities. The design in bold is seen in a schematic below.

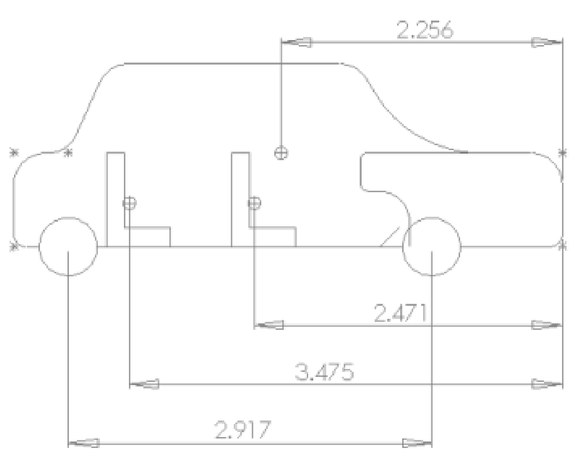

Figure 9 - Schematic of the Handling Subsystem Optimum

The center of gravity for the battery is not noted, but the center of gravity for the overall vehicle is. The drawing looks a little strange (the front end is a little long) but the relationships between the various parts are to scale, and the design looks feasible. This was chosen as the design that would be kept because it gives a good between the length of the wheel base, and comfort constraints, without having any active constraints. This gives us some wiggle room in the design variables without worry of going into the infeasible space.

As mentioned, there seems to be an infinite number of optimal points, all giving the same solution. This can be proved using some basic linear algebra theory. If we manipulate the equation for the K_{US} we can formulate the following linear algebraic equation (with arbitrary constants K_1 through K_5 and the design variables renamed as simply x_1 through x_4). The analysis behind this is seen in Appendix E.

$$K_1x_1 + K_2x_2 + K_3x_e + K_4x_4 = K_5$$

We can also write three generic constraint equations (shown as active):

$$A_1x_1 = b_1$$

$$A_2x_2 = b_2$$

$$A_3x_3 = b_3$$

Or in matrix form:

$$\begin{bmatrix} K_1 & K_2 & K_3 & K_4 \\ A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & A_3 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} K_5 \\ b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

If we want to place the K_{US} at a specific value, and we have three active constraints, then there will be a unique solution because the coefficient matrix **A** created by combining the four equations, will have full rank; however, if even one of those constraints were to become inactive, then **A** would lose rank. The SQP solution shows that we have 0 active constraints for at least one of the solutions, and thus **A** is not full rank which results in an infinite number of solutions.

This also shows why we have unique solutions when we have a least three active constraints (or in the case of the max/min four active constraints).

3.b.5) Parametric Study

It is known that the tire properties ($C_{\alpha f}$ and $C_{\alpha r}$) do affect the understeer coefficient; thus we could tweak those parameters to move our understeer to a more "desirable" place if we weren't happy with our current characteristics without changing the location of any of the mass objects. Table 13 shows how changing these parameters would affect the understeer coefficient, given fixed weight distribution.

	Increase the value	**Decrease the value**
$\mathbf{C_{\alpha f}}$	Decreases K_{US}	Increases K_{US}
$\mathbf{C_{\alpha r}}$	Increases K_{US}	Decreases K_{US}

Table 12 - Relationship between changes in the tire properties and the understeer coefficient given fixed weight distribution

Changing these parameters will affect the minimum and maximum values for the understeer coefficient. This would then change the heuristically determined ideal curve use to determine the objective, resulting in a different optimal K_{US} value and different design variable ranges that could achieve this value. This does not change the result that we have an infinite number of solutions, and thus does not give us any more confidence in what the optimal solution truly is; however, this affect will become very useful after the subsystems have been integrated, as we will see in later sections.

4.) System Integration Study

4.a) Integration Set-up

The battery packaging subsystem results indicates that the optimum design was one where the battery dimensions (and thus its volume) were maximized as much as the constraints allowed. This is expected to conflict with the handling subsystem because it deals so heavily on center of gravities of vehicle components. Thus the combined system optimum will likely be different from the individual subsystem optima.

From the onset of the project we chose design variables that would be common, or at least mathematically related, between the two subsystems. This allowed the use of the All-in-One (AIO) approach for subsystem integration. Many of the constraints in each subsystem are based heavily on physical constraints imposed by the Chevrolet Volt's known dimensions. This leads to overlapping constraints in both subsystems that become redundant when the subsystem is combined. Despite these expected constraint redundancies, both sets of subsystem constraints were integrated into the combined system. This allowed us to see which, if any, set of subsystem's constraints dominate the integrated system.

Another important issue during system integration comes from a design variable specific only to the handling subsystem. The handling subsystem includes a design variable for rear seat position whereas the battery packaging subsystem assumes the rear seat is fixed to the rear of the cabin. The battery packaging subsystem's objective function depends on rear seat position in that it increases the objective function value based on how close the location comes to the Hamza optimal rear seat locations. In the integration process, we decided to keep the rear seat as a design variable, and thus no longer fix it to the rear of the cabin as it was in the battery subsystem.

Lastly, we had to find a way to relate the increasing volume in the battery to an increasing mass for the battery, as this will affect the handling characteristics. We did this by estimating the density to be a constant value, and that the volume increases are adding matter at that same density, even though this is probably not the case it is a good enough approximation for our model.

The two subsystem objective functions were combined by adding them together. Each subsystem objective had values around each other, so it was decided not to add any additional waiting at first to see what the optimum was. The resulting objective function is seen below.

$$\max f = \left[\frac{1}{3}(AERange/40) + \frac{1}{3}\left(FrontSeat_{optimality}\right) + \frac{1}{3}\left(RearSeat_{optimality}\right)\right] + Handling_{optimality}$$

Using an SQP algorithm we were able to obtain the results seen below. All units are in millimeters.

ArmX	1553.39
ArmY	280
BaseX	1156.61
BaseY	790
BaseZ	150
LfrSeat	2777.39
Lcabin	2710
Cr	2735.06
Active Constraints	Battery subsystem constraints: g2, g3, g4, g5, g7, g8, g9, g12

Table 13 - System Integration results for equal weighting between the Handling Subsystem and the Battery Subsystem

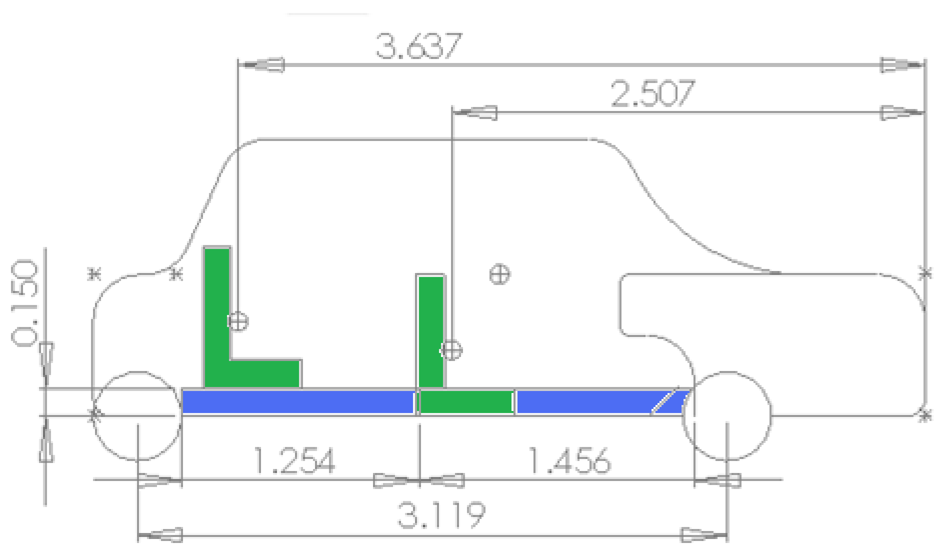

Figure 10 - Schematic of the System Integration Optimum

Note that there are 8 active constraints and 8 design variables. More specifically, all of these active constraints are from the battery packaging subsystem. As was discussed in the battery packaging subsystem results section, this tells us that the design variables for the handling subsystem are not active, but the optimization is still trying to maximize the battery volume as done in the battery subsystem, but do so by placing the rear seat in a position that gives good handling characteristics.

4.b) Objective Weighting Analysis

This result led us to perform an analysis on varying the weightings of the different parts of the combined objective. In the first trial, we increase the affect of everything but the AER value, and thus all the other objectives were scaled up by 10. The second trial only scaled the front and rear seat optimality conditions, to see if the handling and AER would be reasonable if those two conditions were maximized. The results of these trials can be seen below.

	Trial 1	Trial 2	Trial 3
K_{US}	0.0049	0.0047	0.0045
FrontSeat Optimality	0.87875	0.960942	0.998366
RearSeat Optimality	0.9212	0.92143	0.89874
AER (mi)	4.216	4.149	4.069

Table 14 - Results based off different objective weightings

We can see that by changing the weighting schemes we obtain different vehicle characteristics. In the second trial, we increased the weighting on everything but the AER. This allowed us to get closer to the Hamza optimal seat positions, and the ideal K_{US} as found in the subsystem analysis but at the cost of decreasing the AER. By examining the vehicle handling variables for each of these trials we notice a very interesting trend.

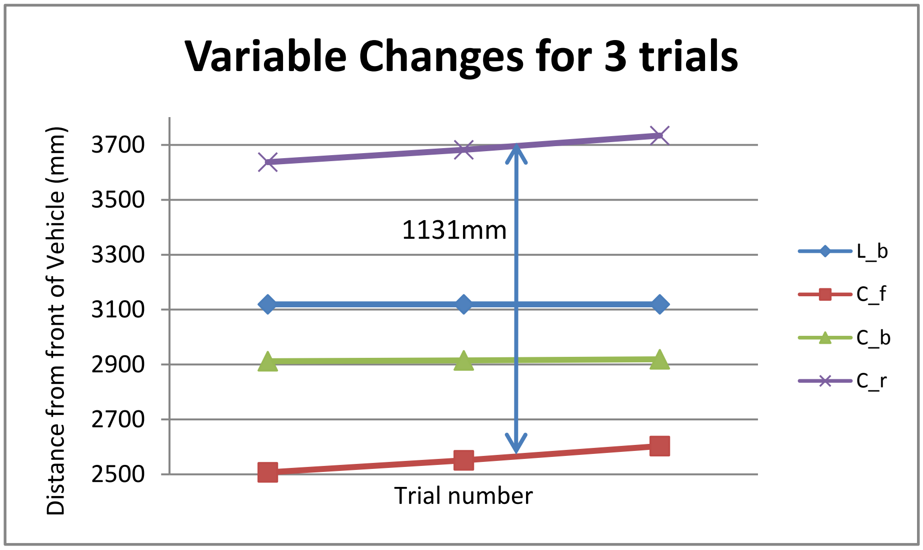

Figure 11 - Handling subsystem design variables for the three trials with different weightings.

In every trial the location of the center of gravity for the battery and the wheelbase stay roughly the same. The only change is the location of the front and rear seats (in response to the Hamza criteria). We notice that the slopes of both C_F and C_R, are similar and it turns out the distance between them is a constant 1131mm. This value is due to the constraint activity in g5 of the battery subsystem which pertains to maintaining minimum rear legroom of 981mm.

More optimizations were performed with the rear leg room constraint relaxed. We wanted to make the constraint inactive, and see if our extra degree of freedom would allow us to place the rear seat in the position that gave us optimality in the Hamza criteria for the rear seat. Trials were performed at different starting positions, and all of the trials gave the same results, seen below.

ArmX	1552mm	**BaseZ**	150mm
ArmY	280mm	**LfrSeat**	2777mm
BaseX	1557mm	**Lcabin**	2710mm
BaseY	790mm	**C_r**	3502mm

Table 15- Optimal design variables after weighting the AER less than the other objectives, and relaxing the rear leg room constraint to be inactive

By looking at how this result affects the variables used in the handling subsystem, we can again analyze in a graph how the objects are placed relative each other. We expected that by relaxing the rear leg room constraint we would be able to move the rear seats further forward: which would subsequently increase the understeer coefficient.

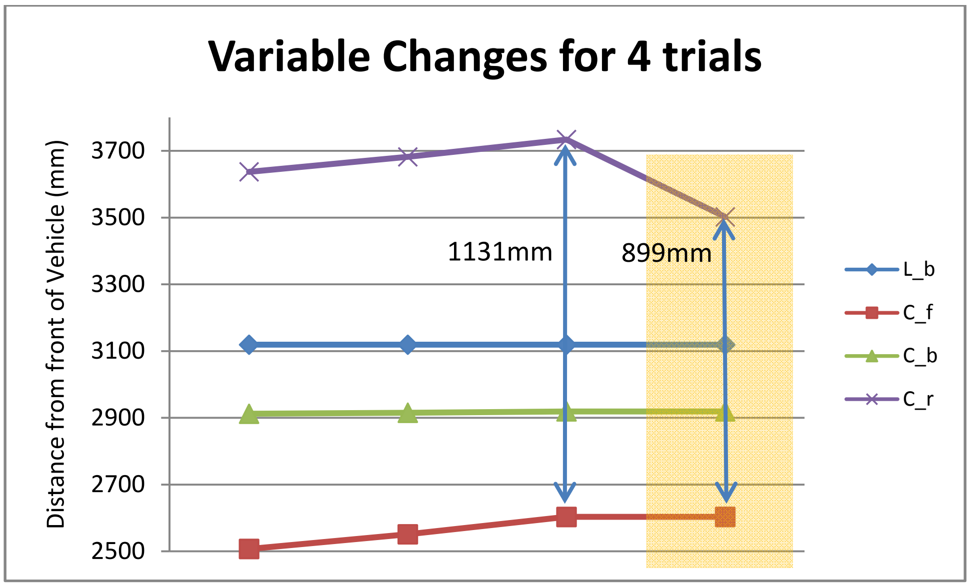

Figure 12 - Figure 12 with the additional relaxed constraint results (highlighted in yellow)

Recall in the third trial that we maximized the front seat Hamza optimality but at a loss in rear seat optimality. We can see in the chart above that relaxation of the rear leg room constraint introduced an additional degree of freedom which allowed us to place the rear seat also at optimality. This dropped the distance between the rear and front seat to be at 899mm, which results in a rear leg room of 749mm. By doing this, we do influence the understeer coefficient which gets changed to be 0.005, as was expected. We could have corrected for this by increasing the wheelbase; unfortunately, the variable is being defined by an active constraint and thus cannot increase anymore.

In all of the solution methods, and with the different weightings, we have unique solutions. While the handling subsystem would suggest that any time we aren't at activity on the handling constraints we have an infinite number of solutions, we can see that the active constraints we do have impose activity in both L_F and C_B. The variable C_R is then placed by the Hamza criteria so that that value is optimal, which means we will have a unique K_{US} value determined by the placement of the wheel base, L_B. These results also continue to give us an AER around 160miles, telling us that these changes don't have too much of an effect on the AER because the battery volume is not changing significantly due to all of the objective weight changes.

4.c) Concluding Remarks

Overall these results tell us some very important details that decision makers could utilize when trying to design this car. The first is that in order to get the most out of the battery, we want to maximize the volume. With all five of the battery dimension variables being defined by active constraints, we know that further analysis of these constraints would be necessary to determine more accurately what the battery volume (and thus all electric range and fuel economy) will be. The second important detail is that the handling can almost doesn't need to be designed for. As long as constraints on things like the wheel base, and the position of the power train and seats are clearly defined based off of comfort, safety or other packaging constraints, it is unlikely that the handling characteristics of the vehicle will be unacceptable. Granted, if designers wanted to stray far from the norm (i.e. placing the powertrain components in the center of the wheelbase) then an optimization study like the one in this paper could be utilized to get a very good idea of where the other major mass components of the vehicle could be placed to still maintain good handling characteristics. As was discussed in the handling subsystem, we could obtain the ideal value for K_{US} by tweaking the tire stiffness parameters. We wouldn't want to consider them as design variables unless we were also including the suspension, as it would add too much complexity to the handling (and thus this integration) without giving us any really useful additional information that designers could use. With the introduction of suspension characteristics we could use the tires as a link between the tradeoffs between handling and ride, and thus would then need to be considered as design variables, but that is for a future optimization study.

5.) Acknowledgements

Steven Hoffenson – PhD Candidate, University of Michigan – Helped with model formulation and optimization trouble shooting.

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Professor A. Galip Ulsoy – University of Michigan – Typical Parameter values for vehicle dynamics, transient dynamic model, professor for ME 568 – Vehicle Control Systems

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Rakesh Patil – PhD student, University of Michigan – Provided data and model parameters from his PSAT simulation model of a Chevrolet Volt.

6.) References

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Appendix A - Method of Determining Dimensions off of an Image of the Chevrolet Volt using Photo Editing Software

To create a more meaningful problem, many of the parameters were taken from the Chevrolet Volt. This came into play many times during the modeling phase of the project. For example, the PSAT vehicle simulation was one specifically modeling the Chevrolet Volt components and dimensions.

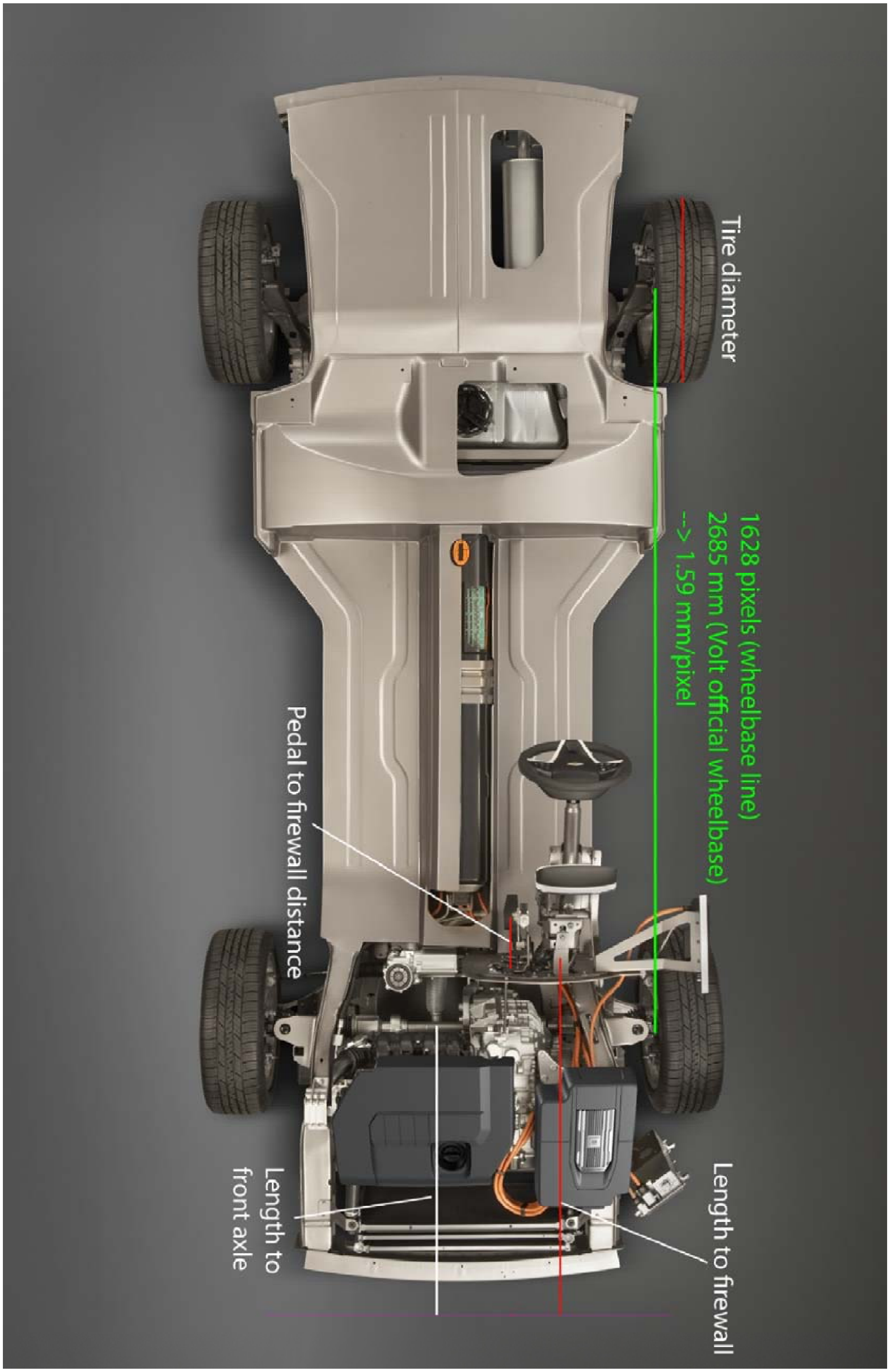
Although General Motors has officially released some of the Volt's specific physical parameters such as wheelbase, overall length, and overall width, many parameters needed for the study were no available. Such unavailable parameters were length from front of vehicle to front axle, outer tire diameter, length from front of vehicle to firewall and distance between accelerator pedal and firewall. These parameters had to be estimated in a method that was at least somewhat scientific and that would yield reasonable estimates. Because General Motors had released many useful images of the Chevrolet Volt's chassis and powertrain layout, we opted for the use of image-based length measurements to estimate the needed parameters.

Image-based length measurement consists of utilizing an image (where the basic unit of imagery is the pixel) along with a known "official" base measurement. By measuring the number of pixels used to represent that base measurement in the image, one can easily calculate a conversion factor with units of distance/pixel that can be applied to estimate unknown distances within the same image. The key image, shown on the next page with additional measurement notes overlaid, was an overhead shot of the Volt powertrain and chassis.

The base measurement was the wheelbase dimensions as described by General Motors to be 2685 mm. By drawing the green line in the image representing the wheelbase measurement we acquire the number of pixels in the length of the wheelbase. This was found to be 1628 pixels. Thus the conversion ratio between pixels and millimeters was found to be 2685 mm/1628 pixels = 1.59 mm/pixel.

To determine the estimated lengths of other components, lines were drawn on the image representing the desired measurements. The number of pixels in these lines multiplied by the conversion ratio yielded the estimated lengths. The table below summarizes the length estimations used in this project.

Symbol	Description	Units	Symbol	Description	Units
e5	Est.Volt tire outer diameter	[mm]	L_{front}	Distance to the front axle	[mm]
e10	Est. Volt pedal to firewall distance	[mm]	R_T	Radius of tire	[m]
e5	Est. Volt tire outer diameter	[mm]	$L_{firewall}$	Length from front to firewall	[mm]


Tire diameter
1628 pixels (wheelbase line)
2685 mm (Volt official wheelbase)
--> 1.59 mm/pixel
Pedal to firewall distance
Length to front axle
Length to firewall

Appendix B – PSAT Model (Rakesh et. Al)

The PSAT simulation key model parameters are listed in the following output file generated by PSAT.

```
The configuration is Series Engine Hybrid
The transmission is a No transmission
The position choice is Single reduction
**** VEHICLE
****************************************
The initialization file of the vehicle is veh_800_f0f1f2_ford_focus
The vehicle mass is 1679.03

**** ENGINE
****************************************
The initialization file of the engine is eng_si_1000_Insight
The original maximum power is  49.57 kW
The original peak efficiency is  39.28 %
This file has not been scaled

**** ELECTRIC MOTOR
****************************************
The initialization file of the motor is mc_pm_55_100_UQM_PowerPhase100
The original maximum mechanical power is 101.49 kW
The original peak efficiency is  94.00 %
This file has not been scaled

**** GENERATOR
****************************************
The initialization file of the generator is gc_pm_58_58
The original maximum power is  58.11 kW
The original peak efficiency is  91.50 %
This file has not been scaled

**** FINAL DRIVE
****************************************
The initialization file of the final drive is fd_307_Mercedes_C220CDI
This file has not been scaled

***** WHEEL
****************************************
The initialization file of the wheel is wh_0317_P195_65_R15

The initialization file of the mechanical accessories is accmech_300

**** MECHANICAL ACCESSORIES
****************************************
**** ELECTRICAL ACCESSORIES
****************************************
The initialization file of the electrical accessories is accelec_300
```

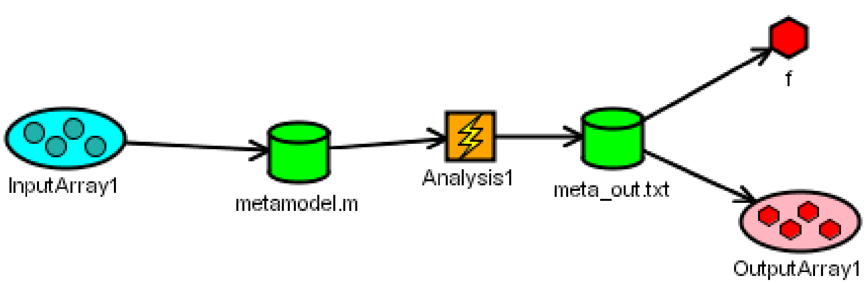
Appendix C – Battery Subsystem Optimization Details

C.1) SQP Method

```
%% Parameters

e1 = 4404;  %Volt overall length [mm]
e2 = 790;   %Max battery BaseY [mm]
e3 = 1064;  %Toyota Prius front leg room [mm]
e4 = 981;   %Toyota Prius rear leg room [mm]\
e5 = 948;   %Prius rear head room [mm]
e6 = 470;   %Estimated Volt tire outer diameter (not for simulation)[mm]
e7 = 1224;  %Volt estimated length from front to firewall [mm]
e8 = 875;   %Calculated leg room required for NHTSA airbag safety [mm]
e9 = 609;   %Typical seat width (y-direction) [mm]
e10 = 50;   %Constraint for minimum fuel economy on UDDS cycle [mpg]
e11 = 150;  %Door panel widths (y-direction) [mm]
e12 = 242;  %Volt est. pedal to firewall distance [mm]
e13 = 1798; %Volt official vehicle width
e14 = 150;  %Battery pack arm height (BattArmZ) [mm] ***WAS 250***
e15 = 150;  %typical back of seat thickness (x-direction) [mm]
e16 = 520;  %typical bottom of seat length (x-direction) [mm]
e17 = 150;  %max height of battery base z

%% Design Variables:
ArmX = 519.9998;
ArmY = 280;
ArmZ = e14;
BaseX = 2190;
BaseY = 790;
BaseZ = 150;
LfrSeat = 1744;
Lcabin = 2710;
Ltrunk = e1-e7-Lcabin; %Not a design parameter due to equ. constraint

%intermediate vars
L_FR_SEAT_HAMZA = 811;  %Hamza's optimal front seat position [mm]
L_RE_SEAT_HAMZA = 1709; %Hamza's optimal rear seat location,
                            %compare to our Lcabin, rear seats assumed to
                            %be constrained to end of cabin

batt_vol = ArmX*ArmY*e14+BaseX*BaseY*BaseZ;

%% Surrogate equations:
%Metamodel creation based on Rakesh Patil PSAT simulation data
AER = 8e-7*batt_vol + 0.8639;
FuelEcon = 5e-16*batt_vol^2 - 2e-8*batt_vol + 57.998;

%% Objective Function
f1 = (AER/40);
f2 = (1-abs(L_FR_SEAT_HAMZA-(LfrSeat-e7))/L_FR_SEAT_HAMZA) ;
f3 = (1-abs(L_RE_SEAT_HAMZA-(Lcabin-e15))/L_RE_SEAT_HAMZA);

%Objective function: MAXIMIZE:
f=1/3*(f1+f2+f3);
```

```
%% Constraints
%subject to:

%g1 = eliminated (equality constraint)
g2 = ArmX + BaseX - Lcabin;
g3 = BaseY - e2;
g4 = -LfrSeat + e15 + e7 + e12 + e3;
g5 = e4 - Lcabin + e15 + LfrSeat - e7;
g6 = e10 - FuelEcon;
g7 = e6 - Ltrunk;
g8 = ArmY + 2*e9 - e13 +2*e11;
g9 = LfrSeat + BaseX - e7 - Lcabin;
g10 = e7 + e16 - LfrSeat;
g11 = e16 - BaseX;
g12 = BaseZ - e17;

%% File output to meta_out.txt
%   Write values of objective function and constraint to a text file
    fid = fopen('meta_outMOD.txt', 'w+');
    fprintf(fid, 'Batt. Vol = %f\n', batt_vol);
    fprintf(fid, 'Fuel Econ. = %f\n', FuelEcon);
    fprintf(fid, 'g2 = %f\n', g2);
    fprintf(fid, 'g3 = %f\n', g3);
    fprintf(fid, 'g4 = %f\n', g4);
    fprintf(fid, 'g5 = %f\n', g5);
    fprintf(fid, 'g6 = %f\n', g6);
    fprintf(fid, 'g7 = %f\n', g7);
    fprintf(fid, 'g8 = %f\n', g8);
    fprintf(fid, 'g9 = %f\n', g9);
    fprintf(fid, 'g10 = %f\n', g10);
    fprintf(fid, 'g11 = %f\n', g11);
    fprintf(fid, 'g12 = %f\n', g12);
    fprintf(fid, 'f1 = %f\n', f1);
    fprintf(fid, 'f2 = %f\n', f2);
    fprintf(fid, 'f3 = %f\n', f3);
    fclose(fid);
```

Optimus model for Battery Packaging subsystem is shown below.



C.1) DIRECT Method

Optimization File

```
clear all
GLOBAL.MaxIter=50000;
GLOBAL.MaxEval=0;
vlb=[0 0 0 0 0 0 0];
vub=[5000 5000 5000 5000 5000 5000 5000 ];
I=[ ];
nc=11;
c_L=-inf*ones(nc,1);
c_U=zeros(nc,1);
A=[ ]; b_L=[ ]; b_U=[ ];
xopt = gclSolve('directFUN', 'directNONLCON', vlb, vub, A, b_L, b_U, c_L,
c_U, I, GLOBAL);
optimalx=xopt.x_k  %optimal point
optimalf=xopt.f_k  %optimal objective
```

Function (objective) File

```
function [fdirect] = directFUN(x)

%% Parameters
e14 = 150;  %Battery pack arm height (BattArmZ) [mm] ***WAS 250***
%% Design Variables:
ArmX = x(1);
ArmY = x(2);
ArmZ = e14;
BaseX = x(3);
BaseY = x(4);
BaseZ = x(5);
LfrSeat = x(6);
Lcabin = x(7);
Ltrunk = e1-e7-Lcabin; %Not a design parameter due to equ. constraint

%intermediate vars
L_FR_SEAT_HAMZA = 811+570;  %Hamza's optimal front seat position [mm]
L_RE_SEAT_HAMZA = 1709+570; %Hamza's optimal rear seat location,
                            %compare to our Lcabin, rear seats assumed to
                            %be constrained to end of cabin

batt_vol = x(1)*x(2)*e14+x(3)*x(4)*x(5);

%% Surrogate equations:
AER = 8e-7*batt_vol + 0.8639;
FuelEcon = 5e-16*batt_vol^2 - 2e-8*batt_vol + 57.998;

%% Objective Function
f1 = (AER/40);
f2 = (1-abs(L_FR_SEAT_HAMZA-(LfrSeat-e7-e15))/L_FR_SEAT_HAMZA) ;
f3 = (1-abs(L_RE_SEAT_HAMZA-(Lcabin-e15))/L_RE_SEAT_HAMZA);

%Objective function: MAXIMIZE:
f=1/3*(f1+f2+f3);
fdirect =-f;
```

Constraints File

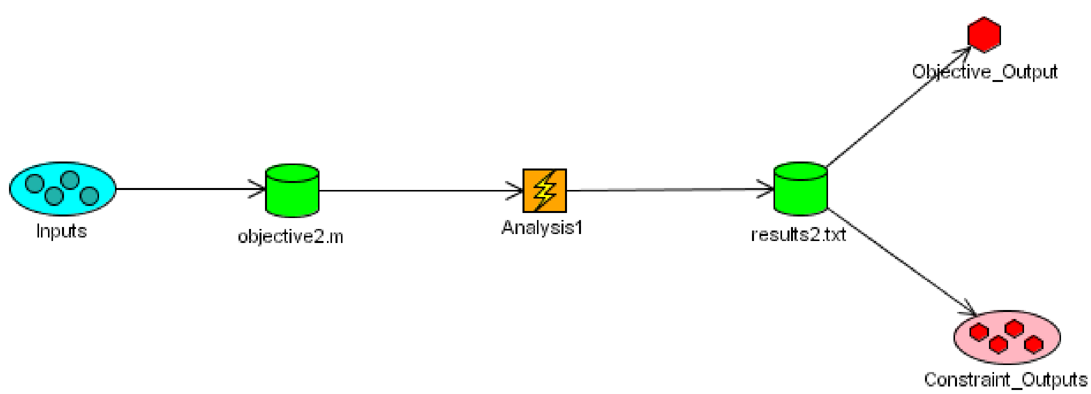
```
function [g,h] = directNONLCON(x)

%% Parameters
e1 = 4404;  %Volt overall length [mm]
e2 = 790;   %Max battery x(4) [mm]
e3 = 1064;  %Toyota Prius front leg room [mm]
e4 = 981;   %Toyota Prius rear leg room [mm]\
e5 = 948;   %Prius rear head room [mm]
e6 = 470;   %Estimated Volt tire outer diameter (not for simulation)[mm]
e7 = 1224;  %Volt estimated length from front to firewall [mm]
e8 = 875;   %Calculated leg room required for NHTSA airbag safety [mm]
e9 = 609;   %Typical seat width (y-direction) [mm]
e10 = 50;   %Constraint for minimum fuel economy on UDDS cycle [mpg]
e11 = 150;  %Door panel widths (y-direction) [mm]
e12 = 242;  %Volt est. pedal to firewall distance [mm]
e13 = 1798; %Volt official vehicle width
e14 = 150;  %Battery pack arm height (BattArmZ) [mm] ***WAS 250***
e15 = 150;  %typical back of seat thickness (x-direction) [mm]
e16 = 520;  %typical bottom of seat length (x-direction) [mm]
e17 = 150;  %max height of battery base z
Ltrunk = e1-e7-x(7); %Not a design parameter due to equ. constraint

batt_vol = x(1)*x(2)*e14+x(3)*x(4)*x(5);

%% Surrogate equations:
AER = 8e-7*batt_vol + 0.8639;
FuelEcon = 5e-16*batt_vol^2 - 2e-8*batt_vol + 57.998;

%ineq. constraints

%g1 = eliminated (equality constraint)
g = [(x(1) + x(3) - x(7));
(x(4) - e2);
(-x(6) + e15 + e7 + e12 + e3);
(e4 - x(7) + e15 + x(6) - e7);
(e10 - FuelEcon);
(e6 - Ltrunk);
(x(2) + 2*e9 - e13 +2*e11);
 (x(6) + x(3) - e7 - x(7));
(e7 + e16 - x(6));
(e16 - x(3));
(x(5) - e17)];

h=[];
```

Appendix D – Handling Subsystem Optimization Details

D.1) SQP Method

```
%Team 3 - ME 555
%Vehicle Handling Subsystem
%Author: Timothy Stutz
%This m-file is to be used with Optimus to perform the optimization for the
%handling subsystem.

%Design Variables
L_b = 2.91681;              %m; length of wheelbase (variable)
C_f = 2.4713;               %m; length to front seats center (variable)
C_r = 3.47541;              %m; length to rear seats center (variable)
C_b = 2.96597;              %m; length to battery center (variable)

%Mass/Inertial
M = 1500;                   %kg; total mass
m_f = 181;                  %kg; mass of front seats w/ passengers
m_r = 181;                  %kg; mass of rear seats w/ passengers
m_p = 250;                  %kg; mass of powertrain system
m_b = 200;                  %kg; mass of battery pack
m_e = M-(m_f+m_r+m_p+m_b);  %kg; mass of everything else

%Distance/Placement
L = 4.404;                  %m; length of vehicle
L_f = 1.05;                 %m; length from front of car to front axle
C_p = 0.8;                  %m; length to powertrain center (fixed)
C_e = L/2;                  %m; length to "everything else" center (fixed)

%Calculate CG location
Cg = (m_b*C_b+m_e*C_e+m_f*C_f+m_p*C_p+m_r*C_r)/M;   %m; length from front

%Calculate distance from Cg to front/wheel axles
a = Cg-L_f;                 %m; length from front axle to Cg
b = L_b-a;                  %m; length from rear axle to Cg

%Tire Properties
Caf = 47000;          %N/rad; cornering stiffness of front wheels
Car = 44000;          %N/rad; cornering stiffness of rear wheels

%This is the calculation for the static Kus value that will be used to
%compare with the ideal at various velocities.
Kus = (M*b)/(L_b*Caf)-(M*a)/(L_b*Car);

%Constraint Equations
g1 = 1.851 - C_f;
g2 = C_f + 0.98 - C_r;
g3 = C_f - C_b;
g4 = C_b - C_r;
g5 = L_f - Cg;
g6 = Cg - L_f - L_b;
g7 = 2.685 - L_b;
```

```
g8 = L_b - L + L_f + 0.47/2;
g9 = C_r - L + 0.47/2;
g10 = -(Kus*1000);

v0 = [0:5:40];
%The Ideal Kus for the velocities in the vector above, the equation used
%was offline and determined to fit an ideal quadratic. The equation is
%Kus_I = (8e-6)*(v0)^2 + (9e-5)*(v0) - 0.0004;
Kus_I = [-0.0004 0.0002 0.0013 0.0027 0.0046 0.0069 0.0095 0.0126 0.016];
R = 1 - sum(sqrt((Kus-Kus_I).^2));    %Least squares estimation
Obj = R;

%Write values of objective function and constraint to a text file
fid = fopen('results2.txt', 'wt');
fprintf(fid, 'Obj = %f\n', Obj);
fprintf(fid, 'g1 = %f\n', g1);
fprintf(fid, 'g2 = %f\n', g2);
fprintf(fid, 'g3 = %f\n', g3);
fprintf(fid, 'g4 = %f\n', g4);
fprintf(fid, 'g5 = %f\n', g5);
fprintf(fid, 'g6 = %f\n', g6);
fprintf(fid, 'g7 = %f\n', g7);
fprintf(fid, 'g8 = %f\n', g8);
fprintf(fid, 'g9 = %f\n', g9);
fprintf(fid, 'g10 = %f\n', g10);
fclose(fid);
```



D.2) DIRECT Method

FUN.m File

```
function [f]=FUN(x)
%Mass/Inertial
M = 1500;                    %kg; total mass
m_f = 181;                   %kg; mass of front seats w/ passengers
m_r = 181;                   %kg; mass of rear seats w/ passengers
m_p = 250;                   %kg; mass of powertrain system
m_b = 200;                   %kg; mass of battery pack
m_e = M-(m_f+m_r+m_p+m_b);   %kg; mass of everything else

%Distance/Placement
L = 4.404;                   %m; length of vehicle
L_b = x(1);                 %m; length of wheelbase (variable)
L_f = 1.05;                  %m; length from front of car to front axle
C_p = 0.8;                   %m; length to powertrain center (fixed)
C_e = L/2;                   %m; length to "everything else" center (fixed)
C_f = x(2);                   %m; length to front seats center (variable)
C_r = x(3);                   %m; length to rear seats center (variable)
C_b = x(4);                   %m; length to battery center (variable)

%Calculate CG location
Cg = (m_b*C_b+m_e*C_e+m_f*C_f+m_p*C_p+m_r*C_r)/M;   %m; length from front

%Calculate distance from Cg to front/wheel axles
a = Cg-L_f;                  %m; length from front axle to Cg
b = L_b-a;                   %m; length from rear axle to Cg

%Tire Properties
Caf = 47000;           %N/rad; cornering stiffness of front wheels
Car = 44000;           %N/rad; cornering stiffness of rear wheels

%Kus will be used to calculate the "steering sensitivity" value and used to
%determine the stability. As Kus increases, the vehicle becomes more
%directionally stable.
Kus = (M*b)/(L_b*Caf)-(M*a)/(L_b*Car);

v0 = [0:5:40];
%The Ideal Kus for the velocities in the vector above;
Kus_I = [-0.0004 0.0002 0.0013 0.0027 0.0046 0.0069 0.0095 0.0126 0.016];
R = 1 - sum(sqrt((Kus-Kus_I).^2));
f = -R;
end
```

NONLCON.m File

```
function [g,h]=NONLCON(x)
L_b = x(1);
C_f = x(2);
C_r = x(3);
C_b = x(4);

%Mass/Inertial
M = 1500;                    %kg; total mass
I = 2420;                    %kg*m^2; Moment of Inertia about Yaw Axis
m_f = 181;                   %kg; mass of front seats w/ passengers
m_r = 181;                   %kg; mass of rear seats w/ passengers
m_p = 250;                   %kg; mass of powertrain system
m_b = 200;                   %kg; mass of battery pack
m_e = M-(m_f+m_r+m_p+m_b);   %kg; mass of everything else

%Distance/Placement
L = 4.404;                   %m; length of vehicle
L_f = 1.05;                  %m; length from front of car to front axle
C_p = 0.8;                   %m; length to powertrain center (fixed)
C_e = L/2;                   %m; length to "everything else" center (fixed)

%Calculate CG location
Cg = (m_b*C_b+m_e*C_e+m_f*C_f+m_p*C_p+m_r*C_r)/M;   %m; length from front

%Calculate distance from Cg to front/wheel axles
a = Cg-L_f;                  %m; length from front axle to Cg
b = L_b-a;                   %m; length from rear axle to Cg

%Tire Properties
Caf = 47000;           %N/rad; cornering stiffness of front wheels
Car = 44000;           %N/rad; cornering stiffness of rear wheels

%Kus will be used to calculate the "steering sensitivity" value and used to
%determine the stability. As Kus increases, the vehicle becomes more
%directionally stable.
Kus = (M*b)/(L_b*Caf)-(M*a)/(L_b*Car);

%Constraint Equations
g(1) = 1.851 - C_f;
g(2) = C_f + 0.98 - C_r;
g(3) = C_f - C_b;
g(4) = C_b - C_r;
g(5) = L_f - Cg;
g(6) = Cg - L_f - L_b;
g(7) = 2.685 - L_b;
g(8) = L_b - L + 0.47/2;
g(9) = C_r - L + 0.47/2;
g(10) = -(Kus*1000);
end
```

DIRECT.m File

```
clear all
GLOBAL.MaxIter=0;
GLOBAL.MaxEval=1000;
vlb=[2.685 1.851 2.831 1.851];  %determined based off constraint activity
vub=[4.169 3.189 4.169 4.169];  %determined based off constraint activity

nc=10;
c_L=inf*ones(nc,1);
c_U=zeros(nc,1);
A=[];b_L=[];b_U=[];
I=[];

xopt=gclSolve('FUN','NONLCON',vlb,vub,A,b_L,b_U,c_L,c_U,I,GLOBAL)
optimalx=xopt.x_k
optimalf=xopt.f_k
```

Result Example

optimalx =

3.4270

2.4209

4.0947

4.0402

optimalf =

-0.9588

Appendix E – Mathematical Calculations for "Proof" of Infinite Solutions.

Given

$$K_{US}\left(C_b, C_f, C_r, L_{wheelbase}\right) \text{ and } C_{\propto f} = C_{\propto r} = C_{\propto}$$

Then

$$K_{US} = \frac{\left(M + \frac{2ML_{front}}{L_{wheelbase}} - \frac{1}{L_{wheelbase}}\left(m_b C_b + m_e C_e + m_f C_f + m_p C_p + m_r C_r\right)\right)}{C_\alpha}$$

$$L_{wheelbase}(C_{\propto}K_{US} - M) = 2ML_{front} - m_e C_e - m_p C_p - m_b C_b - m_f C_f - m_r C_r$$

$$m_b C_b + m_f C_f + m_r C_r + (C_{\propto}K_{US} - M)L_{wheelbase} = 2ML_{front} - m_e C_e - m_p C_p$$

Let

$$K_1 = m_b \quad K_2 = m_f \quad K_3 = m_r \quad K_4 = C_{\propto}K_{US} - M \quad K_5 = 2ML_{front} - m_e C_e - m_p C_p$$

$$x_1 = C_b \quad x_2 = C_f \quad x_3 = C_r \quad x_4 = L_{wheelbase}$$

Thus:

$$K_1 x_1 + K_2 x_2 + K_3 x_3 + K_4 x_4 = K_5$$

We also know that all of the constraints are linear. If we have a design variable that is defined by an active constraint, then a new linear could be written for that constraint that is only a function of the design variable associated with the activity. Thus for *n* variables and *n* active constraints:

$$A_i x_i - b_i = 0 \quad i = 1 \dots n$$

This could also be seen by letting $A_i = 1$ that you are simply letting b_i be the value of the variable at activity.

Appendix F – System Integration Optimization Details

Matlab .m file

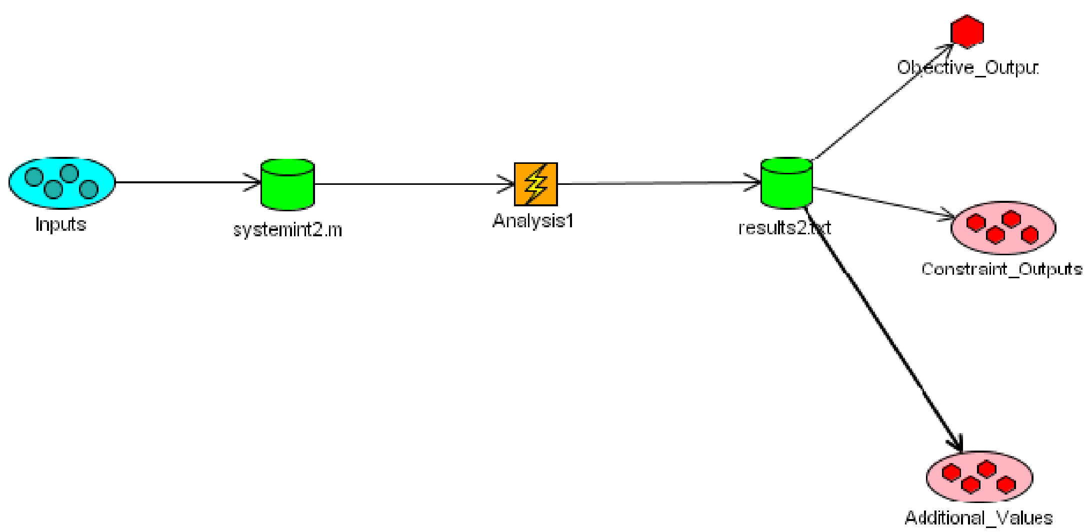
```
%System Level Design Optimization for Group #3
%The goal of this optimization is to maximize the fuel economy and handling
%characteristics for an extended range electric vehicle while maintaining
%appropriate packaging constraints and comfort constraints.

%Design Variables - These are the variables optimus will change to find the
%maximum
ArmX = 1553;         %mm, the length of battery arm
ArmY = 280;          %mm, the width of battery arm
BaseX = 1157;        %mm, the length of battery base
BaseY = 790;         %mm, the width of battery base
BaseZ = 150;         %mm, the height of the battery base
LfrSeat = 2777;      %mm, the distance b/w front end and back of front seat
Lcabin = 2710;       %mm, the distance b/w firewall and end of cabin
C_r = 3502;          %mm, the distance b/w front end and rear seat C.G

%Vehicle Parameters
%Mass/Inertial
m_b = (ArmX*ArmY*150+BaseX*BaseY*BaseZ)*2e-6;   %kg; mass of battery
M = 1300+m_b;                   %kg, total mass
I = 2420;                       %kg*m^2, Moment of Inertia about Yaw Axis
m_f = 181;                      %kg, mass of front seats w/ passengers
m_r = 181;                      %kg, mass of rear seats w/ passengers
m_p = 250;                      %kg, mass of powertrain system
m_e = 688;                      %kg, mass of everything else
ma = ArmX*ArmY*150*2e-6;        %kg, mass of battery arm
mb = BaseX*BaseY*BaseZ*2e-6;    %kg, mass of battery base

%Distance/Placement
L_f = 1.05*1000;            %mm, distance b/w front end and front axle
C_p = 0.8*1000;             %mm, distance b/w front end and powertrain C.G

%Tire Properties
Caf = 47000;          %N/rad, cornering stiffness of front wheels
Car = 44000;          %N/rad, cornering stiffness of rear wheels

%Constraint Parameters
e1 = 4404;  %Volt overall length [mm]
e2 = 790;   %Max battery BaseY [mm]
e3 = 1064;  %Toyota Prius front leg room [mm]
e4 = 981/2; %Toyota Prius rear leg room [mm]
e5 = 948;   %Prius rear head room [mm]
e6 = 470;   %Estimated Volt tire outer diameter (not for simulation)[mm]
e7 = 1224;  %Volt estimated length from front to firewall [mm]
e8 = 875;   %Calculated leg room required for NHTSA airbag safety [mm]
e9 = 609;   %Typical seat width (y-direction) [mm]
e10 = 50;   %Constraint for minimum fuel economy on UDDS cycle [mpg]
e11 = 150;  %Door panel widths (y-direction) [mm]
e12 = 242;  %Volt est. pedal to firewall distance [mm]
e13 = 1798; %Volt official vehicle width
e14 = 150;  %Battery pack arm height (BattArmZ) [mm] ***WAS 250***
e15 = 150;  %typical back of seat thickness (x-direction) [mm]
```

```
e16 = 520;  %typical bottom of seat length (x-direction) [mm]
e17 = 150;  %max height of battery base z
C_e = e1/2; %mm; length to "everything else" center (fixed)

%Intermediate Calculations
%Calculations necessary to translate the battery/packaging variables into
%the handling variables.
L_b = (e7+Lcabin)-L_f+(1/2)*470;    %mm, length of wheelbase
C_b = e7+Lcabin-((mb*(0.5)*BaseX+ma*(BaseX+ArmX/2))/m_b); %battery C.G
C_f = LfrSeat-(1/3)*520;            %mm, front seat C.G

%Calculations necessary for battery subsystem objective function
L_FR_SEAT_HAMZA = 811+570;  %mm, Hamza's optimal front seat position
L_RE_SEAT_HAMZA = 1709+570; %mm, Hamza's optimal rear seat location
batt_vol = ArmX*ArmY*e14+BaseX*BaseY*BaseZ; %battery volume
FuelEcon = 5e-16*batt_vol^2 - 2e-8*batt_vol + 57.998;

%Other necessary calculations
Ltrunk = e1-e7-Lcabin; %Not a design parameter due to equ. constraint
Cg = (m_b*C_b+m_e*C_e+m_f*C_f+m_p*C_p+m_r*C_r)/M;  %mm, vehicle C.G

%Calculate distance from Cg to front/wheel axles
a = Cg-L_f;                 %mm; length from front axle to Cg
b = L_b-a;                  %mm; length from rear axle to Cg

%Calculation of Kus - the understeer coefficient
Kus = (M*(b/1000))/((L_b/1000)*Caf)-(M*(a/1000))/((L_b/1000)*Car);

%Constraint Equations
%Handling Subsystem

%Note: Constraints with a -1 given were used to remove the redundant,
%non-dominant constraint. Most of these were determined either with
%pre-calculations or by analyzing the first few results and realizing they
%weren't active. It also turns out, most of these are from the handling
%subsystem, as the constraints determined through there were much
%"rougher" and not necessarily as accurate. Also, not all redundant
%constraints are given a -1, some were left redundant, and some -1s were
%due to constraints being "unimportant" like the constraint on the battery
%c.g. from the handling subsystem.

g1 = -1;%1851 - C_f;                %How far forward front seat can go
g2 = -1;%C_f + 980 - C_r;           %How close rear seat can get to front
g3 = -1;%C_f - C_b;                 %How far forward battery could be
g4 = -1;%C_b - C_r;                 %How far back battery could be
g5 = L_f - Cg;                      %Cg must lie between the front
g6 = Cg - L_f - L_b;                %and rear axles
g7 = 2685 - L_b;                    %minimum wheelbase
g8 = L_b - e1 + L_f + (1/2)*470;    %maximum wheelbase
g9 = C_r - e1 + (1/2)*470;          %maximum rear seat location
g10 = -(Kus*1000);                  %Kus must be positive

%Battery Subsystem
%g11 = eliminated (equality constraint)
g12 = ArmX + BaseX - Lcabin;            %Battery exists in the cabin
```

```
g13 = BaseY - e2;                        %Maximum base width
g14 = -LfrSeat + e15 + e7 + e12 + e3;                %Front Leg room
g15 = e4 - (C_r-e7+(1/3)*e16) + e15 + LfrSeat - e7; %Rear Leg room
g16 = e10 - FuelEcon;
g17 = e6 - Ltrunk;
g18 = ArmY + 2*e9 - e13 +2*e11;
g19 = LfrSeat + BaseX - e7 - Lcabin;
g20 = -1;%e7 + e16 - LfrSeat;
g21 = e16 - BaseX;
g22 = BaseZ - e17;

%Objective Calculations
w1 = 10;    %front seat optimality weight
w2 = 10;    %rear seat optimality weight
w3 = 1;     %AER optimality weight
w4 = 10;    %handling weight
%Handling
%The Kus_I vector is the "ideal" Kus value at various velocities ranging
%from 0 to 40 m/s. The f_hand is the least squares average between the
%constant Kus determined by the design variables, and this curve created
%heuristically.

Kus_I = [-0.0004 0.0002 0.0013 0.0027 0.0046 0.0069 0.0095 0.0126 0.016];
f_hand = w4*(1 - sum(sqrt((Kus-Kus_I).^2)));

%Battery/Packaging
AER = 8e-7*batt_vol + 0.8639;

%f1 is the AER objective, normalized to be 1 at 40miles. f2 is the front
%seat optimality condition, normalized to be 1 at optimum, same as f3 only
%for the rear seat.
f1 = w3*(AER/40);
f2 = w1*(1-abs(L_FR_SEAT_HAMZA-(C_f-e7))/L_FR_SEAT_HAMZA);
f3 = w2*(1-abs(L_RE_SEAT_HAMZA-(C_r-e7))/L_RE_SEAT_HAMZA);

f_batt=f1+f2+f3;

%Our overall objective is going to be equal weighting between f_batt and
%f_hand, given w1->w4 are all 1.
Obj = f_batt/3+f_hand;

%Result Output
%Write values of objective function and constraint to a text file, as well
%as intermediate calculations.
fid = fopen('results2.txt', 'wt');
fprintf(fid, 'Obj = %f\n', Obj);
fprintf(fid, 'f_hand = %f\n', f_hand);
fprintf(fid, 'f_batt = %f\n', f_batt);
fprintf(fid, 'f1 = %f\n', f1);
fprintf(fid, 'f2 = %f\n', f2);
fprintf(fid, 'f3 = %f\n', f3);
fprintf(fid, 'FuelEcon = %f\n', FuelEcon);
fprintf(fid, 'Kus = %f\n', Kus);
fprintf(fid, 'g1 = %f\n', g1);
fprintf(fid, 'g2 = %f\n', g2);
```

```
fprintf(fid, 'g3 = %f\n', g3);
fprintf(fid, 'g4 = %f\n', g4);
fprintf(fid, 'g5 = %f\n', g5);
fprintf(fid, 'g6 = %f\n', g6);
fprintf(fid, 'g7 = %f\n', g7);
fprintf(fid, 'g8 = %f\n', g8);
fprintf(fid, 'g9 = %f\n', g9);
fprintf(fid, 'g10 = %f\n', g10);
fprintf(fid, 'g12 = %f\n', g12);
fprintf(fid, 'g13 = %f\n', g13);
fprintf(fid, 'g14 = %f\n', g14);
fprintf(fid, 'g15 = %f\n', g15);
fprintf(fid, 'g16 = %f\n', g16);
fprintf(fid, 'g17 = %f\n', g17);
fprintf(fid, 'g18 = %f\n', g18);
fprintf(fid, 'g19 = %f\n', g19);
fprintf(fid, 'g20 = %f\n', g20);
fprintf(fid, 'g21 = %f\n', g21);
fprintf(fid, 'g22 = %f\n', g22);
fclose(fid);
```

Optimus Layout



Note: the "Additional_Values" block collects the values for parameters which aren't the constraints nor the objective, but intermediate calculations and the different sub-objectives for the battery subsystem.