OPTIMAL DESIGN OF MULTI-PLATE CLUTCH SYSTEM

by

Abhijit Khare

Adrien Yee

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Instructor: Professor Michael Kokkolaras

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Abstract

Our project aims to optimize the clutch configuration in power transmission system, specifically for automotive application. The optimization problem is split into two subsystems i.e. system packaging and thermal analysis. System packaging involves minimizing the mass of the friction plates and hence improving compactness while at the same time upholding its key performance requirements such as allowable slip time, packaging constraints and etc. The thermal analysis subsystem serves as a design tradeoff against system packaging. The greater the dimension of the disc and lining, the greater is the mass but the better is the heat dissipation. Also the greater the slip time the greater is the heat produced. The analyses for each of these subsystems have been carried out separately and will be combined as an integrated system optimization.

The division of work is as follows:

a. System packaging sub-system is done by Adrien Yee
b. Thermal analysis sub-system is done by Abhijit Khare

1 Problem Definition

A friction clutch is a means of transmitting motion from the engine to the gearbox in an automobile. When there is a need to switch to a different gear ratio, the clutch pedal is pushed. This pushes the left hand shaft and the contact between the plates on the two shafts is lost. Consequently, the output shaft is no longer driven by the input shaft and we can shift gears on the downside of the driven shaft. When this gear change operation is completed, the clutch pedal is released. This allows the two sets of plates to contact each other again, allowing the input and output shafts to rotate in sync and transmission of torque between these components.

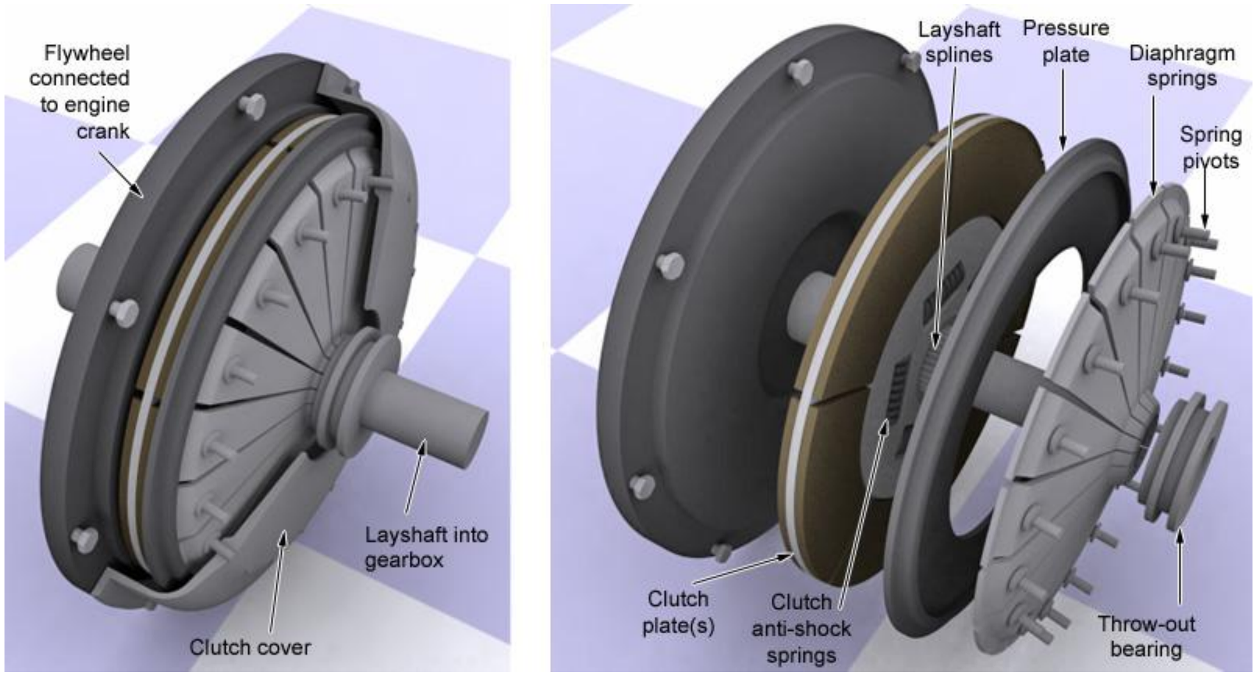

Figure 1: Automotive Clutch System

As discussed above, the operation of the multi-plate clutch depends on the friction between the friction plates and the pressure plates. Two of the main aspects to the clutch design that significantly affect its performance are mass of the clutch and heat dissipation.

Here is an explanation of effect of these aspects in the operation of a friction clutch system. The clutch pedal is released so that the friction discs can make contact when the pressure plates are gradually pressed against the friction plates. However the driven plates are stationary or rotating at lower speed. Hence we are essentially taking an object from low velocity or rest and making it attain a pre-decided velocity equal to that of the driving shaft. Thus here we encounter the first issue. The discs, when they make contact with each other slip against each other at first. We need the driven shaft to rotate at the speed of the driver shaft as quickly as possible. That is we need to reduce this slip time to a low value but high enough to give a smooth transition. In other words we require that the frictional torque of the clutch to be higher. Also as is the case with any subsystem of a larger system, there are packaging constraints which need to be taken care of. The clutch assembly should not occupy too much space. It should fit into the space available in the transmission case. Additionally, the mass of the assembly should be less in order to minimize the overall mass and moment of inertia. On top of this as is the case with any device functioning on the principles of friction, there is production of heat due to the friction between the discs on the driver shaft and the driven shaft. This is undesirable since it leads to wear of the friction material and also affects the efficiency of the clutch. Also since the material wears out, we need to adjust the clutch travel frequently to make up for the distancing of the plates which requires more clamping force. Therefore we need to reduce the time required to cool the disc and the lining to normal temperature.

To handle this problem the optimization of the main system is split into two sub-systems and it will be tackled from four aspects:

1. Packaging of the system – mass of the clutch-plates
2. Reduction of slip time between the pressure plates and the frictions plates
3. Reduction of time required to cool the friction plate
4. Reduction of time required to cool the friction lining

1.1 System Packaging

In designing a product, one of the main issues is often the packaging of the system. The mass of the system needs to as low as possible while satisfying the performance requirements. For an automobile clutch, lesser mass means the lesser load on the engine hence improving the engine response and prolonging the life of the parts. However, the reduction of mass in this case is constrained by the slipping time. There will be four variables that can be looked into in minimizing the mass: inner radius of the disc, outer radius of the disc, clamping force and the thickness of the disc. In addition, several important parameters are required to support the computation e.g. the number of plates with 2 surfaces each, the coefficient of the pad material used. We will also be conducting parametric studies on these two variables and will try to find the optimal solution to this problem for a particular set of values for number of plates and coefficient of friction of the pad material.

1.2 Thermal Analysis

As explained previously we need to build the clutch so that it is easier to cool. Excess heating of the clutch plate and lining affects the performance of the clutch and enhances the wear of the clutch plate/ lining. Here we have to reduce the cooling time for the friction disc and the lining on it. Since optimizing both at once would have turned into a multi-objective optimization problem at the subsystem stage and this was something that we did not wish to handle at this stage, we decided to covert both the limiting temperatures for the lining and the disc into non-linear constraints. There was a common limiting cooling time for both the disc and the lining. We then decided to include both the cooling times as constraints while trying to optimize for maximum cooling time. The variables in this system were taken as the inner radius and outer radius of the friction disc, the angle of lining covering on the disc, the thickness of the disc and the thickness of the lining. Also we decided to do parametric studies on various material combinations for the disc and the lining.

2 Nomenclature

2.1 System Packaging

2.1.1 Parameters and Intermediate variables

δ = distance between discs when unloaded [mm],

Δr = minimum difference between radii [mm],

μk = dynamic coefficient of friction

dmax = maximum disc thickness [mm],

Fmax = maximum clamping force [N].

Iz = moment of inertia of the working machine [kg mm2],

Lmax = maximum length [mm], N = input speed [rev/min],

nmax = maximum number of discs,

Pmax = maximum allowable pressure on the disc [Mpa],

ri min = minimum inner radius [mm],

ro max = maximum outer radius [mm],

s = factor of safety,

Tf = frictional resistance torque [Nm],

Ts = static input torque [Nm],

tmax = maximum slipping time [s],

Vs = maximum relative speed of the slip [m/s],

ρ_D = Density of disc material [Kg/m^3]

2.1.2 Variables

ri = inner radius [mm],

ro = outer radius [mm],

d = thickness of the discs [mm],

F = clamping force [N],

n = number of friction surfaces. (this is a discreet variable and will be considered for parametric study)

2.2 Thermal Analysis

2.2.1 Parameters and intermediate variables

R = Tire radius [m]

M = Mass of the vehicle [Kg]

V = Velocity of the vehicle [m/s]

V_{max} = Maximum velocity of the vehicle [m/s]

uk = Coefficient of friction between disc and lining

ρ_D = Density of disc material [Kg/m^3]

ρ_L = Density of lining material [Kg/m^3]

CpD = Constant pressure specific heat of the disc [J/Kg K]

CpL = Constant pressure specific heat of the lining [J/Kg K]

t = desired slip time

s = factor of safety

σL = yield stress of lining [Pa]

σD = fatigue stress after 10^6 cycles of the disc [Pa]

T_{max} = Maximum temperature of lining [K]

T_{atm} = Ambient temperature [K]

T_1 = Final temperature after slip stops [K]

wc = Wear constant [N/m^4]

N = number of cycles before slip stops

L = Characteristic length of lining [m]

L_D = Characteristic length of disc [m]

Φ = Angle of disc exposed to air [rad]

A_S = Lining surface area [m^2]

A_{SD} = Effective heat dissipation area for the disc [m^2]

T_2 = Maximum lining temperature after slip stops [K]

V_L = Volume of lining [m^3]

V_D = Volume of disc [m^3]

ω = Angular velocity of the driving shaft [rad/sec]

ω_{max} = Maximum angular velocity [rad/sec]

τ = Torque applied by clutch [Nm]

F = Clamping force [N]

P_{max2} = Maximum pressure between lining and disc [Pa]

2.2.2 Variables

t_{max} = The longer time required to cool the lining or the disc

r_i = Inner radius of lining/ disc

r_o = Outer radius of lining/ disc

θ = Angle covered by the lining [rad]

d_L = Lining thickness [m]

d = Thickness of disc [m]

3 Mathematical Model

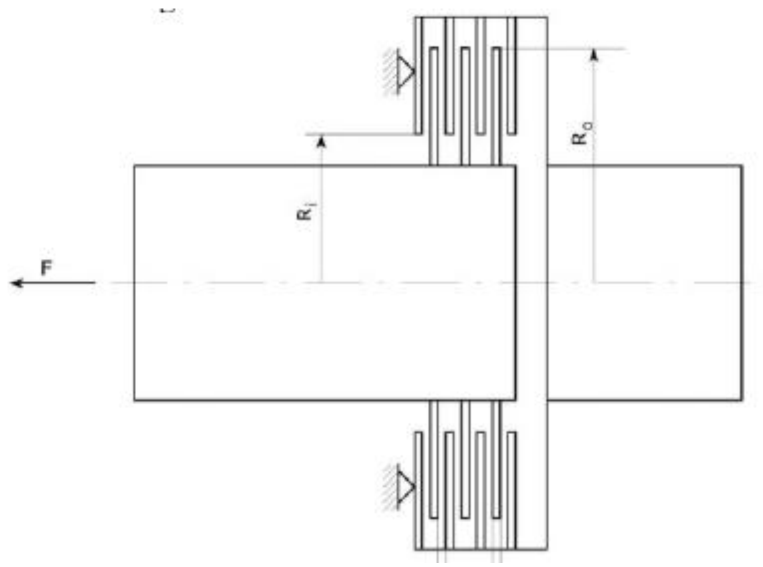

Figure 2: Schematic Diagram of Multi-plate Clutch

3.1 System Packaging

The objective function aims to minimize the mass of the friction plates which is simplified as hollow center plates with inner and outer diameters. The minimum inner diameter of the plate is constrained by the diameter of the inner shaft that goes through the friction plates. On the other hand, a maximum limit is imposed on the outer diameter of the plate by the inner diameter of the clutch cover. The total mass of the friction plates will be the mass of each plate multiply by the number of plates. The minimization of mass will have to satisfy the slip time, which increases when the working machine's moment of inertia increases or when the sum of braking and frictional resistance torque decreases. The braking torque provided by the clutch plates is proportional to the friction coefficient, clamping force, number of plates and the difference between the plate outer and inner diameter. A safety factor of 1.8 is applied to the input torque to account for some room of uncertainty errors. As the revolution of the input shaft increases, the slip time increases linearly. The clamping force is constrained by the maximum allowable pressure which is determined by the leverage of pedal force. There is also a temperature constraint since the slipping of friction plates generates enormous heat energy. It is proportional to the clamping pressure and relative speed between the slipping bodies. Lastly, the total length of the clutch plates is constrained by the available space between the flywheel and the inner surface of the clutch cover.

$g1 = (n + 1)(d + \delta) - L_{max} \leq 0$

The total length of the clutch plates should not be more than the specified maximum.

$g2 = p_{rz} - p_{max} \leq 0$

The clamping pressure should be less than the maximum allowable pressure.

$g3 = p_{rz.} V_{sr} - p_{max}. V_{sr\ max} \leq 0$

The heat generated is constrained in order to control the maximum temperature of the components.

$g4 = V_{sr} - V_{sr\ max} \leq 0$

The relative speed should be less than the maximum allowable relative speed to avoid overrunning the pad material.

$g5 = \frac{I_Z \omega}{T_b + T_f} - t_{max} \leq 0$

In this constraint we limit the amount of slip time before both plates start rotating at the same speed.

$g6 = F - F_{max} \leq 0$

The clamping force should be less than the specified minimum.

$g7 = -t \leq 0$

The slip time must be larger than zero.

$g8 = s\ T_s\ V_{sr} - T_b \leq 0$

The generated torque must be greater than the input torque times the safety factor.

$g9 = r_i - 0.8r_o \leq 0$

The outer radius should be less than the specified maximum.

$g10 = \frac{dF}{\pi E\,(r_o^2 - r_i^2)} - d$

The deflection of the disc due to the stress should not be more than the disc thickness.

3.2 Thermal Analysis

The goal of this section is to minimize the cooling time for the disc or the lining, whichever is more. When the clutch is disengaged to allow gear shifting, the driven shaft and the friction plates slow down as compared to the driving shaft and the pressure plates. After the suitable gear is selected, the clutch is re-engaged and the pressure plates start spinning against the friction plates which are rotating at slower speed. The heat that is produced during this time at which the pressure plates are trying to make the friction plates rotate at the same speed is large. This affects the friction lining on the friction plates and causes wear. Over a prolonged period of operation this wear reaches tangible proportions and the performance of the clutch reduces. Even after engaging the clutch there is less contact between the friction and pressure plates and this is not desirable. Hence we need to find the optimal dimensions of the lining/ disc and the angle covered by the lining on the disc so that we can optimize the system for lesser cooling time.

We are trying to minimize the cooling time for the disc or lining, whichever is more.

$\min f = t_{max}$

Before defining the constraints we need to define the intermediate variables:-

The characteristic length is the mean length of the disc/ lining in the tangential direction.

$$L = \left(\frac{r_i + r_o}{2} \right) \theta$$

$$L_D = \left(\frac{r_i/2 + r_o}{2} \right) \phi$$

The angle of the disc exposed to air is the angle of the full circle minus the angle covered by the lining.

$$\phi = 2\pi - 6$$

The one side surface area of disc and lining is given as:

$$A_S = \frac{\left(r_0^2 - r_i^2\right)\theta}{2}$$

$$A_{SD} = \frac{\left(r_0^2 - \left(r_i/2 \right)^2 \right)\phi}{2}$$

The volume of the disc and lining is given by:

$$V_L = \frac{(r_o^2 - r_i^2)\theta d L}{2}$$

$$V_D = \pi \left(r_o^2 - (\frac{r_i}{2})^2 \right) d$$

ω = V/R

Total energy input is equal to the kinematic energy loss in the clutch while slipping

$$\tau = \frac{1}{8\,t^2} M\,R^2 \omega$$

Clamping force in terms of torque is given by

$$\mathrm{F} = 2\frac{\tau}{\mu(\mathrm{r_o} + \mathrm{r_i})}$$

Under constant wear the relation between the clamping force and the maximum pressure is given by

$$P_{Max} = \frac{F}{r_i\theta(r_o - r_i)}$$

The final temperature in the disc/ lining is given by

$$T_2 = T_1 + M\frac{R^2\omega^2}{8(2\rho_L V_L Cp_L + \rho_D V_D Cp_D)}$$

Derivations of objective function:

According to Frank Incropera the time required to cool the lining pad is:

$$t = \frac{\rho V C p}{h A s} \ln\left(\frac{T_2 - T_\infty}{T_1 - T_\infty}\right)$$

where: *p* and Cp are design parameter (determined by the materials of the disk and lining).

In this case, *p* is equal to 1161.4 and Cp is equal to 1.007 for the linings, and *p* is equal to 7854 and Cp is equal to 434 for the disks.

V and As are design variables, h is heat convection rate, which is derived as following:

The velocity after deceleration is assumed kept constant at 40km/h. Therefore:

v =40Km/h =11.11 m/s

u =V =11.11m/s

The environment temperature is:

T =27°C =*300K*

The kinematic viscosity of air in the environment condition and the heat conduction coefficient of the lining are:

$v = 30.84*10^6 m^2 / s$

k = 26.3 *10-3

The Reynolds number and Nusselt number of the lining are:

$$Re_L = \frac{u_\infty L}{\upsilon} = \frac{11.11 * L}{30.84 * 10^{-6}} = 3.6 * 10^5 * L$$

$$\overline{Nu_L} = 0.664\, Re_L^{\frac{1}{2}}\, Pr^{\frac{1}{3}} = 354.916 L^{\frac{1}{2}}$$

And those numbers of the disk are:

$$Re_D = \frac{u_\infty L}{\upsilon} = \frac{11.11 * L}{30.84 * 10^{-6}} = 3.6 * 10^5 * L_D$$

$$\overline{Nu_D} = 0.664\, Re_D^{\frac{1}{2}}\, Pr^{\frac{1}{3}} = 354.916 L_D^{\frac{1}{2}}$$

Where the Prandtl Number of the air is:

Pr =0.707

Therefore, the average heat conduction coefficient rate, h, can be shown as:

$$\bar{h} = \frac{\overline{Nu_L} * k}{L} = 9.334 * L^{\frac{-1}{2}}$$

$$\bar{h}_D = \frac{\overline{Nu_D} * k_D}{L_D} = 21472.2 * L_D^{\frac{-1}{2}}$$

Substitute into the function given by Incropera, then we get

$$t_L = \left(\frac{\rho V Cp L^{\frac{1}{2}}}{9.334 As} \ln\left(\frac{T_2 - T_\infty}{T_1 - T_\infty} \right) \right)$$

and

$$t_D = \left(\frac{\rho_D V_D Cp_D L_D^{\frac{1}{2}}}{21472.2 As_D} \ln\left(\frac{T_2 - T_\infty}{T_1 - T_\infty} \right) \right)$$

Constraints:

The time required for heat dissipation should be more than the time required to cool the disc and the lining

g1: $$\left\{ \frac{\rho_L V_L Cp_L L^{\frac{1}{2}}}{9.334 * As} \ln\left(\frac{T_2 - T_\infty}{T_1 - T_\infty} \right) \right\} - t_{Max} \leq 0$$

g2: $$\left\{ \frac{\rho_D V_D Cp_D L_D^{\frac{1}{2}}}{21472.2 * 2 * A_{SD}} \ln\left(\frac{T_2 - T_\infty}{T_1 - T_\infty} \right) \right\} - t_{Max} \leq 0$$

The total thickness of the disc and the two linings should be less than approximately an inch and a half.

$g3 = 2\ dL + d - 0.0381 \leq 0$

Inner radius should be greater than 0.

g4 = $-r_i \leq 0$

The outer radius should be greater than the inner radius

g5 = $r_o - r_i \leq 0$

The outer radius of the lining is limited by specification to:-

g6: $1.5r_o - R < 0$

Stress caused by clamping should not exceed the maximum allowable stress in the lining

g7: $P_{Max} - \frac{\sigma_{lining}}{SF} \leq 0$

Maximum temperature should not exceed the maximum allowable temperature for the lining.

g8: $T_2 - T_{Max} \leq 0$

The angle covered by the lining should be greater than zero

g9: $-\theta < 0$

From the point of view of geometric tolerances the angle covered by the lining should be greater than 90 degrees.

g10: $\theta - \pi/2 \leq 0$

According to James Halderman the lining should be replaced when the thickness reduces below 0.063 inches

g11: $-d + 1.6*10^{-3} + \frac{P_{Max}}{A_s} \times w \times N \leq 0$

Assuming that the diameter of the shaft is 95% of the inner diameter of the clutch disc (taking into account the keys) the shear stress in the disc and the shaft after 1000000 cycles should not exceed the fatigue stress.

Also the shear stress should not exceed the yield stress of steel which is the material used for the disc.

g12 = $\frac{\mu F}{\pi * 0.9 * 2 * d} - \frac{\sigma}{2s}$

g13 =16 $\frac{\tau}{\pi (0.9\, r_i)^3} - \frac{\sigma}{2s}$

The thickness of the lining and the disc should be greater than 0

g14 = -dL $\leq$ 0

g15 = -d $\leq$ 0

3.3 Summary Model

3.3.1 System Packaging

min $f = \pi d\rho\,(r_o^2 - r_i^2)(n + 1)$

subject to:

$g1 = (n + 1)(d + \delta) - L_{max} \leq 0$

$g2 = p_{rz} - p_{max} \leq 0$ where $p_{rz} = \frac{F}{\pi(r_o^2 - r_i^2)}$

$g3 = p_{rz.} V_{sr} - p_{max}. V_{sr\,max} \leq 0$ where $V_{sr} = \pi R_{sr} n/30$ & $R_{sr} = \frac{\frac{2}{3}(r_0^3 - r_i^3)}{r_o^2 - r_i^2}$

$g4 = V_{sr} - V_{sr\,max} \leq 0$

$g5 = \frac{I_Z\,\omega}{T_b + T_f} - t_{max} \leq 0$ where $T_b = \frac{2}{3}\mu\,F\,n\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}$ & $\omega = 2\pi N/60$

$g6 = F - F_{max} \leq 0$

$g7 = -t \leq 0$

$g8 = T_b - s\,T_s\,V_{sr} \leq 0$

$g9 = r_i - 0.8r_o \leq 0$

$g10 = \frac{F}{\pi E\,(r_o^2 - r_i^2)}$

3.3.2 Thermal Analysis

min $f = t_{max}$

subject to:

g1: $$\left\{\frac{\rho_L V_L Cp_L L^{\frac{1}{2}}}{9.334 * As}\ln\left(\frac{T_2 - T_\infty}{T_1 - T_\infty}\right)\right\} - t_{Max} \leq 0$$

g2: $$\left\{\frac{\rho_D V_D Cp_D L_D^{\frac{1}{2}}}{21472.2 * 2 * A_{SD}}\ln\left(\frac{T_2 - T_\infty}{T_1 - T_\infty}\right)\right\} - t_{Max} \leq 0$$

g3 = 2 dL + d -0.0381 ≤ 0

g4 = $-r_i \le 0$

g5 = $r_o - r_i \le 0$

g6: $1.5r_o - R < 0$

g7: $P_{Max} - \frac{\sigma_{lining}}{SF} \le 0$

g8: $T_2 - T_{Max} \le 0$

g9: $-\theta < 0$

g10: $\theta - \pi/2 \le 0$

g11: $-d + 1.6*10^{-3} + \frac{P_{Max}}{A_S} \times w \times N \le 0$

g12 = $\frac{\mu F}{\pi * 0.9 * 2 * d} - \frac{\sigma}{2s}$

g13 =16 $\frac{\tau}{\pi(0.9\, r_i)^3} - \frac{\sigma}{2s}$

g14 = -dL ≤ 0

g15 = -d ≤ 0

4 Model Analysis

4.1 System Packaging

	r_o	r_i	F	d
f	+	-		+
g1				+
g2	-	+	+	
g3	+	-		
g4	+	-		
g5	-	+	-	
g6	+	-	+	
g7				
g8	+	-	+	
g9	-	+		
g10	-	+		

Table 1: Monotonicity Table for System Packaging

The observation of the monotonicity table showed that one of the variables, thickness d can be eliminated by replacing constraint g1into the objective function. The new objective function becomes the following after substituting g1:

min $f = \pi d\rho\,(r_o^2 - r_i^2)(n + 1)$ where $d = L_{max}/(n + 1) - \delta$

Looking at the other three variables, one out of g2, g5 and g9 is active for ro; one out of g2, g5 and g9 is active for ri; Hence MP1 is satisfied. Also g2, g5, g6 and g8 are monotonic for F and g1 is monotonic for d. However this analysis does not give us much information about the variables. We can, however, find the tentative lower and upper bounds and the initial values for the optimization study, shown in the numerical results section.

4.2 Thermal Analysis

	t_{max}	r_o	r_i	θ	dL	d
f	+					
g1	-					-
g2	-		+		-	
g3			+		+	+
g4						
g5		-	-			
g6		+	+			
g7		-		-		
g8		-	+		-	-
g9				-		
g10				+		
g11		-	-	-	-	
g12		-	-			-
g13			-			
g14		+			-	
g15						-

Table 2: Monotonicity Table for Thermal Analysis

For the variable t_{max}, the constraints g1 and g2 are active constraints. The objective function contains only one variable t_{max} and hence one out of g1 and g2 is active. Hence MP1 is satisfied. From the table it can be seen that for g1, the variables ri and dL are monotonic and for g2 the variables ri and d are monotonic. On observing the table it is clear that for ri, the constraints g4, g11, g12, g13 (and perhaps g7 as well) are active. Also by MP2 the constraint g3 is active for dL and d. However all other variables in g1 and g2 are non-monotonic. Hence no further answers can be garnered from monotonicity analysis. Using g3 and g16 we can find the upper and lower bound for these non monotonic variables.

Hence we have decided to go through with optimization without simplifying the problem using monotonicity analysis.

5 Optimization Study

5.1 System Packaging

The system packaging subsystem optimization is successfully carried out using optimization software (LMS Optimus) linked with Matlab. The optimization model was based on an example done by former ODE research student Kuei-Yuan Chan in 2005. In this approach, the optimization process requires Matlab to act as a server for generating inputs and outputs while Optimus is runs the variable optimization process. Two of the three gradient-based algorithms i.e. Nonlinear Programming Quadratic Line Search (NLPQL) and Sequential Quadratic Programming (SQP) are experimented initially for difference in results, optimization run time and convergence. The final results for both methods are found to be similar but SQP was selected because of better rate of convergence. Since the constraints g8 and g10 are active, it is a boundary solution and hence a local result.

	ro (mm)	ri (mm)	F (N)	d (mm)	Mass (kg)
Ub	0.2	0.15	1500	0.008	10
Lb	0.17	0.001	50	0.003	0
Nominal	0.18	0.1	500	0.005	7.917
Optimal	0.179	0.112	490.873	0.003	4.141
change (%)	-0.56	12	-1.83	-40	-47.69

Table 3: System Packaging optimization results and its boundary & nominal values

5.2 Thermal Analysis

For optimizing the thermal aspect of the system we decided to optimize using the Fmincon function in Matlab and to verify the results using Optimus. The upper and lower bounds along with the initial values are given in the numerical results section.

The results are given below:

	ro	ri	theta	dL	d	tc_{max}
Ub	0.2	0.2	π/2	0.02	0.04	200
Lb	0.001	0.001	0.0001	0.001	0.001	0.0001
Nominal	0.19	0.04	0.03	0.01	0.015	200
Optimal	0.2	0.538	0.776	0.002	0.034	72.4085

Table 4: Thermal optimization results with nominal and optimal values.

Since the value for ro is the upper limit, this is a local minima that is reached due to manufacturing and space constraints. The algorithm for both Optimus and Matlab converged to the same value for different starting points. With the exception of some initial values where there were matrix errors or violations of mathematical rules, the code was fairly robust.

6 Parametric Study

6.1 System Packaging

In system packaging, the parameters –number of friction surfaces and coefficient of friction are explored for possible change in optimum. The number of friction surfaces has to be an even number with nominal value of N=8. The same optimization model was used with only N varied until the objective function reaches an optimum point. The compilation of results below showed that the mass decreases as the number of friction surfaces are raised until the optimum number of friction surfaces is 14. Beyond this optimum, the mass increases again because the outer radius (ro) has reached its lower bound of 0.17mm while the inner radius (ri) is constrained by g9. The end result at N=14 is a mass reduction of approximately 4.8% from 4.142kg at N=8.

N	ro (mm)	ri (mm)	F (N)	d (mm)	Mass (kg)	% change
4	0.200	0.038	1015.242	0.003	4.551	9.87
6	0.184	0.088	654.498	0.003	4.295	3.69
8	0.179	0.112	490.874	0.003	4.142	-
10	0.178	0.127	392.699	0.003	4.050	-2.22
12	0.176	0.134	327.249	0.003	3.988	-3.72
14	0.176	0.141	280.500	0.003	3.944	-4.78
16	0.170	0.136	245.444	0.003	4.168	0.63

Table 5: Parametric analysis for number of friction surfaces

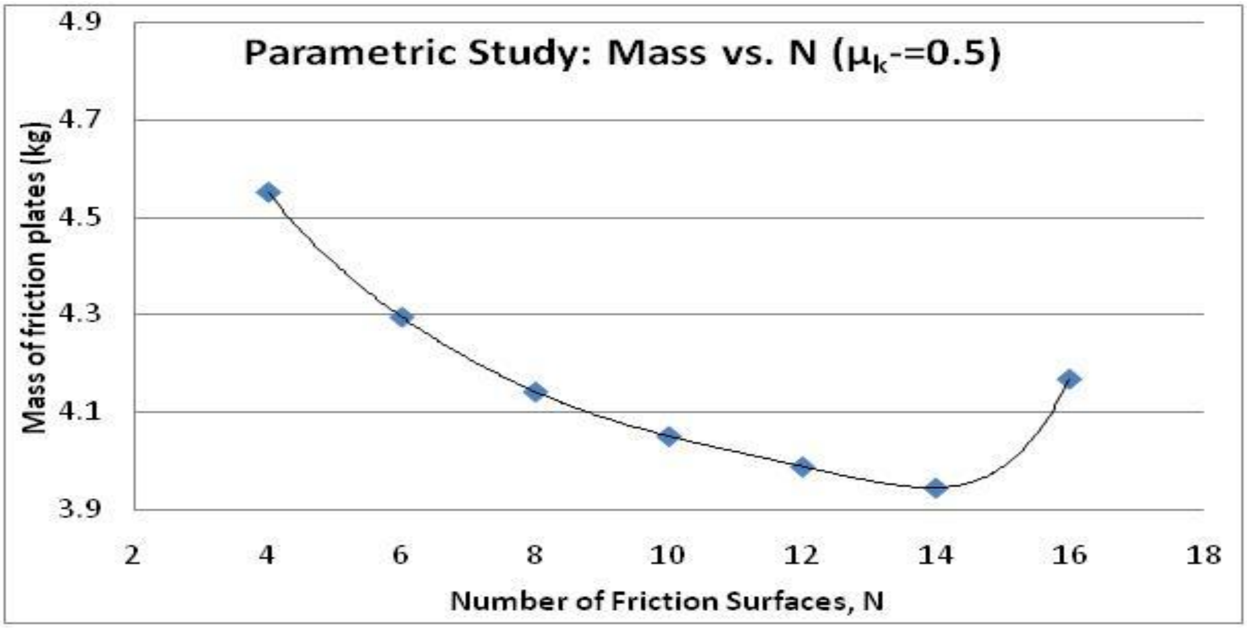

Figure 3: Mass of friction plates vs. number of friction surfaces

Using the optimum number of number surfaces, the coefficient friction (μ) parameter is varied keeping all other parameters constant. As μ is increased, the outer diameter decreases while inner diameter increases, causing the overall contact area to decrease hence reducing the mass. Again, the reduction stops when the outer diameter reaches its lower bound while ri is constrained by g9 at μ = 0.55. The mass is further reduced by 6.8% from 3.9kg to approximately 3.7kg.

μ	ro (mm)	ri (mm)	F (N)	d (mm)	Mass (kg)	Change (%)
0.40	0.178	0.133	350.624	0.003	4.931	25.0
0.45	0.177	0.137	311.666	0.003	4.383	11.1
0.50	0.176	0.141	280.500	0.003	3.944	-
0.55	0.170	0.136	255.004	0.003	3.677	-6.8
0.60	0.170	0.136	233.760	0.003	3.677	-6.8

Table 6: Parametric analysis for various coefficients of friction

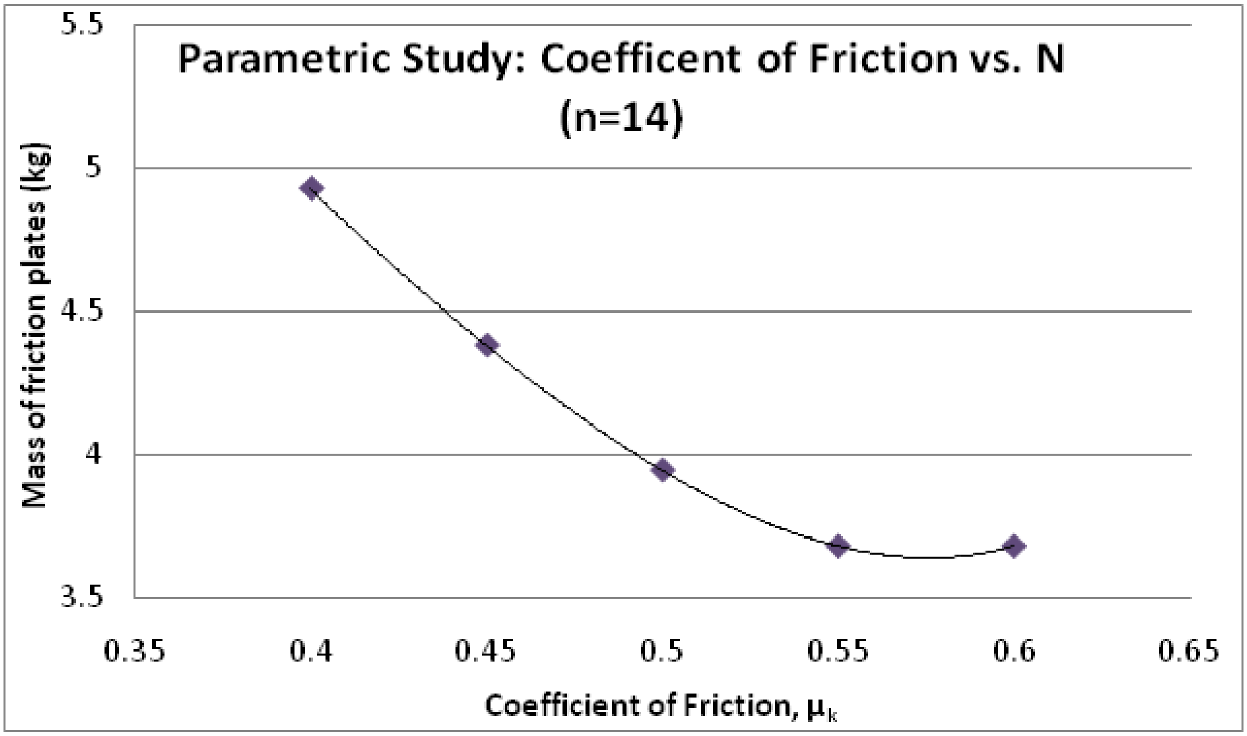

Figure 4: Mass of friction plates with various coefficients of friction

As explained earlier, the optimal value of the mass is reached at around 0.55 value of coefficient of friction.

6.2 Thermal Analysis

Parametric studies were also conducted for different vehicle weights, different vehicle speeds and different slip time. Temperature change was also considered.

The results are given below:

Variation of coefficient of friction with cooling time.

μ	ro	ri	theta	dL	d	f=tcmax
0.3	0.2	0.0597	1.0417	0.0021	0.034	80.88
0.35	0.2	0.0576	0.9647	0.00206	0.034	77.87
0.4	0.2	0.0564	0.9016	0.00204	0.034	75.35
0.45	0.2	0.0552	0.8499	0.00202	0.034	73.19
0.47	0.2	0.0539	0.778	0.002	0.034	72.403

Table 7: Variation of cooling time with coefficient of friction

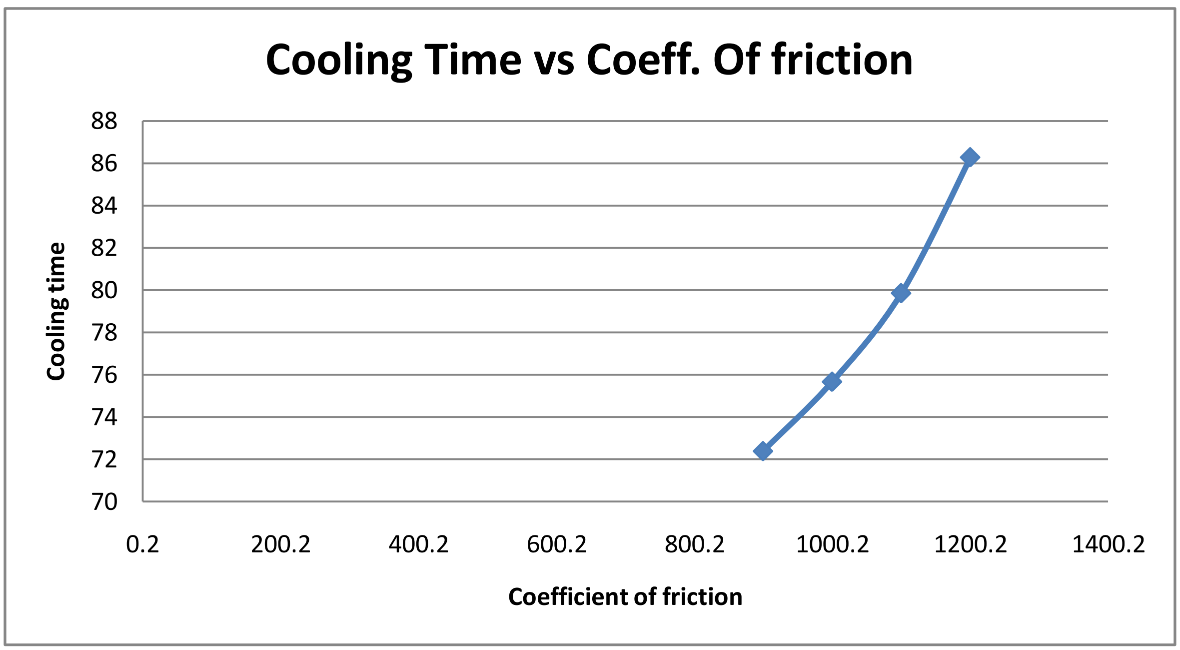

Figure 5: Variation of cooling time with coefficient of friction

As is seen from the table, the objective function decreases with increase in coefficient of friction. This is because the slip decreases as friction increases and hence the temperature doesn't go too high thus making it easier to cool.

Variation of mass of vehicle with cooling time.

Mass	ro	ri	theta	dL	d	F=tc$_{max}$
900	0.2	0.0538	0.778	0.002	0.34	72.4035
1000	0.2	0.0563	0.865	0.00205	0.34	75.68
1100	0.2	0.059	0.946	0.00209	0.34	79.864
1200	0.2	0.0612	1.035	0.00212	0.34	86.279

Table 8: Variation of cooling time with mass of the vehicle.

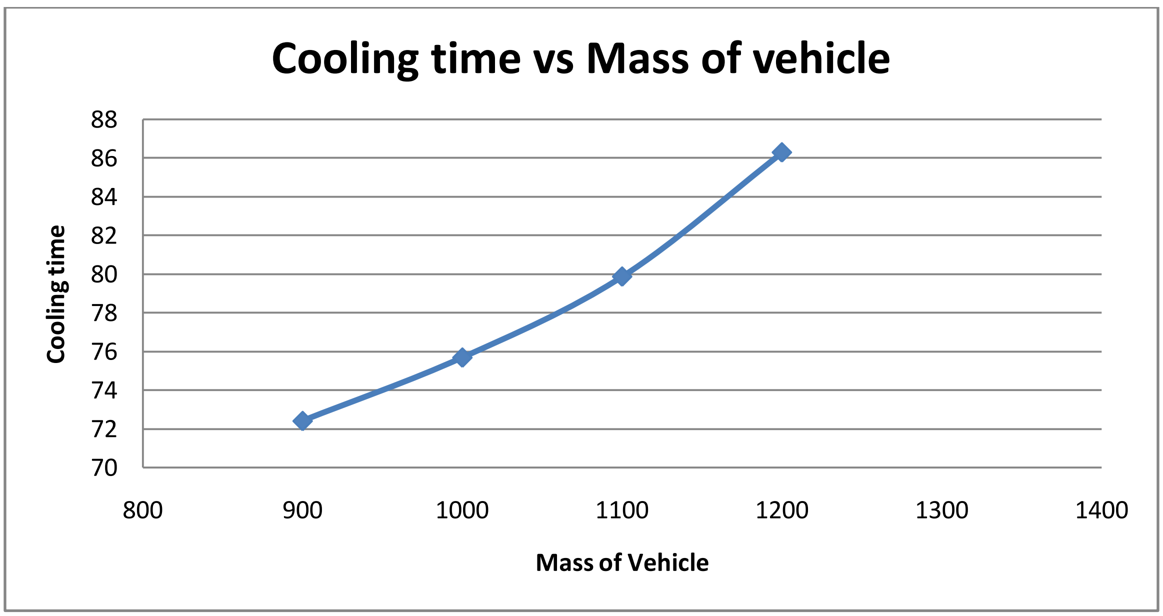

Figure 6: Variation of cooling time with vehicle mass

As is evident from the table given above, the cooling time of the clutch increases with mass of the vehicle. This is because the inertia increases thus increasing the slip time and heat generated. Thus the temperature increase is more and the cooling time increases.

7 Discussion of Results

In optimizing the two sub-systems of weight reduction and cooling time reduction we have made certain tradeoffs. Actuating force which is a vital variable in the weight analysis is just an intermediate result in the thermal analysis. The weight sub-system is optimized for a coefficient of friction equal to 0.6 while the thermal part for a value of 0.47. The engine speed values considered, the slip time, the materials considered, all were different in the two sub-systems. In simple words the two sub-systems have been optimized not considering the other sub-system.

The weight part arrives at an optimal solution of around 4 Kgs for an outer radius of 0.18m, inner radius 0.1m thickness of 3mm. However the constraints to this problem are such that the upper bound for the outer radius is 0.18 too. Hence this solution is a boundary optima and on closer inspection it can be seen that it is a local optima. While this solution is a good one considering the constraint imposed on the system it suffers from a drawback. Boundary optima are not robust solutions. Manufacturing defects and reasons beyond human control (environmental etc) can easily manufacture a part with dimension required dimension 0.19 m as more or less than the required value. Hence it is not advisable to have a value on the limit of the permissible region. On analyzing the results for various values of coefficient of friction and number of plates it can be seen that the weight decreases as coefficient of friction increases. The weight reaches an optimal value at 0.6 coefficient of friction. Also on observing the variation of weight with number of friction plates it is found that the weight is optimal for 14 friction plates.

In the thermal part an expression was first derived to find the cooling time for both the lining and the friction disk. Since it was not known which one the two would take longer to cool down the two cooling times were converted to constraints with tc_{max} as the variable. Hence this was a single objective minimization problem. The optimal solution in this case too was a boundary optima since the outer radius that the solution gave was a boundary value. As is the case with the weight part, this solution is hence is a local optimum defined by constraint of the material, the stress values and manufacturing limitations. On conducting a parametric study of the thermal part it is observed that the cooling time is optimal for a coefficient of friction equal to 0.47. Also on observing the variation of mass with the cooling time the cooling time increases with mass of the vehicle.

Linking these sub-systems is a multiobjective optimization since the thermal cooling time reduces as the surface area increases. This means increases dimensions. However, the weight part tries to reduce the dimensions since the lesser the dimensions, the lesser the weight.

Hence we will be using multi-objective optimization with weights to plot a pareto curve and select an optimal combination of values.

8 Integration

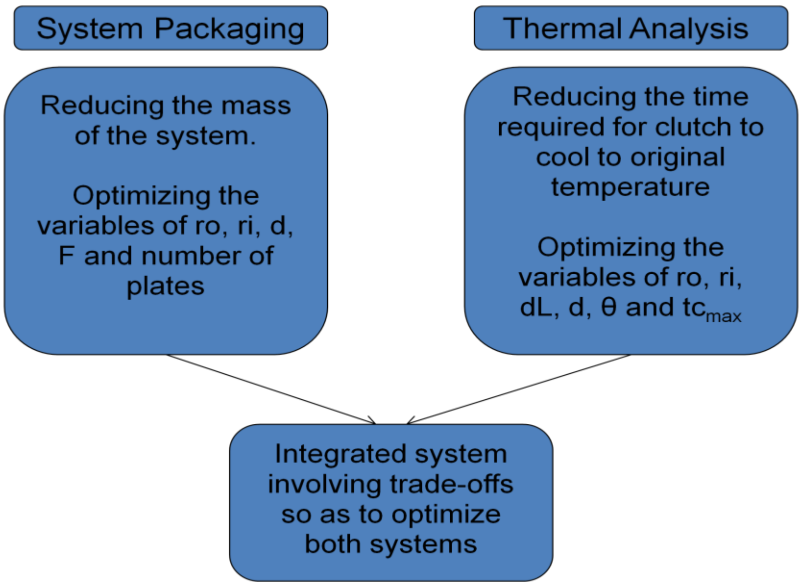

Figure 7: Schematic of system integration

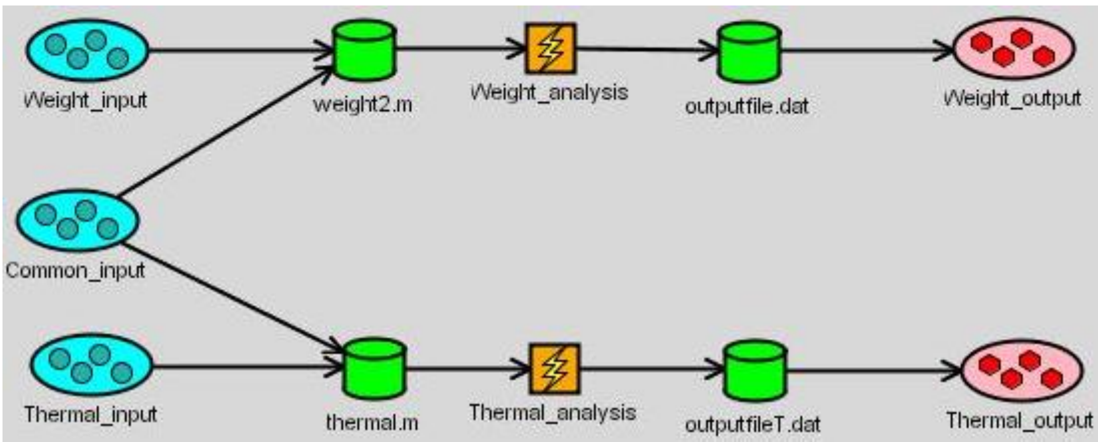

Figure 8: Integrated Optimization in Optimus

Onto the system integration. As explained before, the two sub-systems basically work against each other. One tries to increase the dimensions while the other tries to decrease them. We hence decided to use weights to solve this optimization problem. Two separate M files were prepared, one for the thermal part and one for the cooling time. In both, care was taken to maintain the same nomenclature so that this does not cause any issues. We used Optimus the optimize the problem. Common inputs were separately put in one input array while inputs particular to the individual systems were put in separate input arrays. The results of the optimization are given below.

	Initial	**W1**	**W2**	**W3**	**W4**	**W5**
F	**450**	**244.2766**	**254.0827**	**251.7776**	**253.0076**	**260.6076**
theta	**0.3**	**1.035266**	**1.051951**	**1.046599**	**1.042208**	**0.942663**
dL	**0.01**	**0.002112**	**0.002005**	**0.002002**	**0.001972**	**0.002195**
tmax	**200**	**200**	**168.9172**	**160.5531**	**145.107**	**122.5469**
router	**0.18**	**0.175398**	**0.182471**	**0.183153**	**0.18681**	**0.199903**
rinner	**0.1**	**0.085202**	**0.076349**	**0.073083**	**0.070544**	**0.032256**
thickness	**0.005**	**0.003**	**0.003677**	**0.004038**	**0.004742**	**0.005821**

Table 9: System integration results showing values of variables at various weights

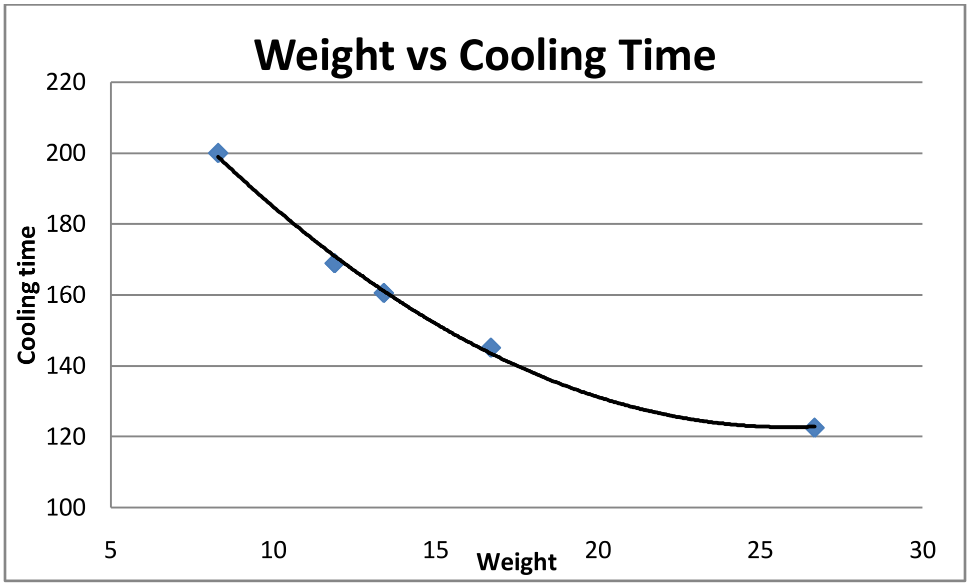

Figure 9: Pareto curve of Weight vs cooling time

	Initial	Step 1	Step 2	Step 3	Step 4	Step 5
Mass (Kg)	13.19469	8.307356	11.89722	13.41579	16.71454	26.69149
Weight1		1	0.75	0.5	0.25	0
Cooling time	200	200	168.9172	160.5531	145.107	122.5469
Weight2		0	0.25	0.5	0.75	1

Table 10: Optimal values for various combinations of weights.

It can be seen that the two functions form a pareto curve which defines the relationship between the two objectives. Since we have to make a choice between weight and mass we chose to give preference to mass since wear due to improper cooling takes place over and extended period of time and not immediately. **Hence we decided to select Weight as 11.9 Kg and cooling time as 170 seconds as optimum.** The values for variables are as follows:

Actuation force = 254N

Theta= 1 radian

Lining thickness = 0.002m

Disc thickness = 0.004m

Outer radius = 0.18m

Inner radius = 0.076m

This is not a boundary solution since none of the values are on the limit. Hence this is an interior optimum and can be considered to be fairly robust.

9 References

1. Stanislaw Krenich, Andrzej Osyczka, Optimal Design Of Multiple Clutch Brakes Using a Multistage Evolutionary Method, Cracow University of Technology, 2004.
2. David W. South, Jon R. Mancuso, Mechanical Power Transmission Components, Marcel Dekker, Inc., 1994, p.127-207
3. Joseph Edward Shigley, Charles R. Mischke, Mechanical Engineering Design, McGrawHill Inc., 1989, p.646-647.
4. Frank P. Incropera and David P. De Witt, Introduction to Heat Transfer, John Wiley and Sons, 1990

10 Appendices

10.1 Subsystem Matlab Codes

10.1.1 System Packaging

```
% Initialization
ro=0.18; %m outer - casing
ri=0.1; %m inner - shaft
F=500; %N - actuating force
d=0.005; %m - thickness of discs
rhoD=2500; %kg/m3
N=3000; %input speed
n=14; %friction surfaces
uk=0.6;
Pmax=1e6; %max allowable pressure
delta=0.0005; %distance between unloaded discs
Lmax=0.1; %max length of assembly
tmax=5; %max slipping time
Vsrmax=10; %max relative speed
Ts=75; %static input torque
%s=1.8; %factor of safety
Iz=55; %moment of inertia
Tf=10; %frictional resistance torque
Fmax=1500; %max clamping force
w=2*pi*N/60;
Tb=2*uk*F*n*(ro^3-ri^3)/(ro^2-ri^2)/3; %braking torque
t=Iz*w/(Tb+Tf); %slipping time
Prz=F/(3*(ro^2-ri^2)); %real pressure force/area
Rsr=2*(ro^3-ri^3)/(ro^2-ri^2)/3; %mean friction radius
Vsr=2*pi*N*Rsr/60; %relative speed of slip
deltar=0.02;
nmax=12;
E=8000;

% Calculations
f = pi*d*rho*(ro^2-ri^2)*(n+1);

g1=(n+1)*(d+delta)- Lmax;
g2=Prz-Pmax;
g3=Prz*Vsr-Pmax*Vsrmax;
g4=Vsr-Vsrmax;
g5=t-tmax;
g6=F-Fmax;
g7=-t;
g8=(Ts*Vsr)-Tb;
```

```
g9=ri-0.8*ro;
g10=(d*F/pi/E/(ro^2-ri^2))-d;

% Write values of objective function and constraint to a text file
fname = 'outputfile.dat';
fid = fopen(fname,'w');
fprintf(fid, 'f = %f\n', f);
fprintf(fid,'ro = %g\n',ro);
fprintf(fid,'ri = %g\n',ri);
fprintf(fid,'F = %g\n',F);
fprintf(fid,'d = %g\n\n',d);
fprintf(fid,'t = %g\n\n',t);
fprintf(fid, 'g1 = %f\n', g1);
fprintf(fid, 'g2 = %f\n', g2);
fprintf(fid, 'g3 = %f\n', g3);
fprintf(fid, 'g4 = %f\n', g4);
fprintf(fid, 'g5 = %f\n', g5);
fprintf(fid, 'g6 = %f\n', g6);
fprintf(fid, 'g7 = %f\n', g7);
fprintf(fid, 'g8 = %f\n', g8);
fprintf(fid, 'g9 = %f\n', g9);
fprintf(fid, 'g10 = %f\n', g10);

fclose(fid);
```

10.1.2 Thermal Analysis

```
% Initialization
ro=1.7000000000000e-001;
ri=1.5371441356488e-002;
theta=1.0015622324501e-002;
dL=1.9206476568660e-002;
d=3.4646063554312e-003;
tmax=9.8685800552336e+001;
R=0.3;
M=900;
V=30;
Vmax=80;
uk=0.6;
rhoD=2500;
rhoL=2100;
CpL=470;
CpD=440;
t=10;
s=2;
sigL=100e6;
sigD=320e6;
Tmax=550;
```

```
Tatm=300;
T1=310;
wc=1e-13;
N=3000;
L=(ri+ro)*theta/2;
phi=2*pi-theta;
Ld=((ri/2)+ro)*phi/2;
As=(ro^2-ri^2)*theta/2;
Asd=(ro^2-(ri/2)^2)*phi/2;
VL=(ro^2-ri^2)*theta*dL/2;
VD=pi*(ro^2-(ri/2)^2)*d;
w=V/R;
wmax=Vmax/R;
tau=M*R^2*w/(8*t^2);
F=2*tau/(uk*(ro+ri));
Pmax=F/(ri*theta*(ro-ri));
T2=T1+((M*R^2*w^2)/(8*((2*rhoL*VL*CpL)+(rhoD*VD*CpD))));

% Calculations
fT = tmax;

g1=(rhoL*VL*CpL*L^0.5*log((T2-Tatm)/(T1-Tatm))/(9.334*As))-tmax;
g2=(rhoD*VD*CpD*Ld^0.5*log((T2-Tatm)/(T1-Tatm))/(21472.2*2*Asd))-tmax;
g3=2*dL+d-38.1e-3;
g4=-ri;
g5=ri-ro;
g6=1.5*ro-R;
g7=Pmax-(sigL/s);
g8=T2-Tmax;
g9=-theta;
g10=theta-(pi/2);
g11=-dL+1.6e-3+(Pmax/As)*wc*N;
g12=(uk*F/(pi*0.95*ri*2*d))-(sigD/(2*s));
g13=(16*tau/(pi*ri^3))-(sigD/(2*s));
g14=-dL;
g15=-d;

% Write values of objective function and constraint to a text file
fname = 'outputfileT.dat';
fid = fopen(fname,'w');
fprintf(fid, 'fT = %f\n', fT);
fprintf(fid,'ro = %g\n',ro);
fprintf(fid,'ri = %g\n',ri);
fprintf(fid,'theta = %g\n',theta);
fprintf(fid,'dL = %g\n',dL);
fprintf(fid,'d = %g\n',d);
fprintf(fid,'tmax = %g\n\n',tmax);

fprintf(fid, 'g1 = %f\n', g1);
```

```
fprintf(fid, 'g2 = %f\n', g2);
fprintf(fid, 'g3 = %f\n', g3);
fprintf(fid, 'g4 = %f\n', g4);
fprintf(fid, 'g5 = %f\n', g5);
fprintf(fid, 'g6 = %f\n', g6);
fprintf(fid, 'g7 = %f\n', g7);
fprintf(fid, 'g8 = %f\n', g8);
fprintf(fid, 'g9 = %f\n', g9);
fprintf(fid, 'g10 = %f\n', g10);
fprintf(fid, 'g11 = %f\n', g11);
fprintf(fid, 'g12 = %f\n', g12);
fprintf(fid, 'g13 = %f\n', g13);
fprintf(fid, 'g14 = %f\n', g14);
fprintf(fid, 'g15= %f\n', g15);

fclose(fid);
```

10.2 Matlab Server Code

```
while(1)
file_not_found = 1;
while(file_not_found)
if exist('opt_matlab.stop')
delete('opt_matlab.stop');
return
end
if exist('opt_matlab.start')
delete('opt_matlab.start');
file_not_found=0;
else
pause(1);
end
end
% Provide your own matlab function, for example inputfile.m
weight2
clear weight2
fname='opt_matlab.done';
fid=fopen(fname,'w');
fclose(fid);
dir
thermal
clear thermal
fname='opt_matlab.done';
fid=fopen(fname,'w');
fclose(fid);
dir
end
```