Final Report

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ABSTRACT

Policymakers are increasingly acting on the need to reduce emissions of greenhouse gases but in the case of automobiles, these policies often lead to increased cost to producers and consumers, and lower consumer utility for available vehicles. Furthermore, the resulting fuel economy of vehicles in a market depends not only on government objectives but also on the producer objectives of maximizing profit. This study modeled an automotive market as a collection of producers choosing engineering variables and prices in order to maximize profit in the presence of policy scenarios. The first part of the study decomposes the problem into three separate perspectives: engineering design, the automobile market, and government. The engineering model found optimal shape variables and engineering design variables to maximize fuel economy of a subcompact vehicle. In the market model, engineering design variables were found that maximized profits for firms in the automotive market. The policy model optimized policy variables to maximize the market average fuel economy. The three perspectives were then combined to solve the market model with the updated engineering design under specific policy case studies.

1 INTRODUCTION

Many different decision makers influence the resulting fuel economy of vehicles in an automotive segment. These decision makers include producers who are interested in maximizing profit and policymakers who are interested in minimizing greenhouse gas emissions without creating adverse economic consequences. Both are restricted by engineering constraints on vehicle design. This study will optimize vehicle design from three perspectives of increasing scope. The first concentrates only on the engineering design, maximizing fuel economy while satisfying constraints of consumer comfort and design feasibility. The second broadens the scope to include the producer who sets engineering characteristics and price to maximize profit. The third includes policymakers who set regulations that encourage producers to increase fuel economy while limiting the increase in cost to producers and consumers, and limiting the decrease in consumer utility. We expect that the optimization of each independent subsystem will result in a different vehicle design than the overall system optimization.

Much of the work in this study will be based off of Michalek et al. [1] who implemented a producer model that determined market equilibrium given various policy scenarios. This model will be used in subsystem 3 to optimize policy variables (such as the amount of tax on gasoline). Subsystem 2 will use the framework of Michalek et al. but will modify the demand model that is used to find market equilibrium. The three subsystems will later be combined in the overall system optimization, solving for policy variables using the updated demand model from subsystem 2 and additional engineering characteristics modeled in subsystem 1.

2 OVERALL ANALYSIS

Several intuitive tradeoffs between the three subsystems can be identified. The engineering design will require minimum power consumption for the accessories, while the consumer model will impose an increase in this parameter. Moreover, engineering design and producer model will impose different values on the scaling parameter of the engine.

3 NOMENCLATURE

B, C	Observed and unobserved demographic interaction coefficients
$D_{\_rear}$	Distance of closest rear visibility point on the ground (m) (5m)
$D_{\_front}$	Distance of closest front visibility point on the ground (m) (4m)
E_{TM}	Engine torque map (N/m)
E_{SM}	Engine speed map (rad/s)
E_{FM}	Engine fuel map
$H_{\_cabin}$	Vehicle cabin height (m)
$H_{\_front}$	Vehicle front height (m)
$H_{\_ground}$	Vehicle ground clearance (m) (0.331m)
$H_{\_person}$	Height driver (m) (1.94m)
$H_{\_rear}$	Vehicle trunk height (m)
$H_{\_windshield}$	Vehicle windshield height (m)
J	Set of all vehicle designs produced
J_k	Set of all vehicle designs produced by producer k
L	Demographic variables in the demand model
$L_{ji}(\beta)$	Logit Probability Function
$L_{\_front}$	Vehicle front length (m)
$L_{\_max}$	Maximum length of the vehicle (m) (5.5m)
$L_{\_rear}$	Vehicle rear length (m)
$L_{\_wheelbase}$	Vehicle wheel base (m) (2.045m)
P_{ACC}	Power requirement for accessories (W)
P_j	Probability that design j will have higher utility than all other designs
$W_{\_total}$	Vehicle width (m)
X_h, Y_h	Driver's hip point location (m)
X_e, Y_e	Driver's eyes point location (m)
Z	Vehicle characteristics in the demand model
a	Constants from the BLP model.
b_M	Base size of engine type M
c^B	Base manufacturing cost per vehicle (without engine)
c^I	Investment cost
c_C	Additional cost to consumer resulting from policy

c_C^{UB}	Allowable upper limit for additional cost to consumers
$c^E{}_j$	Engine manufacturing cost for design j
c_k	Total cost for producer k
c_{Pj}	Total production cost for design j
c_{Rk}	Total regulation cost for producer k
$c_{Rk}{}^{UB}$	Allowable upper limit for regulation cost per vehicle to a producer
c_{Vj}	Variable manufacturing cost per vehicle for design j
f^E	Market equilibrium model
f_M	ADVISOR simulation for engine type M
j	Vehicle design index
k	Producer index
p_{CAFE}	Penalty for regulation violation
p_d	Price of diesel including tax
$p_d{}^B$	Base price of diesel not including tax
p_f	Price of fuel including taxes
p_g	Price of gasoline including tax
$p_g{}^B$	Base price of gasoline not including tax
p_j	Selling price of design j
q_j	Demand for design j
s	Size of car buying market
$\mathbf{t}$	Fuel tax vector $(t_g, t_d)^T$
t_d	Tax on diesel in cents per gallon
t_g	Tax on gasoline in cents per gallon
v_j	Observable component of utility for design j
v_{LB}	Allowable lower limit for observable consumer utility
veh_{FA}	Vehicle frontal area (m^2)
veh_{CD}	Vehicle coefficient of drag
$\mathbf{x}$	Design variable vector
x_1	Engine scaling parameter
x_{1j}	Engine scaling parameter for design j
x_{2j}	Final drive ratio for design j
$\mathbf{z}$	Product characteristics vector

z_1	Fuel economy (mpg)
z_{1j}	Fuel economy of design j
z_{2j}	Acceleration time (0-60 mph) of design j
z_{CAFE}	CAFE fuel economy requirement
Π_k	Total profit for producer k
α	Vehicle frontal angle to the windshield (rad)
β	Demand model coefficient parameter
δ	Cost model parameter
ϕ_1	Upward visibility angle (rad)
ϕ_2	Downward visibility angle (rad)
θ	Vehicle back angle (rad)

4 SUBSYSTEM 1: ENGINEERING DESIGN

by Monica Toma

4.1 Problem statement

As the increase in fuel efficiency is a constant demand for the automotive industry, the actors of the vehicle market pay a particular attention to the minimization of fuel consumption. The first level at which a considerable difference can be made is the engineering design. The choice of the shape, drive train type and configuration of the vehicle will directly affect the future fuel consumption of the vehicle. As the design parameters play also the key part in the performance and in the comfort of the passengers, a first optimization process is required to identify the values of these design parameters which will provide the optimal fuel consumption, while meeting the performance and quality requirements.

Under these considerations, the present subsystem will optimize the fuel economy of a conventional vehicle subjected to comfort of the passengers, performance and design constraints. The engineering point of view has been chosen to approach this problem, as it will be detailed in the following sections. The environment that will be used to model the vehicle is Advisor. The program includes several vehicle models and is able to compute the fuel economy as well as the performance of the vehicle in terms of acceleration time, gas emissions etc. The optimization process will be run over a classical highway-city driving cycle based on the Federal Test Procedure (FTP-75) drive cycle used by the Environmental Protection Agency (EPA) to certify the fuel economy and emissions performance of consumer vehicles.

There are many vehicle parameters that affect fuel economy but for simplification reasons the present problem will take into account only the variables related to the shape of the vehicle and the engine size. In a future version the weight of the vehicle could also be included. The case study chosen for the optimization problem is the typical passenger car and the model used was Ford Focus.

In order to proceed to the problem optimization several assumptions have been made. To simplify the problem the internal configuration of the vehicle studied is as follows: the drive train configuration is conventional, and the constitutive components are: fuel converter, clutch, gearbox, final drive and wheels. The optimization problem will only take into account total power of the engine, the other component's characteristic being only parameters of the design. The results from Hamza et al. [10] on the interior design for optimal passenger comfort have been included as assumptions. These assumptions refer mainly to the position of the passengers inside the car compartment, as it will be described in detail in the next sections.

A schematic representation of the shape of the vehicle considered, as well as the design parameters taken into account are presented in Figure 4-1.

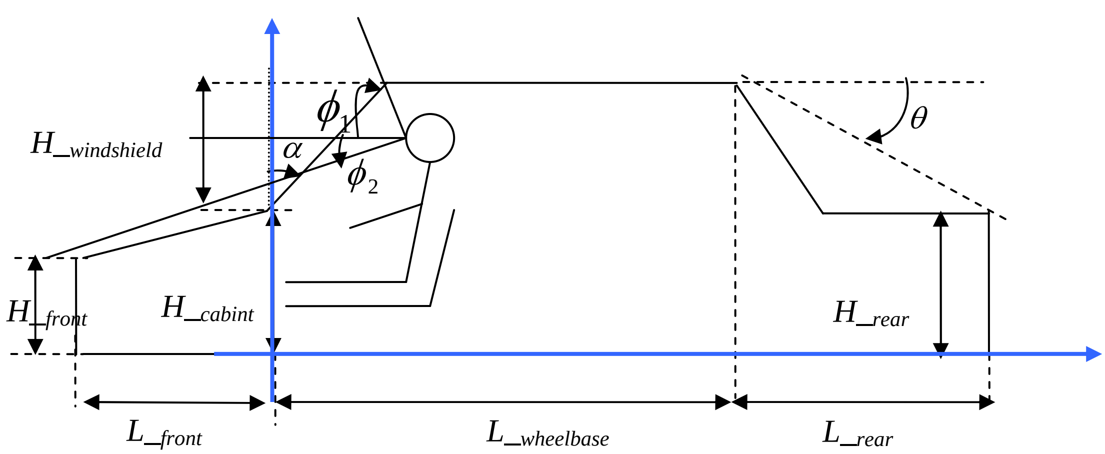


Figure 4-1 - Vehicle Design Variables

The subsystem constraints have been divided in three categories: performance constraints, comfort constraints and design constraints. The vehicle performance has to meet several constraints such as the acceleration time, maximum speed attainable, cruising grade, maximum acceleration etc. The comfort constraints are those imposed by the consumer's need, as angle of visibility, head clearance from the roof, but also the power requirement for the accessories. An important aspect is the platform in which the producer company includes the vehicle to be designed, and which imposes constraints on the maximum engine power, size of vehicle etc.

The analysis of the problem reveals several trades-offs that motivate the optimization problem. First of all the design variables will have different impacts on the coefficient of drag of the vehicle. Moreover, a high windshield will increase the frontal area of the car and by this decrease the aerodynamic performances of the vehicle. However the driving process requires a certain frontal and rear visibility. In the same time these parameter will influence the comfort of the passengers. The "quality" of the vehicle will require large dimensions as well as a large number of accessories. These requirements will have a strong impact on the coefficient of drag of the vehicle and, on the second hand, will increase the power drain from the engine. The consequence of this is the increasing in vehicle's fuel consumption.

Substantial work has been lately performed in the area of the hybrid powertrain optimization and is less concerned with the design aspect of the vehicle. Nevertheless, the scaling parameter of the engine appears constantly as an optimization parameter, for example in M. Kokkolaras et al. (2004) and Michalek et al. Moreover, Assanis et al. (1999) developed an optimization approach on a hybrid system, using ADVISOR. Chan, Frostick, King, and Zhang in their ME589 project, performed a statistical view of how the variation of vehicle in-use operation parameters will change the overall fuel consumption and various gas emissions. They concluded the engine size and the coefficient of drag are among the parameters which have the strongest impact on fuel consumption. Important considerations on design parameters that influence the aerodynamic behavior of the road vehicles are made in Bosch, 1996 [2] as well as in Wong, 1993 [3] and Hucho, 1998 [9]. Hamza et al. [10] analyzed the optimal design for interior comfort of the passengers. These papers together with the documentation of ADVISOR were used as resources in developing this design optimization problem.

4.2 Mathematical Model

Objective function

The purpose of the engineering design optimization is the maximization of the fuel economy of the vehicle. The fuel economy is calculated based on the ADVISOR model, which provides the combined fuel economy as follows:

$$z_1 = f_1(\underline{x}_{D1}, \underline{x}_{P1}) = \left[\frac{1}{0.55/mpg_{urban}} + \frac{1}{0.45/mpg_{highway}} \right], \quad (4.1)$$

where f_1 is a function of design variables and parameters which describe each component of the vehicle.

The fuel economy is produced by complex computer simulation, and an explicit description of the mathematical model goes beyond the requirements of the present project. However, for our optimization, several changes were made to the modeling environment. More precisely, ADVISOR, in its present form, does not capture in detail the influence of design on the fuel economy. Variables as: α, θ, $H\_{windshield}$, L_{front}, $H\_{curvature}$ etc. are not included in the model and neither is their effect on the performance of the vehicle. As discussed in Wong, 1993 [3] and in Hucho, 1998 [9], the design variables play a key part in the coefficient of drag which is one of the main causes for increase fuel consumption. The effect of these variables on the design will be included in the model by using curves that describe the contribution of each variable to the coefficient of drag. In a future iteration, an evaluation of the glider mass as a function of the design variables will also be included.

The negative null form of the objective will thus be:

$$f(\underline{x}_{D1}, \underline{x}_{P1}) = -z_1 \quad (4.2)$$

where, as seen above, z_1 is the fuel economy computed by ADVISOR.

Constraints

As described in the problem statement, the constraints of the optimization problem have been divided into three categories: performance constraints, quality constraints and design constraints. The physical constraints of the present subsystem include both design constraints and quality constraints. The so called “quality constraints” impose limits on the design variables as a function of consumer’s comfort. The practical constraints of the optimization problem are the performance constraints. A baseline design will be selected as lower limit on the performance, expressed in terms of: acceleration time to reach a 60mph.

g1: $z_2 - z_{2BASE} \leq 0$

where $z_{2BASE} = 13.5s$ and the performance z_2 is computed using ADVISOR. A parametric study could be performed based on different values of the limit z_{2BASE}.

g2: Upward visibility.

$\phi_{1\min} - \phi_1 \leq 0$

where $\phi_1 = \arctan\left(\dfrac{H_{cabin} + H_{windshield} - Y_e}{Y_e - H_{windshield} \cdot \tan(\alpha)}\right)$, $\phi_{1\min} = 12°$ *(4.3)*

g3: Downward visibility.

$\phi_{2\min} - \phi_2 \leq 0$

where $\phi_2 = \min\left\{\arctan\left(\dfrac{Y_e - H_{cabin}}{X_e}\right), \arctan\left(\dfrac{Y_e - H_{front}}{X_e + L_{front}}\right)\right\}$, $\phi_{2\min} = 10°$ *(4.4)*

g4: Driver's and passenger head clearance

$0.47 \cdot H_{person} + Y_h - H_{cabin} - H_{windshield} \leq 0$

g5: Closest visible point on the ground

$\dfrac{\left(H_{ground} + Y_e\right)}{\tan(\phi_2)} - X_e - L_{front} - D_{front} \leq 0$, $D_{front} = 4m$

g6: Crash worthiness

$L_{front} + L_{wheelbase} - 3.6 \le 0$

g7: Positive of angle of attack

$H_{front} - H_{cabin} \le 0$

g8: Maximum allowed length

$L_{front} + L_{wheelbase} + L_{rear} - L_{\max} \le 0$, $L_{\max} = 5.5m$

g9: Windshield allowable length

$$\frac{H_{windshield}}{\tan(\alpha)} - L_{wheelbase} \le 0$$

g10: Backward visibility

$$H_{rear} \frac{L_{wheelbase} + L_{rear} - X_e}{Y_e - H_{rear}} - D_{rear} \le 0,\ D_{rear} = 5m$$

where $H_{rear} = H_{cabin} + H_{windshield} - L_{rear} \cdot \tan(\theta)$

g11: Overall design of the car

$L_{rear} \le 0.36 \cdot (L_{front} + L_{wheelbase})$

h1:

veh_CD = f(L_front ,H_front ,H_cabin ,H_windshield ,L_wheelbase ,H_curvature ,L_rear ,H_rear α θ)

h2: *veh_FA = W_total *(H_cabin + H_windshield + H_curvature)*

Design variables and parameters

The optimization problem has ten variable: $\underline{x}_{D1}$, the list of variables being presented in Table 4-1. The power requirement of the accessories was kept as a fixed parameter (P_{ACC} = 700W) for the purposes of this study, but it will be included as a variable in the overall system. The number of degrees of freedom is eight as *veh_CD* and *veh_FA* are included in a the equality constraints h(1) and h(2).

The optimization study performed by Hamza et al. [10] has been used to establish the parameters describing the comfort of the passengers. In the above mentioned paper, the following values have been found: $X_h = 0.811m$, $Y_h = 0.415m$. Consequently, the driver's eyes coordinates are:

$X_e = X_h + 0.406*H\_{person} \sin(20°);$

$Y_e = Y_h + 0.406*H\_{person} \cos(20°);$

under the assumption that the angle of the driver's seat is of 110° and the height of the driver is of *1.95m*.

Table 4-1 - Variables subsystem 1

veh_CD	Vehicle coefficient of drag
veh_FA	Vehicle frontal area (m^2)
H_{front}	Vehicle front height (m)
H_{cabin}	Vehicle cabin height (m)
$H\_{windshield}$	Vehicle windshield height (m)
L_{rear}	Vehicle rear length (m)
L_{front}	Vehicle front length (m)
α	Vehicle frontal angle to the windshield (rad)
θ	Vehicle back angle (rad)
x_1	Engine scaling parameter

The optimization problem uses a highly complicated simulation that requires large computations. For this reason, we have chosen transform the initial model by eliminating the two variables that are computed by the equality constraints. The other variables have been bounded as it follows:

	H_{front}	H_{cabin}	$H\_{windshield}$	L_{front}	L_{rear}	α	θ	x_1
LB	0	0	0	0	0	0	0	0.75
UB	3	3	3	3	3	1.58709	1.58709	2.5

With this in mind, a feasible solution has been found at:

H_{front}	H_{cabin}	$H\_{windshield}$	L_{front}	L_{rear}	α	θ	x_1
0.5738	0.8585	0.473	1.5555	1.2204	1.076	0.4449	1.442

and *veh_CD* = 0.3307, *veh_FA* = 2,2626. The value of the constraints and of the objective function is:

f	***g1***	***g2***	***g3***	***g4***	***g5***	***g6***	***g7***	***g8***	***g9***	***g10***	***G11***
36.87	-0.3504	-0.4899	-0.0438	0.000	-0.7041	0.000	-0.2847	-0.6791	-1.7898	-1.0008	-0.0758

Summary model

$$\min_{\underline{x}_{D1}} f_1(\underline{x}_{D1}, \underline{x}_{P1}) = -\left[\frac{1}{0.55/mpg_{urban}} + \frac{1}{0.45/mpg_{highway}}\right]$$

subject to: $\underline{g}_d(\underline{x}_{D1}, \underline{x}_{P1}) \le 0$ Design constraints

$\underline{g}_q(\underline{x}_{D1}, \underline{x}_{P1}) \le 0$ Quality constraints

$\underline{g}_p(\underline{x}_{D1}, \underline{x}_{P1}) \le 0$ Performance constraints

where the constraints and the variables have been detailed previously.

The current model evolved from a more complex idea such that concrete results could be obtained. The design of the vehicle is a highly complex problem and many of the parameters that describe the shape of the vehicle had to be ignored. A more detailed description of changes that were made is included in Appendix A

4.3 Model Analysis

As the objective is provided by a highly complicated simulation, a monotonic analysis cannot be performed. However, several intuitive dependences can be developed. As discussed in Wong, [3] the fuel economy decreases with respect to an increase in frontal area, coefficient of drag and power requirement of the accessories. In order to reduce the fuel consumption, scaling of the engine is necessary. We can thus deduct that the optimization process will try to minimize both the coefficient of drag and the frontal area while finding an engine size that will provide the required performance.

The monotonicity table for the problem is presented in Table 4-2. The empirical form of the coefficient of drag and its nonlinear dependence on the design variables, do not permit us to deduct other monotonic dependences or to conclude on the activity of the constraints.

Table 4-2 - Monotonicity table

	H_front	H_cabin	$H\_windshield$	L_front	L_rear	α	θ	x_1
f	U	U	U	U	U	U	U	U
g 1	U	U	U	U	U	U	U	U
g 2		-	U			-		
g 3	$+^*$	$+^*$		$+^*$				
g 4		-	-					
g 5	$-^*$	$-^*$		U^*				
g 6				+				
g 7	+	-						
g 8				+	+			
g 9			+			-		
g 10		+	+		U		-	
g 11				-	+			

4.4 Optimization Study

The model has been implemented in MATLAB and the following optimization results were obtained using *fmincon*.

MATLAB results

Due to large amount of computation required by ADVISOR, only four iterations of the *fmincon* function can be computed in one run. As a consequence, after these four iterations the current point was saved, as well as the configuration of the vehicle, and the next computation had as a starting point, the final result from the previous one. For this reason, multiple starting point optimizations could not be performed. The optimum point for the value of the parameter P_{ACC} = 700W is presented in Table 4-3.

Table 4-3 Optimum point

H_front	H_cabin	$H\_windshield$	L_front	L_rear	α	θ	x_1

M	m	M	m	m	rad	rad	
0.5631	0.8563	0.4752	1.555	1.296	0.7152	0.3902	1.4

The value of the constraints and of the objective at the termination point is included in Table 4-4. As mentioned before, the equality constraints were used to transform the model, with the purpose of reducing the number of variables. In the analysis we will thus be concerned only with the inequality constraints.

The analysis of the value for the constraints at the optimum shows that the constraints g_1, g_4, g_6, g_{10} and g_{11} are active at the optimum, while the corresponding Lagrange multipliers are positive. The Lagrange multipliers associated with the unactive constraints are zero, which proves that, the transversability conditions: $\mu_i g_i = 0$ at the optimum is satisfied. Moreover, all the constraints are satisfied, which means feasibility is also satisfied and the Lagrange multipliers are positive. From the nature of the SQP algorithm, the stationarity condition is always satisfied at the computed optimum, we can thus conclude that the optimum is a KKT point. This does not guarantee, however that the point is a minimum.

Table 4-4 Value of the objective and constraints at the optimum

f	***g1***	***g2***	***g3***	***g4***	***g5***	***g6***	***g7***	***g8***	***g9***	***g10***	***g11***
-37.41	0.00	-0.0430	-0.0477	0.00	-0.8113	0.00	-0.2932	-0.6040	-1.4979	0.00	0.00

Table 4-5 Lagrange multipliers computed at the optimum

g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1.2289	0	0	6.4706	0	0.1457	0	0	0	0.0673	0.4046

The Matlab *fmincon* function allows us to compute the gradient of the objective as well as the Hessian matrix of the objective at the optimum point. Using the Hessian we can compute the determinant of the principal minors for which we obtain: H_1 = 37.1864, H_2 = 215.1175, H_3 = 393.0487, H_4 = 393.0487, H_5 = 393.0487, H_6 = 393.0487, H_7 = 410.3029, H_8 = 0.1767, H_9 = 0.1767. The analysis of the Hessian reveals that it is a positive-definite matrix, and thus **the KKT point is a minimum**.

Table 4-6 - Gradient of the objective at the optimum

$\frac{df}{dH\_front}$	$\frac{df}{dH\_cabin}$	$\frac{df}{dH\_windshield}$	$\frac{df}{dL\_front}$	$\frac{df}{dL\_rear}$	$\frac{df}{d\alpha}$	$\frac{df}{d\theta}$	$\frac{df}{dx_1}$
0	3.8157	3.8157	1.6384	1.6776	0	0	13.7078

Table 4-7 - Hessian matrix of the objective at the optimum

37.1864	80.2441	80.2441	0	0	0	31.3589	7.9315	17.9543
80.2441	178.9427	177.9427	0	0	0	69.5417	17.5706	39.8139
80.2441	177.9427	178.9427	0	0	0	69.5417	17.5706	39.8139
0	0	0	1.0000	0	0	0	0	0
0	0	0	0	1.0000	0	0	0	0
0	0	0	0	0	1.0000	0	0	0
31.3589	69.5417	69.5417	0	0	0	28.1521	7.0242	15.5596
7.9315	17.5706	17.5706	0	0	0	7.0242	1.7605	3.9313
17.9543	39.8139	39.8139	0	0	0	15.5596	3.9313	9.9082

4.5 Parametric Study

The parametric study was motivated by the activity of constraint g_1, the only one that includes the variable associated with the size of the engine: the engine scaling parameter. The upper bound for the acceleration time was lowered from 13.5s to 12s, and the results of the optimization process are included in Table 4-8.

Table 4-8 - Optimum point for more improved acceleration time

H_{front}	H_{cabin}	$H\_{windshield}$	L_{front}	L_{rear}	α	θ	x_1
M	m	M	m	m	rad	rad	
0.5631	0.8563	0.4752	1.555	1.296	0.7152	0.3902	1.588

The value for the objective function obtained in the new conditions is fuel economy = 34.7657. The comparison of this result with the optimum of the initial problem shows that the constraint whose bound was reduced is critical in determining the value for the scaling of the engine. As it was suspected, the optimized shape is rather decoupled from the optimized engine size. The objective is, however, diminished due precisely to the larger engine size.

A two step optimization might be performed for the current model. The first will optimize fuel economy with respect to the shape parameters, and in a second step, shape parameters will be held fixed and the optimization variable will become the size of the engine. However, we strongly believe that this behavior could be eliminated of variables that couple the design parameters of the vehicle, and the model used in ADVISOR would be included in the optimization process. As mentioned earlier, one of these variables is the weight.

4.6 Discussion of Results

The starting point for the optimization problem was considered to be the current Ford Focus model, to which the above mentioned simplified representation has been associated. The starting point corresponds to the design parameters presented in Table 4-9 and the corresponding values for the constraints are included in Table 4-10. It can be easily seen that the starting point is not a feasible point.

Table 4-9 - Starting point for the design optimization

H_{front}	H_{cabin}	$H\_{windshield}$	L_{front}	L_{rear}	α	θ	x_1
0.586	0.771	0.441	1.0361	1.045	1.0268	0.298	1

Table 4-10 - Contraints avlues at starting point

g1	***g2***	***G3***	***g4***	***g5***	***g6***	***g7***	***g8***	***g9***	***g10***	***g11***
??	0.0602	-0.0897	0.1195	-1.2518	-0.5189	-0.1850	-1.3739	-1.7783	1.6788	-0.0642

In Figure 4-2 we have represented, at the same scale, the initial point and the comparison between the initial model and the optimized model.

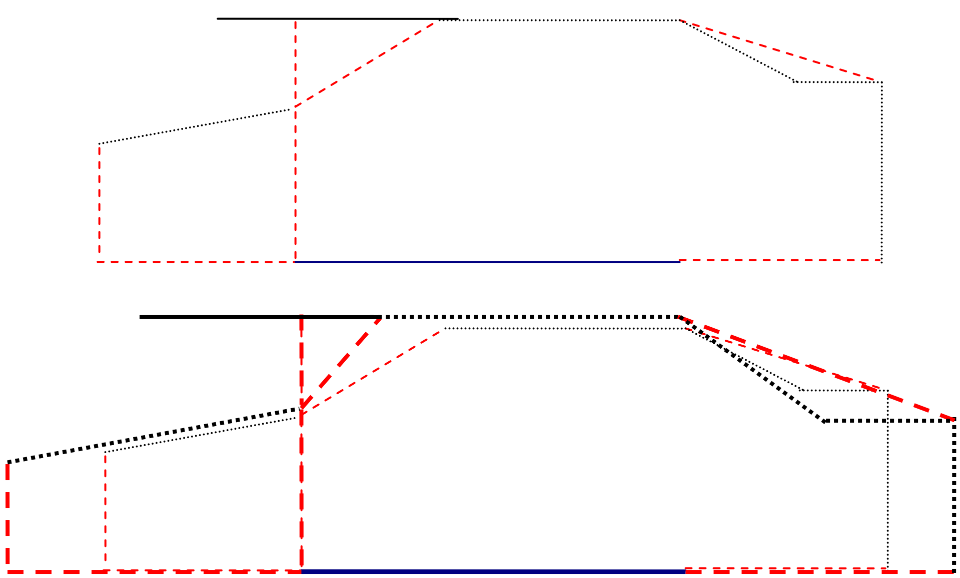


Figure 4-2 Design evolution

A first analysis of the two models points out the optimal model is larger than the initial one, the reason for this evolution could partially be explained through the activity of the constraints. As mentioned in the above section, at optimum, the active constraints are g_1, g_4, g_6, g_{10} and g_{11}. The constraint g_1 refers mostly to the performance of the vehicle, and its activity will be explained in a future paragraph. From Table 4-10 we see that the constraint on the backward visibility angle g_{10} is strongly violated, and we see that at optimum this constraint becomes active. We can thus conclude that this constraint is a rather restrictive on the design. Because the constraint is not monotonic with respect to all variables, a more in depth analysis of its activity is difficult to make.

The activity of the constraint g_4 is easier to interpret. As we can see, at the starting point, this constraint is violated. From the monotonicity analysis, in order to establish feasibility, both variables H_{cabin} and $H\_{windshield}$ have to increase. However, this increase will imply a larger FA, and thus an increase in fuel consumption. As a consequence, to limit the negative influence, their values will be limited by the activity of the constraint. This analysis couldn't be made a priory, because the part the two variables play in the coefficient of drag is unknown. The negative contribution of these variables on the CD could, however, be compensated by the variation of other design variables.

The activity of the constraints g_6 and g_{11} is partially related to the compensation above mentioned, but also with the simplified model that was used. Both constraints impose limits on the variables describing the length of the vehicle. In fact, the first objection that could be added to the optimum design, presented in **Error! Reference source not found.**, is precisely the length. It is important to note that the weight of the vehicle was note included in the optimization process and neither was the impact of the size of the design on this weight. It is well known that, the weight is a driving parameter in vehicle design, the general the automotive constructors trying to minimize it at maximum. We believe that the insertion of the weight into the model would have changed the activity of these two constraints up to moving the optimum from the boundary of the feasible domain, to an interior point.

In what concerns the other design variables, as mentioned in Hucho, [9] the angle of the windshield does not play a key part in the coefficient of drag. In fact, the coefficient of drag will stay the same beyond a certain value of this angle which explains the change in this angle. The other parameters are, unfortunately, coupled in more complex relations, and it is difficult to establish this kind of statements.

Another observation that should be made is that the model presents a decoupling between the variables describing the shape of the vehicle, and the engine's size. More precisely, because the ADVISOR model does not include any of the design variables used in the present model, the coupling between the modeling of the vehicle and ADVIOSR was established using the coefficient of drag (CD) and the frontal area (FA) of the vehicle. We are aware that the same CD and FA could be found for different values of the design parameters and the optimum engine size if then found for the computed CD and FA. However, the discussion on the analysis of the optimization study proved that the optimum found is also a minimum.

The coupling could be strengthened if additional existing ADVISOR parameters, exp glider_mass, will be expressed in terms of the design parameters. The activity of constraint g_1 shows that, in order to obtain the maximum fuel economy, while meeting the performance constraints, the size of the engine has to be as small as possible.

5 SUBSYSTEM 2: MARKET MODEL

by Esra Suel

5.1 Introduction

Description

A good market model is needed in order to understand the effects of different regulation and policy scenarios. The results from the overall policy analysis will be reliable only if we can predict the real market conditions under different policy scenarios. In this subsystem, the aim is to create a model that would predict the design decisions of a certain firm given the design decisions of other firms. Those decisions will be based on the predictions of resulting market shares, expected profits and costs for a given design decision. Firms are modeled purely as profit-driven agents in the model and this is the first big assumption. Consumers are assumed to make their purchasing decisions by the utility they are getting from a certain vehicle and that each consumer will purchase the vehicle which maximizes their utility. The utility is determined by a demand model of their preferences.

The market model in this subsystem is based on an existing model used in a study by Michalek, Papalambros and Skerlos (MPS) in 2004 [1]. The aim is to improve the existing model by using a better demand model. The logit model used in their study assumes that every consumer in the market will get the same utility from a certain vehicle, which obviously is not true. Our subsystem replaces the logit model with a mixed-logit one, which introduces a variation of utilities for different users. There are various studies that are using mixed logit models for the automotive industry in the literature. One of those models was created in a study by Berry, Levinsohn & Pakes (BLP)[5][6]. The utility coefficients for estimating the tastes of different population segments in their model are determined by using historical data based on various demographic variables. Their data includes 23 different vehicle attribute coefficients. The aim was using that BLP model which would let us use a mixed logit demand model with increased attributes taken into account. The detailed description for this model can be found in the Appendix. However, we were not able to get reasonable results when we used that model.[1] The second model we used was the Boyd and Mellman[4] (BM) which was used in the MPS study. We introduced variance on top of BM's estimated coefficient for different population segments for our mixed logit model. The drawback with the BM model was the fact that we were not able to integrate more attributes to the existing model, we had to use the three attributes that the BM model estimated coefficients for: price, fuel economy and acceleration.

We are using game theory to model competition in the market, which is the case in the MPS study as well. A game is simulated on top of the profit maximization algorithm. We are modeling five firms making design decisions to maximize their profit. Those firms make those decisions sequentially by taking turns (i.e. Firm 1 makes a design decision to maximize its profit given the design decisions of the other. Then Firm 2 makes

[1] There is a discussion for the results from the BLP model in the Results section.

its design decision to maximize its profit, then Firm 3, then Firm 4, then Firm 5 and then Firm 1 again.) The game continues until market equilibrium is reached. We define market equilibrium as the point at which no firm can change its design decisions to get more profit. We will use this game model to integrate our subsystems. Competition between firms is crucial for a good market model, and we need that market model to evaluate different policy scenarios. Costs from different policy scenarios (i.e. cost of regulation for each firm) is another addition we have on top of this subsystem for our overall system.

Problem Statement

The objective of this sub-system optimization is to maximize the profit of a particular vehicle configuration given vehicle design decisions of other firms and a demand model. The resulting vehicle configuration determined by the design decisions will affect the cost and the market share of the firm. The optimal design will give the price and design decisions of the optimal vehicle that maximizes the profit of a given firm.

5.2 Mathematical Model

Objective Function

The objective for each producer is modeled as profit maximization. The profit function Π_k for each producer k is calculated as revenue minus cost. Assuming that the production volume is equal to the quantity sold, the revenue depends on the price of the vehicle and the demanded quantity. The revenue is modeled simply as being equal to price time quantity sold.

$$\max \Pi_\text{k} = \sum_{j \in J_k} q_j \times p_j = \sum_{j \in J_k} q_j \left(p_j - c_j^V \right) \qquad (5.1)$$

The quantity sold q depends on the utility that a consumer gets from that vehicle. The quantity will be calculated by the demand model as a function of price p and other selected vehicle characteristics $\mathbf{z}$. Vehicle characteristics are defined as the attributes observed by the consumer, affecting their purchasing decisions such as acceleration, top speed, weight and size. These attributes will depend on the design decisions, such as the drive ratio, the engine size and materials used. The relation between the attributes and design variables will be provided by the engineering model.

Variables

Each firm will be varying its design variables $\mathbf{x}$ and price p to find the optimum configuration which maximizes the expected profit. The selected design variables are the engine scaling parameter x_1 and the final drive ratio x_2. The fourth variable we wanted to include is the engine type M (diesel or gasoline). Since this is a discrete variable, a parametric study is conducted. The optimization algorithm for design variables $\mathbf{x}$ runs twice for each firm, one for the optimum diesel engine and one for the optimum gasoline engine. The resulting values are compared and the optimum one is selected.

Constraints

The only inequality constraints will come from the engineering model. Those impose limits on the design variables – the engine scaling parameter x_1 and the final drive ratio x_2. Design variables $\mathbf{x}_j$ determine the performance characteristics $\mathbf{z}_j$ observed by the purchasing population using a mapping function. Basically constraints on the design variables will impose constraints on the performance characteristics which affect the demand and the cost of the vehicle. The equations are explained in more detail in the following section.

$g_1 : 0.75 \leq x_1 \leq 1.50$ *The Engine Scaling Parameter should be in the range [0.75,1.5]*

$g_2 : 0.2 \leq x_2 \leq 1.3$ *The Final Ratio Should be in the range [0.2, 1.3]*

5.3 Cost Model

We are using the cost model from the paper by Michalek et al. The cost is divided into three subparts: the investment cost c^I, variable cost c^V and the regulation cost c^R. For the subsystem we do not have a regulation cost, the regulation cost will be added in the overall analysis since it is needed for the policy optimization. The sub-system is an optimization under *no regulation* case so regulation cost c^R is zero. The equation for the cost to manufacture the engine c^E is:

$$c_E(M,x) = \begin{cases} \delta_1 \exp(\delta_2 b_M x_1) & if \quad M \in SI \\ \delta_3 (b_M x_1) + \delta_4 & if \quad M \in CI \end{cases} \tag{5.2}$$

where, $\delta_1 = 670.51$, $\delta_2 = 0.0063$, $\delta_3 = 26.23$, and $\delta_4 = 1642.8$.

The final equation for the cost for the manufacturer k when producing q vehicles:

$$c_k = q\left[c_B + \begin{cases} \delta_1 \exp(\delta_2 b_M x_1) & if \quad M \in SI \\ \delta_3 (b_M x_1) + \delta_4 & if \quad M \in CI \end{cases} \right] \tag{5.3}$$

$c_B = \$7500$ cost to manufacture the rest of the vehicle

5.4 Engineering Model

The engineering model created using Advisor in subsystem 1 will provide the mapping function we need to calculate vehicle characteristics $\mathbf{z}$ given design variables $\mathbf{x}$. We have two different mapping functions for the two different engine types we have. The first type is the spark ignition (gasoline) engine and the second is the compression ignition (diesel) engine.

$$\mathbf{z} = f_M(\mathbf{x}) \tag{5.4}$$

5.5 Consumer Demand Model

The main difference between this study and the previous study by Michalek et al. is the demand model that is being used. We will base our model on discrete choice analysis (DCA) and use a random utility model. DCA approach can be followed by various probabilistic choice models such as the logit model, the probit model or the mixed logit model. In a logit model every consumer in the vehicle market is modeled as if they get the identical utility from a certain vehicle. Mixed logit allows us to model different market segments with different utility functions. You can find a discussion for the DCA approach and the mixed logit model in the Appendix. The demand model is used to estimate the market shares P and the quantity demanded: q. The quantity is equal to the size of the car-buying population s times the probability of purchasing that vehicle:

$$q_j = sP_j = \times \int L_{ji}(\beta) f(\beta \mid \theta) d\beta \qquad (5.5)$$

β's are the coefficients in the utility function that determine the importance of a certain attribute in the purchasing decision. $L_{ji}(\beta)$ is the probability of purchasing a certain vehicle given the coefficients (the logit probability). The β's are varied given certain demographic information about the population using the function: $f(\beta \mid \theta)$. The same probability is simulated using an integral approximation: (see the Appendix)

$$P_{ni} = \sum_r L_{ni}(\beta^T) / R \qquad (5.6)$$

Parameters

We have parameters both in our cost model and the demand model. In the cost model, the investment cost (c^I), the cost to manufacture the rest of vehicle other than the engine (c^B) and all the coefficients (δ's) as well as the base engine size (b_M) in the engine cost model are fixed. In the demand model, the coefficients (β's) are determined by the specific demand model we are using (BM or BLP), the size of the car buying population (s) and the integral approximation trials (R) are fixed. All of these values are parameters in the sub-system.

5.6 Summary Model

$$\max \Pi_k = \sum_{j \in J_k} q_j \left(p_j - c_j^V \right) \qquad (5.7)$$

With respect to: $\{M_j, \mathbf{x}\}$ where all $j \in J_k$

Subject to:

$$g_1 : 0.75 \le x_1 \le 1.50$$

$$g_2 : 0.2 \le x_2 \le 1.3$$

$$h_1 : q_j = sP_j = s \times \sum_r L_{ni}(\beta^T, \mathbf{z}) / R$$

$$h_2 : \mathbf{z} = f_M(\mathbf{x})$$

$$h_3 : c_{Vj} = q\left[c_B + \begin{cases} \beta_4 \exp(\beta_5 b_M x_1) & if \quad M \in SI \\ \beta_6 (b_M x_1) + \beta_7 & if \quad M \in CI \end{cases} \right]$$

$$h_3 : p^* \text{ - } Price\ equilibrium$$

5.7 Model Analysis

Monotonicity Analysis

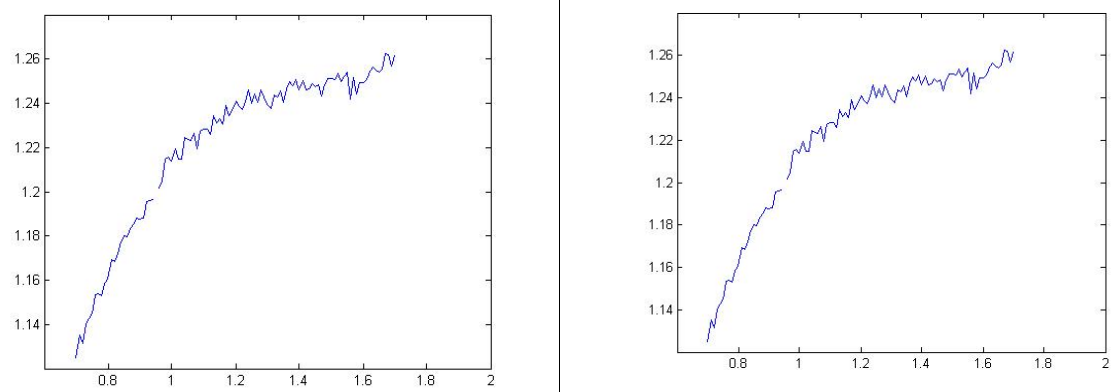
The model is based off of a simulation, so traditional monotonic analysis cannot be performed. However, we do have some expectations for the resulting relationships in our model. The design variables we have will affect the variable cost and demand, which will influence the profit.

As the engine size is increased, the fuel economy decreases. The decrease in the fuel economy will cause a decrease in the demand. The increase in the fuel economy will also cause a price increase, which would decrease the demand. However, an increase in the engine size will result in a decrease the acceleration time which will increase the demand. So there is a trade-off between fuel economy, acceleration and price. This will be determined by the demand model structure and consumer preferences. As the drive ratio is increased, fuel economy seems to increase up to a point, then it starts decreasing. The increase in the final drive ratio will cause a decrease in the acceleration time. Most probably, the consumers will be interested in fuel efficient cars with low acceleration times. So there will be a trade-off between them as well –depending on the consumer preference values. Consumer preferences will be crucial in determining the optimum design decisions.

Model Transformations

Randomness is introduced on the demand when the mixed logit model is used. The values of the coefficients in the utility function are specified randomly based on a mean and standard deviation value or some other demographic properties. Due to this randomness, the function that is used to estimate the expected profit became noisy. Here are some graphs showing this noisy function with respect to design variables.

Figure 1-1 – Noise in Expected Profit Function



Profit vs. Final Drive Ratio

Profit vs. Engine Scaling Parameter

The algorithm fails to find the optimum value because of the noise and it took too much time to evaluate the profit function every time fmincon calls it, due to the integral approximation. We decided to fit a quadratic function to the data we get from this expected profit given design variables using least squares approximation. The matlab function is evaluated at four points to estimate a continuous function, then fmincon is called to minimize that resulting function.

Equilibrium Price

In the game model we have, fmincon is used to minimize the negative expected profit. For a given design decision, equilibrium price is computed. Equilibrium price is defined as the price value at which the firm cannot change the price to increase its expected profit. Fixed point iteration is used to calculate the equilibrium price. Once the equilibrium price is found for a given set of design decisions, the expected profit is calculated at that point. fmincon finds the maximum expected profit value by varying design decisions. In other words, we optimize price for every design decision and then maximize the expected profit using those results.

The Game

In order to model the market and competition between the firms, we are using concepts from game theory. The market is modeled as a game of five players (firms) and the objective functions of each player are identical. All of them are modeled as profit-maximizing agents. The players take turns to take an action and they select the action that maximizes the objective function at each turn. The idea of Nash equilibrium is used to estimate the market equilibrium point. The equilibrium point is defined as the point at which no producer is capable of increasing its own objective by any possible action.

In our model, we have five firms and their objective function is the profit maximization function from sub-system 2. The market equilibrium is the point at which the firms stop changing their vehicle characteristics, so that the resulting expected profits are not changing anymore. In our game simulation, we look at the norm of the difference between the current vehicle characteristics and the previous vehicle characteristics. When the norm is smaller than a certain ε value, we terminate the game and assume that the resulting market situation is the market equilibrium. We assume that this market equilibrium point will give us a reasonable prediction of producers' design decisions. The termination condition for the game is:

$$\left\| \mathbf{z}_{current} - \mathbf{z}_{previous} \right\|_{\infty} \leq \varepsilon \tag{5.8}$$

Alternatively, we can use a terminating condition for the game using the norm of design variables. This will define the market equilibrium as the point at which the firms stop changing their design decisions, so the resulting expected profits are not changing anymore.

$$\left\| \mathbf{x}_{current} - \mathbf{x}_{previous} \right\|_{\infty} \leq \varepsilon \tag{5.9}$$

5.8 Optimization Study

As described in the Model Transformation section, a function was fitted to find the minimum point of the negative profit with respect to design decisions. This minimum is calculated at each round of the game for each firm. (e.g. if it takes 10 rounds to converge for 5 firms, fmincon is called 50 times). This section will list the results in terms of design decisions and vehicle characteristics at the market equilibrium. Then, the properties of the minimum point at some calls of fmincon will be presented to prove that our algorithm finds a KKT point. We'll then show our results with different initial points and lastly, a comparison of the market equilibrium with the mixed logit and the logit demand models will be presented.

Results

The optimum design solutions we get all have gasoline engines. So, for the list of results from this point on all have gasoline engines unless stated otherwise.

Termination Condition on Vehicle Characteristics

If $\left\| \mathbf{z}_{current} - \mathbf{z}_{previous} \right\|_{\infty} \leq 0.1$ is used as a termination condition for the game, it does not terminate within 100 rounds. It starts to jump around certain values, but the norm is never below 0.1. Below are a set of results at the end of 100 rounds, the results are very close to the ones in the next section, when the termination condition is imposed on the design variables.

Table 5-1 – Initial Design Variables and Vehicle Characteristics

	Engine Scaling	Final Drive Ratio	Fuel Economy	Acceleration Time	Price (Cost)
Firm 1	0.7	0.8	29.2101	13.3471	$8551
Firm 2	0.8	1	27.3277	10.8221	$8621
Firm 3	0.9	1.2	25.2833	9.3241	$8696
Firm 4	1.1	1.4	21.6444	8.0591	$8860
Firm 5	1.2	1.3	20.8118	7.7236	$8950

Table 5-2 – Design Variables at Market Equilibrium

	Engine Scaling	Final Drive Ratio
Firm 1	1.1673	1.2447
Firm 2	1.1531	1.2576
Firm 3	1.1599	1.2509
Firm 4	1.1637	1.2541
Firm 5	1.1312	1.2583

Table 5-3 – Vehicle Characteristics, Price and Market Shares at Market Equilibrium

	Fuel Economy	Acceleration Time	Price (Cost)	Market Shares
Firm 1	21.4071	7.8560	$12029	19.88%
Firm 2	21.5350	7.9003	$11993	20.02%
Firm 3	21.4758	7.8792	$12010	19.95%

Firm 4	21.4171	7.8637	$12020	19.91%
Firm 5	21.8064	7.9830	$11939	20.24%

Termination Condition on Design Variables

Table 5-4 – Initial Design Variables, Vehicle Characteristics and Price

	Engine Scaling	Final Drive Ratio	Fuel Economy	Acceleration Time	Price (Cost)
Firm 1	1	1.3	23.4285	8.5705	$8775
Firm 2	1	1.3	23.4285	8.5705	$8775
Firm 3	1	1.3	23.4285	8.5705	$8775
Firm 4	1	1.3	23.4285	8.5705	$8775
Firm 5	1	1.3	23.4285	8.5705	$8775

Table 5-5 – Design Variables at Market Equilibrium

	Engine Scaling	Final Drive Ratio
Firm 1	1.1174	1.2672
Firm 2	1.1696	1.2614
Firm 3	1.155	1.2463
Firm 4	1.1453	1.2464
Firm 5	1.194	1.2538

Table 5-6 – Vehicle Characteristics, Price and Market Shares at Market Equilibrium

	Fuel Economy	Acceleration Time	Price (Cost)	Market Shares
Firm 1	21.9497	8.0331	$11944	20.36%
Firm 2	21.3192	7.8393	$12067	19.87%
Firm 3	21.5512	7.8992	$12033	20.03%
Firm 4	21.6713	7.9354	$12010	20.13%
Firm 5	21.0549	7.7603	$12126	19.62%

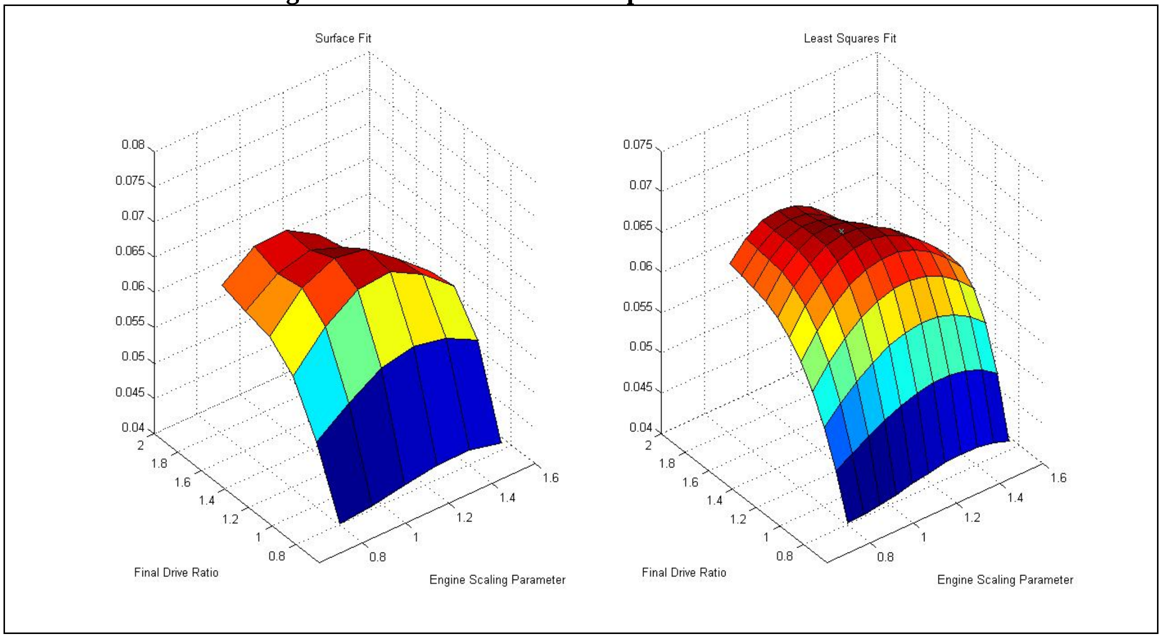
Termination Condition for the game: $\left\| \mathbf{x}_{current} - \mathbf{x}_{previous} \right\|_{\infty} \leq 0.1$

Rounds to terminate the game: 6

Properties of the Optimum Points

As noted before, we evaluate the expected profit using a mixed logit demand model at 36 points and we find the best fit function to the data using least squares. The first plot is the surface plot generated by Matlab and the second one is the least squares fit function together with the optimum point calculated by fmincon.

Figure 5-2 – Function Fit for Expected Profits



Looking at the graphs above, we can say that the calculated solution is actually the optimum point itself. Furthermore, below are the Hessian, the gradient and the Lagrange multipliers at the calculated solution for one of the optimum points:

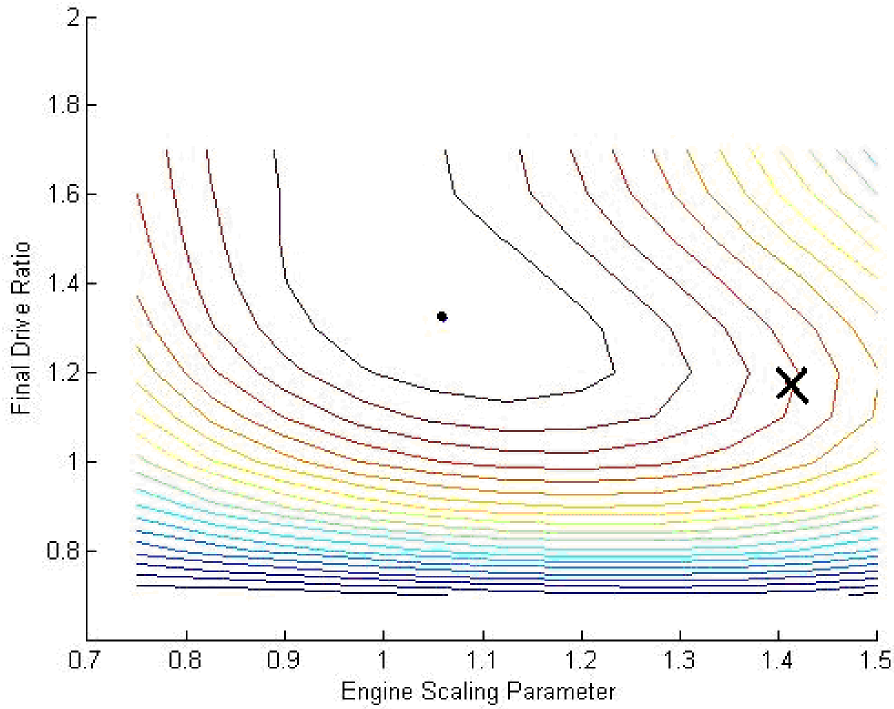
$$\mathbf{x}^* = \begin{bmatrix} 1.0612 \\ 1.3218 \end{bmatrix} \qquad \nabla f = \begin{bmatrix} -0.2366 \times 10^{-5} \\ -0.3095 \times 10^{-5} \end{bmatrix} \qquad H = \begin{bmatrix} 0.0794 & 0.0288 \\ 0.0288 & 0.0867 \end{bmatrix} \quad (5.10)$$

The gradient is almost zero, which makes the gradient of the Lagrangian zero since we have no active constraints and the Hessian is positive definite. Since we know that the multipliers are all zero and we have an interior solution, we can conclude that our solution is a KKT point and the KKT point is a minimum.

Comparison with Simple Logit Equilibrium

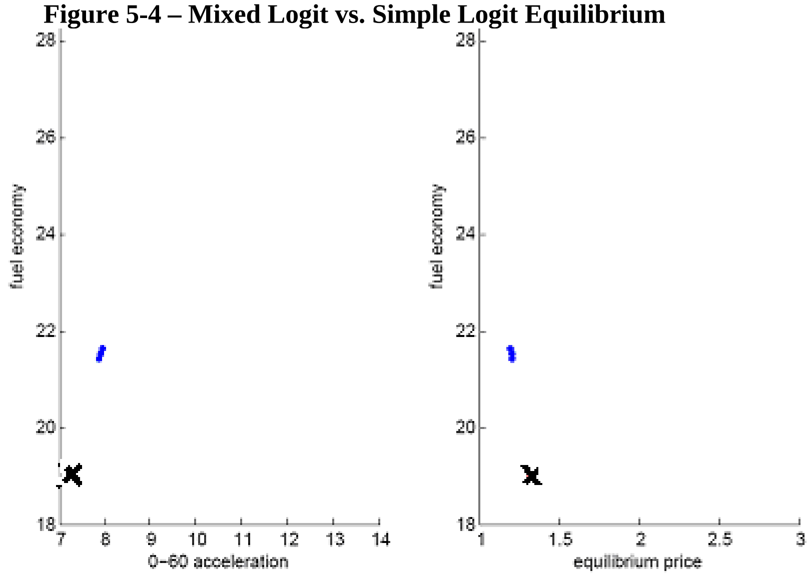
We were hoping to see different equilibrium point when we introduce the mixed logit model as our demand model, since it accounts for the distribution in tastes among consumers. Below is a contour plot where you can see the optimum point from the previous firm optimization problem we had plots for. The red dot is the market equilibrium design variables calculated using the simple logit demand model. Then the first firm chooses its optimum design variables using the mixed logit model. Blue dot is its optimum decision given all other firms decisions as being equal the logit equilibrium. It is obvious that, when we use the mixed logit case, the market equilibrium changes.

Figure 5-3 – Contour plot for the Function Fit



Below are the plots showing the different market equilibrium vehicle characteristics that emerge using the mixed logit model and the simple logit model. The dots are the equilibrium vehicles under the mixed logit case and the crosses are the ones under the simple logit. We see that all firms have vehicles with exact same characteristics, whereas in the mixed logit case there is a small variation. This variation would be more significant if we were using a demand model which is more detailed than the BM model. Furthermore, it is obvious that we have different market equilibrium vehicles under the two different scenarios.

Figure 5-4 – Mixed Logit vs. Simple Logit Equilibrium



Different Initial Conditions

When we start with different initial conditions, we get almost the results. The design variables are accurate up to 0.1; which is consistent with our game's terminating condition. ($\varepsilon = 0.1$) Below is a set of results with a different set of initial conditions.

Table 5-7 – Initial Design Variables, Vehicle Characteristics and Price

	Engine Scaling	Final Drive Ratio	Fuel Economy	Acceleration Time	Price (Cost)
Firm 1	0.7	1.3	28.5394	11.0211	$8551
Firm 2	0.9	0.75	25.2265	11.4512	$8696
Firm 3	1	1	24.1247	9.1067	$8775
Firm 4	1.1	1.4	21.6444	8.0591	$8860
Firm 5	1.3	0.8	20.1853	8.3674	$9046

Table 5-8 – Design Variables at Market Equilibrium

	Engine Scaling	Final Drive Ratio
Firm 1	1.1796	1.2522
Firm 2	1.1321	1.2680
Firm 3	1.1560	1.2505
Firm 4	1.1414	1.2649
Firm 5	1.1598	1.2432

Table 5-9 – Vehicle Characteristics, Price and Market Shares at Market Equilibrium

	Fuel Economy	Acceleration Time	Price (Cost)	Market Shares
Firm 1	21.2319	7.8092	$12111	19.76%
Firm 2	21.7597	7.9744	$12000	20.18%
Firm 3	21.5246	7.8934	$12056	19.99%
Firm 4	21.6542	7.9403	$12022	20.11%
Firm 5	21.5038	7.8837	$12065	19.96%

5.9 Parametric Study

Change the cost parameters

When the cost parameters are increased per vehicle or investment, it is expected that the design variables will stay the same but the prices will increase at the market equilibrium. This is exactly the case. If we change the c^B to be equal to $\$10000$ and run the code with the initial conditions from 1.5.1.1; we get the same design decisions with increase prices. The resulting prices are:

Table 5-10 – Prices for the new cost parameters

	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
Prices	$14475	$14455	$14504	$14541	$14514

Utility Coefficients

Consumer preferences are the most important elements in our model that determines the optimum design variables. If consumers are more interested in fuel economy, firms will most likely start to manufacture vehicles with higher fuel economy. If we were to play with the coefficients in the BM model, we would expect to see a change in the equilibrium designs. However, modifying the coefficients too much could mess up the whole demand model, since those coefficients are estimated so that the demand models fit the real market data.

Let's increase the mean for the fuel economy coefficient and set it to 1.339 instead of 0.339 which would increase the value consumers give to fuel economy. What we would expect to see is having all vehicles with higher fuel economies and longer acceleration times since its importance is decreased in comparison. Our expectation turns out to be true and below is a set of results. However if we continue increasing the coefficient, the results will not change because the design variables hit the bounds. We get the same results when the coefficient is 2 on the fuel economy.

Table 5-11 – Fuel Economy and Acceleration time with new demand coefficients

	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
Fuel Economy	38.4975	38.5110	38.4619	38.4704	38.4939
Acceleration Time	9.6933	9.7010	9.6760	9.6798	9.6914

If we set the fuel economy coefficient to 0.5, here are the results:

Table 5-12 – Fuel Economy and Acceleration time with new demand coefficients

	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5
Fuel Economy	30.5876	31.2577	24.4135	24.0390	24.2634
Acceleration Time	7.6456	7.7836	8.9766	8.7954	8.8840

It is obvious that for a certain range of coefficients determined by the design bounds, utility function will have an effect on the equilibrium vehicles.

5.10 Discussion of the Results

The results of the optimization sub-problem lists a set of expected designs for the vehicles that would exist in market equilibrium where there are five firms producing one vehicle each. The demand model is based on the Boyd and Mellman model in combination with the mixed logit idea. Even though BM model is a very simple model, we still see a variation in the resulting set of vehicles. On the other hand we see that the logit case results in the case in which all firms are producing the exact same vehicle with respect to acceleration, fuel economy and price. Furthermore, it is clear from the results and the corresponding plots that the simple logit equilibrium is different than the mixed logit equilibrium. Since we do know that mixed logit is a better model in reflecting the reality, we can defiantly say that we have improved the model. The solutions are highly influenced by the parameters as we have seen in the previous section. The optimum design is basically the design which would make the consumer utility function has a maximum value for the largest portion of the consumer population. Since the utility function is the key for estimating the utilities, the underlying demand model we are using is crucial. The future step would definitely be to find a way to integrate a better demand model, such as the BLP one, which would capture more demographics and attributes in the utility function and the distribution of the utility coefficients. We can definitely say that the results make sense. First, we are getting a close result to the logit case, which is reasonable since we are using the same BM model with a little more variance. Furthermore, the parametric analysis which shifts the optimum point reasonably when we change the coefficient parameters proves that we have a consistent model. Another thing to do for future work would definitely be to increase the number of products and firms; since there would be much more products and firms in a market situation. Lastly and most importantly, we should include an outside good, which would demonstrate the utility that consumers get when they do not purchase any of the resulting vehicles. In our model, every consumer will buy a car, so the total market share always adds up to 100% which is not a good assumption.

6 SUBSYSTEM 3: POLICY DESIGN

by Katie Kerfoot

6.1 Problem Statement

In addition to engineering characteristics and market preferences, government regulations also affect fuel economy. Taking the perspective of government, this subsection focuses on the design of policies aimed at increasing fuel economy at market equilibrium. Two types of policies will be investigated: taxes on fuel and a Corporate Average Fuel Economy (CAFE) standard. As these policies are implemented to raise fuel economy, the cost incurred by consumers (through fuel taxes) and producers (through CAFE) will increase. Furthermore, due to engineering constraints, consumer utility usually decreases as fuel economy improves. Parametric studies will be performed on cost upper limits and utility lower limits to represent scenarios where the government is more concerned with either cost or utility.

The question of whether CAFE or fuel taxes more efficiently reduce greenhouse gas emissions is heavily discussed in economics (e.g., Kwoka, 1983 [7]; Crandall, 1992 [8]), often with contradictory answers. This debate in economics usually analyzes past responses of automotive firms to policies but rarely links the effect of policies on engineering characteristics.

6.2 Mathematical Model

Objective function

For the purposes of this study, the objective of government is interpreted as maximizing fuel economy at market equilibrium. This value will be determined by running a market equilibrium model developed by Michalek et al. representing an automobile oligopoly. In this model, producers make design and pricing decisions in order to maximize profit. The market (Nash) equilibrium is reached when each producer cannot increase its own profit by any decision given the decisions of its competitors. Policy scenarios will be imposed upon this model, affecting the resulting market equilibrium. The government objective can then be defined as:

$$max\ z_1^E = f^E(z_{CAFE}, p_{CAFE}, \mathbf{t}) \qquad (6.1)$$

where z_1^E is the fuel economy at market equilibrium determined by the equilibrium model f^E. Policy scenario inputs to the model are represented by the CAFE required fuel economy, z_{CAFE}, the CAFE penalty, p_{CAFE}, and fuel taxes, $\mathbf{t}$.

Constraints

The objective of government is maximizing fuel economy without creating adverse economic consequences. For this study “adverse economic consequences” will be described by three quantities: the cost of regulation to the producer per vehicle, the added cost to the consumer, and the decrease in consumer utility. The first constraint will set an upper bound, c_{Rk}^{UB}, for the cost of regulation per vehicle:

$$g_1:\ c_{Rk}^E / q_k^E \le c_{Rk}^{UB} \qquad (6.2)$$

where c_{Rk}^E and q_k^E are the total regulation cost and total quantity of vehicles produced for a producer at market equilibrium. The second constraint will set an upper bound, Δp_{50i}^{UB}, for the additional cost to the consumer. The cost mentioned here will be the purchase price of the vehicle plus the cost of the first 50,000 miles in fuel. The additional cost is measured by subtracting the consumer cost from the base case, p_{50i}^0, where no policies are imposed on the market:

$$g_2:\ \Delta p_{50i}^E \le \Delta p_{50i}^{UB}$$
$$\Delta p_{50i}^E = p^E + 50,000\left(\frac{p_f}{z_1^E}\right) - p_{50i}^0 \qquad (6.3)$$

where $\Delta p_{50i}{}^{E}$, $p_j{}^{E}$, and $z_1{}^{E}$ are the additional cost to the consumer, purchase price, and fuel economy at market equilibrium, and p_f is the price of fuel. The third constraint will set a lower bound (υ_{LB}) for the consumer utility at market equilibrium ($\upsilon_j{}^{E}$):

$$g_3 : \ \upsilon_j{}^{E} \geq \ \upsilon_{LB} \qquad \textit{(6.4)}$$

Design variables and parameters

Similar to subsection 5, producers make decisions about two design variables: the engine scaling parameter, x_1, and the final drive ratio, x_2. These design variables then influence vehicle attributes that are apparent to consumers: fuel economy in mpg, z_1, and the acceleration time from 0-60 mph, z_2. ADVISOR simulation results act as a function, f_M, to map design variables to vehicle attributes given the engine type, M:

$$\mathbf{z} = f_M(\mathbf{x}) \qquad \textit{(6.5)}$$

where $\mathbf{z} = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^{\mathrm{T}}$, and $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^{\mathrm{T}}$. The same engine types described in subsection 5 will be used here: $M = \{\text{SI102, CI88}\}$, where SI is a spark ignition (gasoline) engine and CI is a combustion ignition (diesel). The engineering constraints for the equilibrium model are contained in the set of feasible points of $\mathbf{x}$ as determined by ADVISOR simulations.

Producers also make decisions on production volumes of a vehicle design based on expected demand for that design. Expected design demand is determined through Discrete Choice Analysis, which assumes that consumers purchase products that have the largest utility compared to relevant alternatives. Utility, u, is composed of an observable part, υ, that directly relates to product attributes and a random component, ε, that accounts for factors independent of the product. The probability of consumers purchasing design j of set J is equal to the probability that the utility of that design is greater than the utility of all alternatives, j':

$$P_j = \Pr(\upsilon_j + \varepsilon_j \geq \upsilon_{j'} + \varepsilon_{j'};\ \forall j' \in J) \qquad \textit{(6.6)}$$

This approach differs from subsection 2 in that a logit model, instead of a mixed-logit model, is used to interpret Eq. *(6.6)*. The logit model assumes that the random component of utility, ε, for each alternative is independently and identically distributed (iid), and that it has a double exponential distribution. This allows Eq. *(6.6)* to be rewritten in the form of Eq. *(6.7)*

$$P_j = \frac{e^{\upsilon_j}}{\sum_{j \in J} e^{\upsilon_j}} \qquad (6.7)$$

Discrete Choice Analysis takes the probability of consumers purchasing design j to be equal to the expected market share of that design. Since producers are acting to maximize profit, the quantity of designs produced will equal the expected quantity demanded, which is determined by the size of the car buying market, s, multiplied by the expected market share of design j:

$$q_j = sP_j = s\frac{e^{\upsilon_j}}{\sum_{j \in J} e^{\upsilon_j}} \qquad (6.8)$$

Utility is represented as a linear equation of product attributes and their coefficients. The Boyd and Mellman utility model [4], which was used in Michalek et al., will also be used in this study. In their model, Boyd and Mellman determined coefficients of vehicle attributes including price, fuel economy, and acceleration time (as well as other attributes) such that the predicted purchasing behavior would most closely match observed data from 1977-1978. They used two methods to determine the logit model: one with separate coefficients for price and fuel economy, and one collapsing these attributes into a value they call "price50", the purchasing price plus the price of fuel for the first 50,000 miles. Because the price of fuel was not considered in the Michalek et al. study, the first method (using separate coefficients) was used. However, in this study, policies affecting the price of fuel will be investigated and so the second method, including the price50 attribute, is used. Following this method, utility of design j is calculated as:

$$\upsilon_j = \beta_1 p_{50j} + \beta_2 \left(\frac{60}{z_{2j}}\right) \qquad (6.9)$$

where $\beta_1 = -3.61 \times 10^{-4}$, $\beta_2 = 0.302$, and p_{50} is the price50 attribute. The value of this attribute is dependent upon the purchasing price, p_j, and fuel economy, z_{1j}, of the design as well as the engine type, M, and the tax on gasoline, t_g, and diesel, t_d:

$$p_{50j} = p_j + 50,000 \frac{p_f(M,\mathbf{t})}{z_{1j}}$$

$$p_f(M,\mathbf{t}) = \begin{cases} \frac{t_g}{100} + p_g{}^B & \text{if } M \in SI \\ \frac{t_g}{100} + p_d{}^B & \text{if } M \in CI \end{cases} \tag{6.10}$$

where $\mathbf{t} = \begin{bmatrix} t_g & t_d \end{bmatrix}^{\mathrm{T}}$ in cents per gallon; SI refers to a spark ignition (gasoline) engine and CI, a combustion ignition (diesel) engine.

Producers make choices on design variables, production volumes, and price based on maximizing expected profit. Using the same model for cost as subsection 5, profit for each producer k is calculated by equation *(6.11)* where, and.

$$\Pi_{\mathrm{k}} = \left(\sum_{\mathrm{j} \in \mathrm{J_k}} q_j (p_j - c_{Vj}) - c_I \right) - c_{Rk} \tag{6.11}$$

where Π_{k} is the total profit to the producer, q_j and p_j are the quantity sold and price of design j, c_{Vj} is the variable cost per vehicle, c_I is the investment cost for the production line, and c_{Rk} is the regulation cost resulting from CAFE penalties. Values for c_{Vj} and c_I are defined in subsection 5. The regulation cost, c_{Rk}, is dependent on the CAFE penalty and required fuel economy. This cost can be represented as:

$$c_{Rk} = \max\left(0, \sum_{j \in J_k} q_j p_{CAFE} (z_{CAFE} - z_{1j}) \right) \tag{6.12}$$

In the market equilibrium model, each producer acts to maximize profit under a particular policy scenario:

$$\max \ \Pi_{\mathrm{k}} = \left(\sum_{\mathrm{j} \in \mathrm{J_k}} q_j (p_j - c_{Vj}) - c_I \right) - c_{Rk}$$

Subject to:

engineering constraints on $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$ from Advisor

The model acts so that the above optimization is performed for each producer holding all other producer actions fixed. This procedure is iterated until the producers cannot make any decisions that will result in higher profits given the decisions of the other producers.

The policy model has four degrees of freedom: p_{CAFE}, z_{CAFE}, t_g, and t_d. There are many possible values for these independent variables that result in a feasible solution. An example is the current policy scenario where $p_{CAFE} = \$55$, $z_{CAFE} = 27.5$ mpg, $t_g = 39.5¢$, and $t_d = 45.8¢$. This results in a regulation cost to the producer of $155, no additional cost to the consumer, and a vehicle utility of -4.8401, which satisfies all of the constraints.

Summary model

min $-z_1^E = -f_p(z_{CAFE}, p_{CAFE}, \mathbf{t})$	*Maximize the market average fuel economy at equilibrium*
Subject to:	
$g_1:\ c_{Rk}^E / q_k^E - c_R^{UB} \le 0$	*The regulation cost per vehicle should be less than $500*
$g_2:\ \Delta p_{50i}^E - \Delta p_{50i}^{UB} \le 0$	*The additional cost to consumer should be less than $3,000*
$g_3:\ -\upsilon_j^E + \upsilon^{LB} \le 0$	*The utility for the vehicle should be greater than -6.0*
$g_4:\ -z_{CAFE} \le 0$	*The CAFE standard must be greater than zero*
$g_5:\ -p_{CAFE} \le 0$	*The CAFE penalty must be greater than zero*
$g_6:\ -t_g \le 0$	*The gasoline tax must be greater than zero*
$g_7:\ -t_d \le 0$	*The diesel tax must be greater than zero*
$g_8:\ z_{CAFE} \le 30.0$	*The CAFE standard must be less than 30.0 mpg*
$g_9:\ p_{CAFE} \le 3{,}000$	*The CAFE penalty must be less than $3,000*
$g_{10}:\ t_g \le 250$	*The gasoline tax must be less than 250¢/gal.*
$g_{11}:\ t_d \le 250$	*The diesel tax must be less than 250¢/gal.*

where $c_{Rk}^E,\ \Delta p_{50i}^E,\ \upsilon_j^E = f(z_{CAFE}, p_{CAFE}, \mathbf{t})$.

6.3 Model Analysis

Since this model is based off of a simulation, traditional monotonicity analysis can not be performed. However, there are some relationships that we expect to see in the model. As the CAFE penalty and required fuel economy increase, the regulation cost to the producers will increase, forcing up the market average fuel economy. As the taxes on gasoline and diesel increase, the additional cost to the consumers will increase, increasing the demand for more fuel efficient vehicles and encouraging an increase in the market average fuel economy.

If the relationship between policy variables and market fuel economy is indeed monotonic, it is expected that z_{CAFE} will be bounded by either g_1, g_3, or g_8; p_{CAFE} will be bounded by either g_1 or g_9; t_g will be bounded by either g_2, g_3, or g_{10}; and t_d will be bounded by either g_2, g_3, or g_{11}. Because minimization objective function is expected to be monotonically decreasing with respect to all of the variables, the lower limit constraints— g_4, g_5, g_6, and g_7 —will be inactive and can be removed from the model.

6.4 Numerical Results

The model was solved using the gradient free optimization algorithm Direct. Appendix A includes a discussion of why a gradient free algorithm was chosen for this model. The version of Direct used has no convergence criteria but will automatically stop when the specified maximum number of iterations is reached. A plot of the minimum values of the objective function found in Direct for each iteration is plotted in Figure 6-1, which shows that the solution “converged” by 150 iterations. The solution found after this time is shown in Table 6-6.

Figure 6-1: Best points found at each iteration of Direct

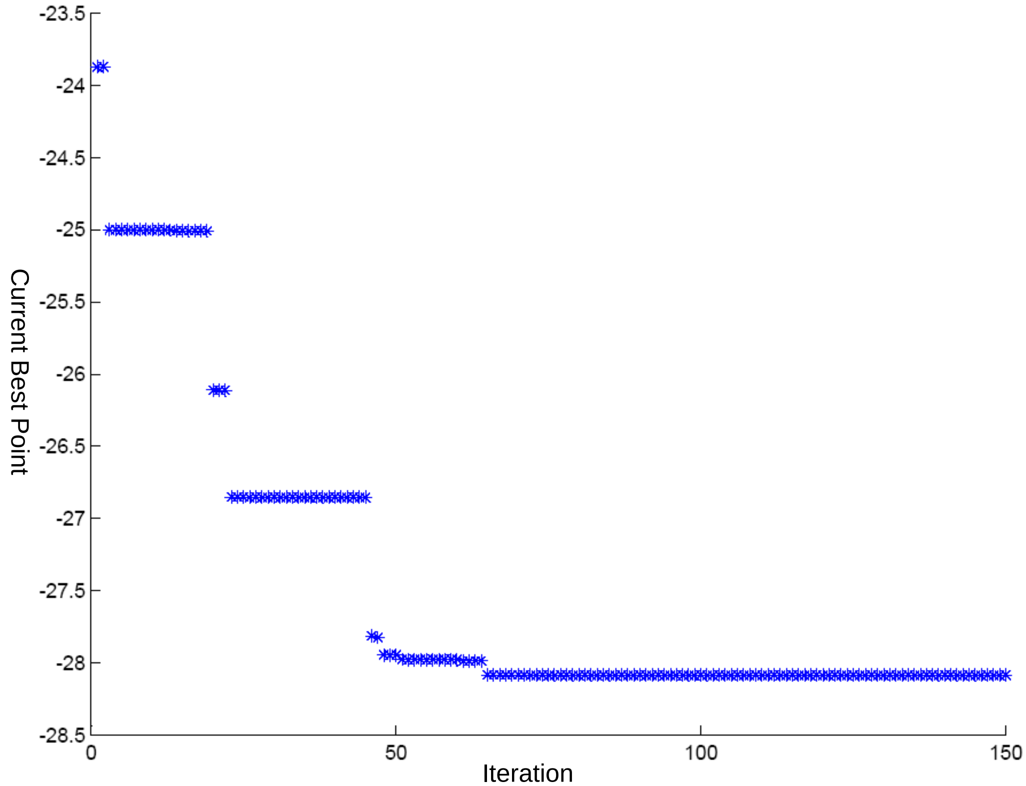


Table 6-6: Best solution found after 150 iterations of the model

CAFE penalty	CAFE standard	Gas tax	Diesel tax	Regulation cost per vehicle	Additional consumer cost	Utility	0-60 accel. time	Fuel economy	Engine scaling parameter	Final drive ratio
p_{CAFE}	z_{CAFE}	t_g	t_d	$c_{Rk}{}^E / q_k{}^E$	$\Delta p_{50i}{}^E$	$\upsilon_j{}^E$	z_1	z_2	x_1	x_2
$	mpg	¢ /gal	¢ /gal	$	$	-	s	mpg	-	-
2,435	28.33	203.09	50.06	489	2,218	-5.99	10.98	28.1066	0.75	1.0999

6.5 Parametric Studies

To check activity of the constraints, a parametric study was performed on the lower bound for utility, and the upper bounds for producer and consumer cost. The lower bound for the consumer utility was relaxed to -6.00 from the previous bound of -5.00; the resulting solution is shown in Table 6-7. A comparison of Tables 6-6 and 6-7 shows that the optimum changed once the consumer utility constraint was relaxed, resulting in a higher market fuel economy. This indicates that the utility constraint, g_3, is most likely active. It should be noted that the change in diesel tax resulting from the relaxed constraint is inconsequential since all firms choose to produce only gasoline vehicles so the diesel tax does not affect the solution.

Table 6-7: Solution found with relaxed utility constraint after 100 iterations of the model

CAFE penalty	CAFE standard	Gas tax	Diesel tax	Regulation cost per vehicle	Additional consumer cost	Utility	0-60 accel. time	Fuel economy	Engine scaling parameter	Final drive ratio
p_{CAFE}	z_{CAFE}	t_g	t_d	$c_{Rk}{}^E / q_k{}^E$	$\Delta p_{50i}{}^E$	$\upsilon_j{}^E$	z_1	z_2	x_1	x_2
$	mpg	¢ /gal	¢ /gal	$	$	-	s	mpg	-	-
596	28.70	248.77	132.41	487	3,024	-6.21	10.99	28.1093	0.75	1.0985

To check the activity of the producer and consumer constraints, the utility constraint was kept at its relaxed lower bound of -6.00. This was done to avoid degeneracy, which occurs when the utility is kept bounded at -5.00 and the other constraints are relaxed. The utility is closely linked to consumer and producer cost because as the consumer cost increases utility decreases, and similarly as producer cost increases, the purchase price increases and utility decreases. Degeneracy occurs because even if the consumer and producer constraints are relaxed, the utility must be further decreased to improve the objective function.

The upper bound for the maximum cost to the producer was relaxed from $500 per vehicle to $700. The optimal solution changed as a result, as shown in Table 6-8. The CAFE standard was allowed to increase, improving the market fuel economy, because the cost to the producer was allowed to rise past $500. Similarly, the upper bound for the maximum additional consumer cost was relaxed from $3,000 to $4,000. The optimal solution changed as shown in Table 6-9. Here, it is unclear whether relaxing the consumer cost constraint influenced the solution point since it is the same as the point obtained from relaxing the utility. Based on this behavior, the consumer cost is most likely not active. Further work should be done to use gradient based optimization near this solution point to confirm the activity of these constraints.

Table 6-8: Solution found with relaxed producer cost constraint after 100 iterations of the model

CAFE penalty	CAFE standard	Gas tax	Diesel tax	Regulation cost per vehicle	Additional consumer cost	Utility	0-60 accel. time	Fuel economy	Engine scaling parameter	Final drive ratio
p_{CAFE}	z_{CAFE}	t_g	t_d	$c_{Rk}{}^E / q_k{}^E$	$\Delta p_{50i}{}^E$	$\upsilon_j{}^E$	z_1	z_2	x_1	x_2
$	mpg	¢/gal	¢/gal	$	$	-	s	mpg	-	-
871	28.70	226.54	158.33	518	2,392	-6.21	10.99	28.1093	0.75	1.0985

Table 6-9: Solution found for relaxed consumer cost constraint after 100 iterations of the model

CAFE penalty	CAFE standard	Gas tax	Diesel tax	Regulation cost per vehicle	Additional consumer cost	Utility	0-60 accel. time	Fuel economy	Engine scaling parameter	Final drive ratio
p_{CAFE}	z_{CAFE}	t_g	t_d	$c_{Rk}{}^E / q_k{}^E$	$\Delta p_{50i}{}^E$	$\upsilon_j{}^E$	z_1	z_2	x_1	x_2
$	mpg	¢/gal	¢/gal	$	$	-	s	mpg	-	-
2,233	28.70	248.77	152.78	487	3,024	-6.29	10.99	28.1152	0.75	1.0952

6.6 Discussion of Results

The optimization results confirmed the speculation that at least two constraints would be needed to bound the policy variables. As expected, the gasoline tax, and the CAFE penalty and standard move to the highest values possible without violating the constraints. It should be noted that it may be possible to reach the market average fuel economy by lowering the CAFE penalty but raising the standard, keeping the same gasoline tax.

The vehicle design produced at market equilibrium has the smallest possible engine size and a relatively low final drive ratio. The lowest final drive ratio was not chosen because reducing this variable lowers the consumer utility of the vehicles, which is constrained by g_3. It is interesting, however, that producers consistently choose the smallest engine size and are more reluctant to reduce final drive ratio, although reducing either variable will increase fuel economy and decrease acceleration performance. A possible explanation for this behavior is that the cost of the engine was modeled as increasing with the size of the engine but independent of the final drive ratio.

A major limitation of the model is that it is restricted to only one vehicle segment. It is likely if producers are faced with very high CAFE standards and consumers are influenced by large gasoline taxes, the market will tend toward smaller types of vehicles, not just smaller engines in current vehicles. This behavior may represent a discontinuity in the solution space where firms choose to produce compact cars with a different engine size range than was included in the model.

7 SYSTEM INTEGRATION STUDY

7.1 Engineering Model

The engineering model simulation was very computationally expensive, so it could not be realistically integrated into the overall system directly. Instead, a surrogate model of the engineering simulation was made, which was entered into the market equilibrium system. The procedure to create the surrogate model was similar to the work in Michalek et al., in which the ADVISOR simulation is integrated as a mapping function. The same variables were used to describe the vehicle: final drive ratio and scaling factor of the engine, while the shape variables computed during the engineering optimization were included as design parameters for the complete system. The maps that were built are surfaces corresponding to the fuel economy, and the acceleration performances, for design variables final drive ratio and scaling parameter in the intervals [0.5, 1.3] and [.75, 2.3], respectively These maps were built for different values of the power accessories.

7.2 Policy Model

The original goal of the system optimization was to integrate the engineering and market model into the policy model and maximize market equilibrium fuel economy. However, the market model was too computationally expensive to integrate it with the policy model given the resources available. Instead, a few specific policy case studies were applied to the market model and the results were compared to those found with the formulation used in the policy subsystem optimization. This comparison will reveal how the policy impact on the market changes using the updated demand model.

8 Total System Optimization: Market Equilibrium

8.1 Description

The overall system combines the market model from subsystem 2 with the engineering model from subsystem 1. Different policy scenarios from subsystem 3 are then applied on top of the market model. The underlying engineering model that was used in subsystem 2 was replaced with the new model from subsystem 1. Figure 8.1 is a pictorial description of our system integration.

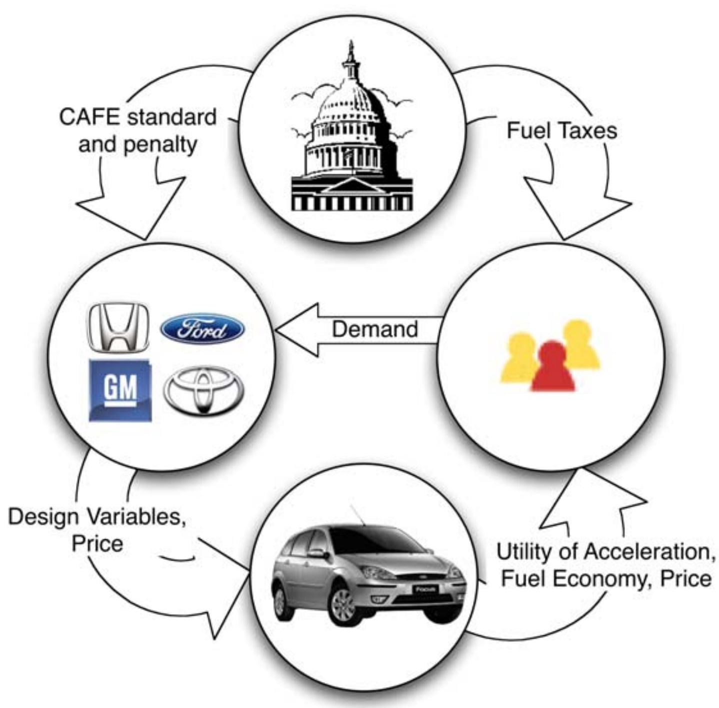


Figure 8-1: Pictorial Description of the System Integration

The demand model uses the engineering model to calculate the resulting attributes fuel economy and acceleration. The demand model uses these attributes to estimate the demand. The scenarios from the policy model feed in to the market model and resulting vehicles under various policy scenarios were observed.

The two main additions to the market model from subsection 2 was the introduction of the number of power accessories as a new attribute and the regulation cost since the firms below the standard will be penalized. In the overall system, we are only looking at gasoline engines in order to reduce the computational time. This is a reasonable reduction since we did not observe any vehicles with diesel engines in subsystem 2 or 3.

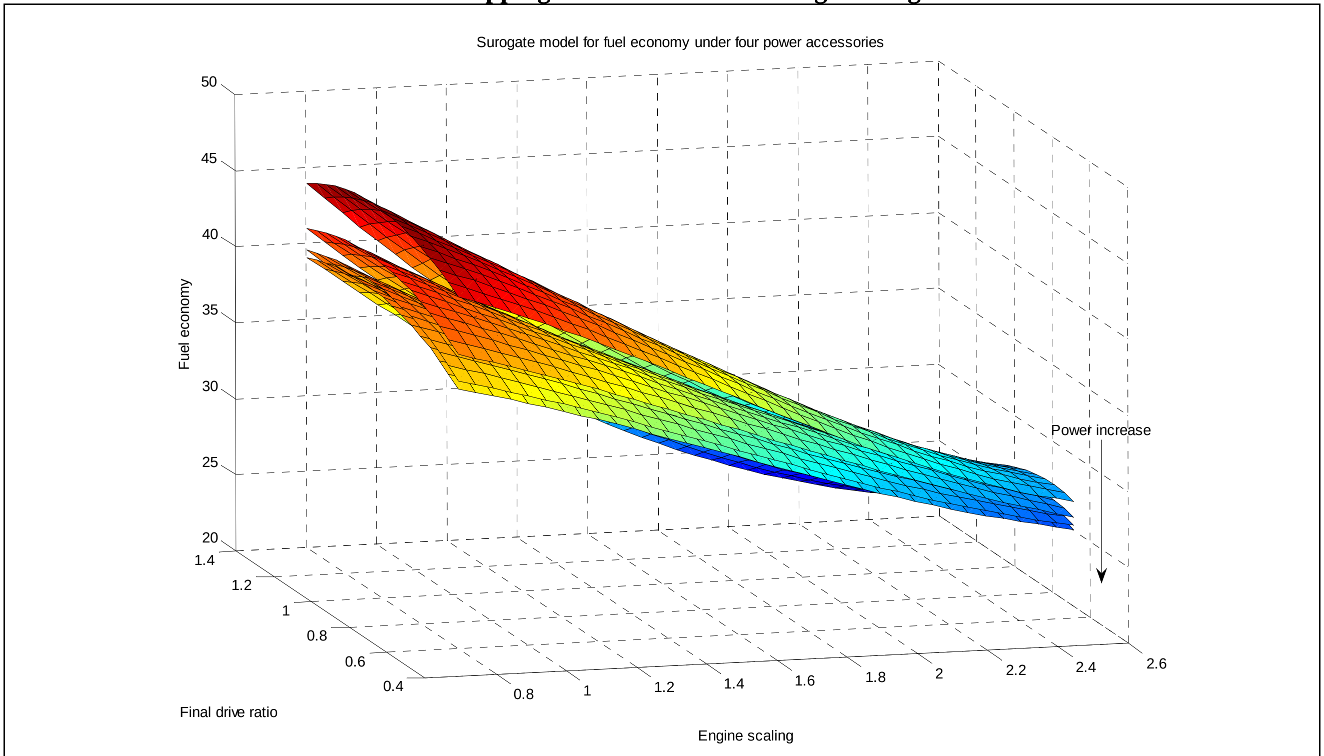
We have a different mapping function for each case of power accessories from our engineering model. We assume that the base electrical load is 700 Watts, including one power accessory increases this to 1200 Watts; including two increases this to 1500 Watts and three, 1600 Watts. For each of those cases, we have a different function mapping the design variables to attributes. Figure 8-2 shows a comparison of the mapping functions from the engineering model. In order to integrate this new attribute to our overall system, we had to assign a utility for power accessories in our demand model. Since we were not able to conduct surveys or collect historical data to estimate the corresponding coefficient, we simply assigned a reasonable value based on the comparison between the BM model and the BLP model.

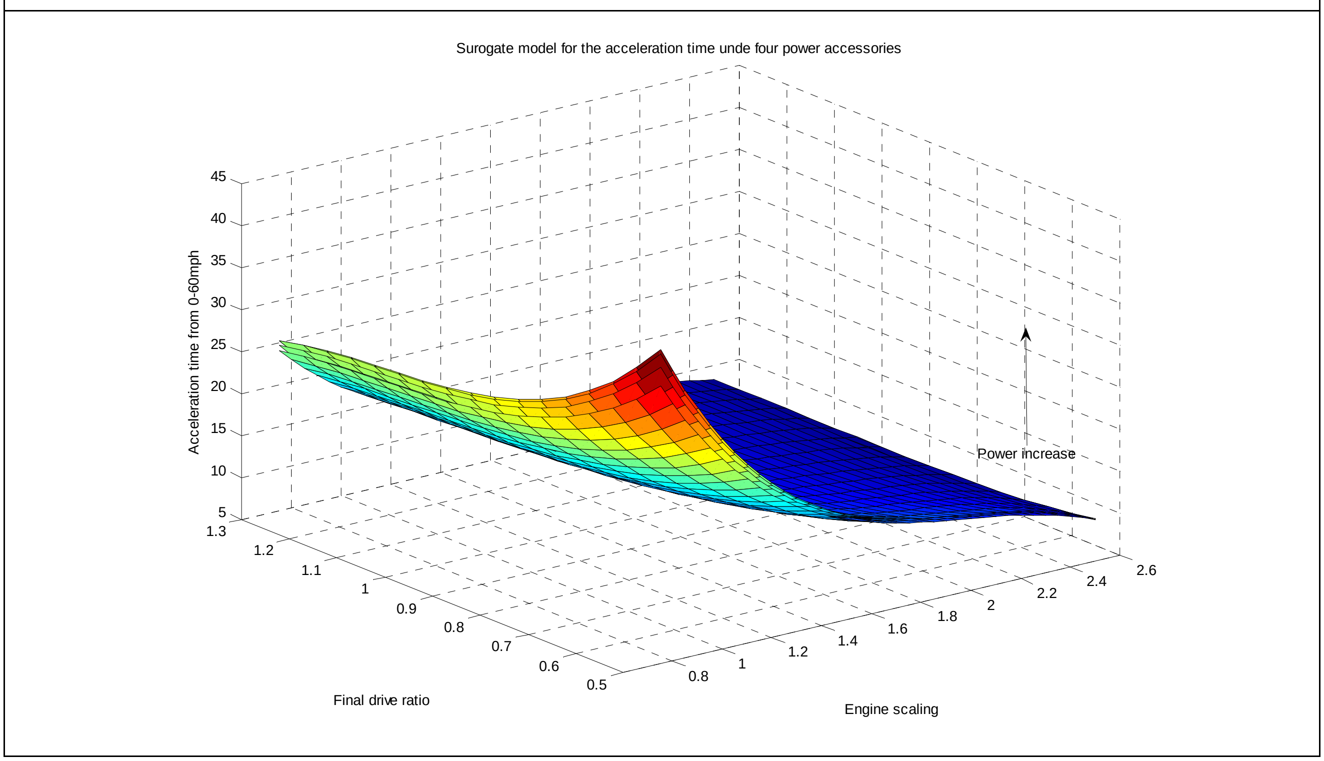
In subsystem 2, we were not imposing any regulations, so the firms were not being panelized based on their vehicles' fuel economies. In the overall system, we included the cost of regulation and the firms are being penalized if they are below the imposed CAFE standards. The cost equation with the introduction of regulation cost is:

$$c_k = c_R + q\left[c_B + \begin{cases} \delta_1 \exp(\delta_2 b_M x_1) & if \quad M \in SI \\ \delta_3 (b_M x_1) + \delta_4 & if \quad M \in CI \end{cases} \right] \qquad (8.1)$$

For the case of gasoline tax, we model the consumers as being sensitive the fuel prices. The BM model that was used in subsystem 3 was used to see the effects of taxation policies.

Table 8-2: Mapping Functions from the Engineering Model





8.2 Summary Model

$$\max \Pi_k = \sum_{j \in J_k} q_j \left(p_j - c_j^V \right) - c_k^R \qquad (8.2)$$

With respect to: $\{Power, \mathbf{x}\}$ where all $j \in J_k$

Subject to:

$g_1 : 0.75 \le x_1 \le 2.50$ *Bounds for the engine scaling*

$g_2 : 0.5 \le x_2 \le 1.3$ *Bounds for the final drive ratio*

Where:

$$c_j^V = c_B + \begin{cases} \delta_1 \exp(\delta_2 b_M x_1) & if \quad M \in SI \\ \delta_3 (b_M x_1) + \delta_4 & if \quad M \in CI \end{cases}$$ *Variable Cost*

Power *# of Power Accessories [0-3]*

8.3 Numerical Results

We ran the market model for several policy scenarios. The base case is the *no regulation* case where there is no CAFE standard and no gas tax. Under this scenario, only one firm chooses to add in more power accessories. The fuel economies are in the range of 25 to 34 mpg. We have active constraints for the engine scaling parameter for two firms. The acceleration time to go from 0 to 60 mph varies between 8 and 13 seconds.

We applied two different policy scenarios to this new market model, which is using the engineering model from subsystem 1. The gasoline tax is 200 cents per gallon for both cases and the standards are 28 mph and 38 mph. The penalty for both cases is $2,500. The first policy case approximates the optimal solution from subsystem 3. In this case, only one firm chooses to violate the standard. Since there is a trade off between paying the penalty and increasing consumer satisfaction and willingness to pay; the firm that is violating the standard chooses to add more power accessories. In the 38 mph case, the standard is much higher and four firms choose to violate the standard. You can find the numerical results in more detail in the following tables.

Table 8-1: No Regulation Case

	Fuel Economy	Acceleration Time	# Power Accessories	Engine Scaling	Final Drive Ratio	Price	Market Shares
	z_1	z_2	z_3	x_1	x_2	p	P
	mpg	s	#	-	-	$	%
Firm 1	33.27	12.59	0	1.40	1.30	11,351	24
Firm 2	25.53	8.23	3	2.50	0.95	14,728	13
Firm 3	33.26	12.58	0	1.40	1.30	11,352	24
Firm 4	27.38	8.28	0	2.50	0.93	14,729	13
Firm 5	33.76	12.88	0	1.36	1.30	11,275	25

Table 8-2: Regulation Case 1: 200 Cents Gasoline tax & 38mpg CAFE Standard w/ $2,500 Penalty

	Fuel Economy	Acceleration Time	# Power Accessories	Engine Scaling	Final Drive Ratio	Price	Market Shares
	z_1	z_2	z_3	x_1	x_2	p	P
	mpg	s	#	-	-	$	%
Firm 1	39.93	18.03	0	1.00	1.30	$10,858	26%
Firm 2	36.49	13.10	0	1.39	1.09	$11,878	21%
Firm 3	27.29	8.28	0	2.50	0.95	$15,835	12%
Firm 4	36.36	13.00	0	1.40	1.08	$11,912	21%
Firm 5	36.44	12.90	0	1.42	1.06	$11,944	21%

Table 8-3: Regulation Case 1: 200 Cents Gasoline tax & 28mpg CAFE Standard w/ $2,500 Penalty

	Fuel Economy	Acceleration Time	# Power Accessories	Engine Scaling	Final Drive Ratio	Price	Market Shares
	z_1	z_2	z_3	x_1	x_2	p	P
	mpg	s	#	-	-	$	%
Firm 1	37.50	15.69	0	1.13	1.30	$10,976	26%
Firm 2	32.34	11.64	0	1.56	1.22	$12,064	20%
Firm 3	31.61	11.71	0	1.52	1.30	$11,958	21%
Firm 4	31.86	11.52	0	1.57	1.24	$12,111	20%
Firm 5	26.21	8.25	2	2.50	0.90	$15,225	13%

8.4 Discussion of Results

As expected, the system results show that increasing the CAFE standard will increase the market average fuel economy. What is interesting is that in all cases, the distribution of types of vehicle designs is similar. In the no regulation case, three firms group around higher fuel economies at 33 mpg, and the other two choose lower fuel economies. In both regulation cases, three firms group around similar fuel economies (36 and 32 mpg, respectively), one firm pushes toward a higher fuel economy and one firm chooses a much lower fuel economy.

Another interesting observation is the relationship of regulation and power accessories. As we might expect, using the formulation of this model, as the CAFE standard increases the number of power accessories will decrease. This indicates that the possible increase in demand from adding power accessories could not outweigh the resulting drop in fuel economy and the regulation costs associated with this. While the current CAFE tests do not include the use of any power accessories, the results of this study indicate that if this changes, producers may choose to reduce the number of power accessories offered in their vehicles to raise the firm's CAFE.

9 ACKNOWLEDGMENTS

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10 REFERENCES

[1] Michalek, J., Papalambros, P. Y., and Skerlos, S., "A Study of Fuel Efficiency and Emission Policy Impact on Optimal Vehicle Design Decisions". Journal of Mechanical Design, Vol. 126, No. 6, 2004, pp. 1062-1070.

[2] "Automotive Handbook, 4th Edition", Robert Bosch GmbH, 2000, pp.37, 721

[3] Wong, J.Y., "Theory of Ground Vehicles, 2nd Edition". John Wiley & Sons, INC, 1993.

[4] Boyd J, and Mellman J., "The effect of fuel economy standards on the U.S. automotive market: a hedonic demand analysis." Transportation Research, Vol. 14, No. 5-6, 1980, pp. 67-378.

[5] S Berry, J Levinsohn & A Pakes. "Automobile Prices in Market Equilibrium." Econometrica 63(4) 1995

[6] S Berry, J Levinsohn & A Pakes. “Differentiated Products Demand Systems from a Combination of Micro and Macro Data: The New Car Market.” The Journal of Political Economy 112(1) 2004

[7] Kwoka, J., "The Limits of Market-Oriented Regulatory Techniques: The Case of Automotive Fuel Economy". Quarterly Journal of Economics, Vol. 98, No. 4, 1983, pp. 695-704.

[8] Crandall, R., "Policy Watch: Corporate Average Fuel Economy Standards". Journal of Economic Perspectives, Vol. 6, No. 2, 1992, pp. 171-180.

[9] “Aerodynamics of Road Vehicles, 4th Edition”, Wolf-Heinrich Hucho, 1998, pp. 131-175.

[10] Hamza, K., Hossoy, I., Reyes-Luna, J. K., and Papalambros, P. Y., "Combined maximization of interior comfort and frontal crashworthiness in preliminary vehicle design ". Journal of Mechanical Design, Vol.35, No. 3, 2004, pp. 167-185

APPENDIX A: Model evolution in Subsystem 1

During the optimization process several simplifications had to be made on the initial approach. Most of these simplifications were motivated by the lack of information, the others by the difficulty of building a pertinent model or just for computational reasons. The weight of the vehicle is one of the key parameters in determining the fuel consumptions. As the dimensions of the vehicle are correlated with its weight, in an initial approach, one of the model variables was its glider_mass, which corresponds to the weight of the structure. As a consequence, models had to be found to model the coefficient of drag of the vehicle as well as the variation of its weight, as functions of the variables describing the shape of the car. If for the coefficient of drag there are many papers, books, articles that deal with the aero-dynamics of ground vehicle, no resources were found that could model the evolution of the weight. Unfortunately, the body of the vehicle doesn't have a uniform structure, and the modeling of the change in mass as a consequence of dimension change proved itself very difficult. Moreover, no concrete data could be found so that a continuous relation could be established between the change in the design and the corresponding change in weight. Due to the short amount the time, we have decided not to include this variable into the final model. Another variable that was excluded from the model was the curvature of the vehicle's roof. The initial choice of including this variable was motivated by the tradeoff between the frontal area of the vehicle and its coefficient of drag that the presence of this parameter was introducing. No other concrete constraint has been found on this variable, and the computational complexity introduced would not justify its use.

During the optimization process, the main problem was due to the computational complexity and also to the limitation of ADVISOR. In one of advanced iteration on the model, the results presented in the following paragraphs were obtained. Two methods were used: a SQP algorithm: *fmincon* and a gradient based algorithm: Direct.

MATLAB results

The termination point after 48 iterations is presented in Table 4-31.

Table A-11 Temination point after 48 iterations

H_{front}	H_{cabin}	$H\_{windshield}$	$H\_{curvature}$	L_{front}	L_{rear}	α	θ	P_{ACC}	x_1
M	m	m	M	m	m	rad	Rad	W	
0.3631	0.3097	2.1420	1.6859	1.0168	1.0380	1.6281	1.4230	700	0.7759

Note that P_{ACC} was not treated as a variable in these results. The termination point is not feasible as the angle of the windshield α is larger than the imposed bound 1.5708. The value of the constraints and of the objective at the termination point is included in Table A-2. The analysis of these value shows that *g2* and *g7* are not respected. Moreover, the value of *g1* poses questions on the validity of the result provided by ADVISOR. This constraint, which is supposed to describe the acceleration performance of the vehicle, is: $z_2 - z_{2BASE} \leq 0$ with $z_{2BASE} = 13.2s$. The value -14.2 reveals that the acceleration time computed by ADVISOR is -1*s*. We could assume that ADVISOR is not able to provide a valid answer for the given input variables. A change in the model should be made, to invalidate this value.

Table A-2 Value of the objective and constraints at the termination point

f	***g1***	***g2***	***g3***	***G4***	***g5***	***g6***	***g7***	***g8***	***g9***	***g10***
-23.35	-14.20	0.1758	-0.1879	-1.1202	-2.6271	-0.5382	0.0534	-1.4002	-2.1678	-6.5928

Table A-3 - Gradient of the objective at the termination point

$\frac{df}{dH\_front}$	$\frac{df}{dH\_cabin}$	$\frac{df}{dH\_windshield}$	$\frac{df}{dH\_curvature}$	$\frac{df}{dH\_front}$	$\frac{df}{dH\_rearn}$	$\frac{df}{d\alpha}$	$\frac{df}{d\theta}$	$\frac{df}{dx_1}$
-5.3151	6.9176	1.8543	1.7547	0.0413	0	-1.4748	0	12.4212

The Lagrange multipliers are included are as follows:

g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
0	0.8438	0	0	0	0	0	0	0	0

We can see from these values that the Lagrange multiplier for *g2* is not zero, even though g2 is also non zero. We thus have $\mu_2 g_2 \neq 0$ which confirms once again that the termination point is not a feasible point.

DIRECT results

Similar overall results have been obtained with the DIRECT algorithm. After 187 evaluation of the function, the algorithm reaches the following optimum point:

Table A-4 Termination point for the DIRECT algorithm

H_{front}	H_{cabin}	$H\_{windshield}$	$H\_{curvature}$	L_{front}	L_{rear}	α	θ	P_{ACC}	x_1
m	m	m	m	m	m	rad	Rad	W	
0.1111	0.3333	1.000	1.0000	0.7500	0.8333	0.7854	0.6109	700	0.8889

This point is feasible, but the analysis of the constraints reveals once again inconsistence of the result provided by ADVISOR on the acceleration time.

Table A-5 Value of the constraints and objective at termination point

f	*g1*	*g2*	*g3*	*g4*	*g5*	*g6*	*g7*	*g8*	*g9*	*g10*
-25.325	-14.20	-0.9229	-0.3451	-0.0018	-3.5305	-0.8050	-0.2222	-1.8717	-1.0450	-1.7074

Once again, we see that a change in the model is required in order to invalidate inconsistent ADVISOR computation.

APPENDIX B: Gradient based optimization for subsystem 3

The model for subsystem 3 was originally coded into MATLAB and solved using fmincon. The feasible point discussed above corresponding to the current policy scenario was used as an initial point. After 100 iterations, the model terminated without finding a solution. The termination point is described below:

Table B-1: Termination point found after 100 iterations of the model

CAFE penalty	CAFE standard	Gas tax	Diesel tax	Regulation cost per vehicle	Additional consumer cost	Utility	0-60 accel. time	Fuel economy	Engine scaling parameter	Final drive ratio
p_{CAFE}	z_{CAFE}	t_g	t_d	$c_{Rk}{}^E / q_k{}^E$	$\Delta p_{50i}{}^E$	$\upsilon_j{}^E$	z_1	z_2	x_1	x_2
$	mpg	¢ /gal	¢ /gal	$	$	-	s	mpg	-	-
100	27.82	39.595	45.914	88.00	-718	-4.87	10.55	27.68	0.75	1.266

The additional consumer cost is negative because the fuel economy increase outweighed the extra cost of gasoline in this scenario. The sign of utility is not important since utilities are only relative measures; the utility reported for this point can be understood as being more desirable than a Ford Focus, which has a utility of approximately -5.24 using this model.

The model was then restarted at this point, and again terminated after 100 iterations without finding a solution. Furthermore, the termination point at the end of this second round was extremely close to the termination point in the first round. Because the model is optimizing over a game theory model, there is a high probability that the objective function contains significant noise. This would prevent gradient based methods from converging to a solution, and would explain why fmincon could not find any solution after 200 iterations. Based on this behavior, the non-gradient based algorithm, Direct, was chosen to perform the optimization.

APPENDIX C: Discrete Choice Analysis and Demand Models

The main difference between this study and the previous study by Michalek et al. is the demand model that is being used. We will base our model on discrete choice analysis (DCA) and use a random utility model. In random utility models, every individual has its own utility function for each and every product that is available in the market. Each individual makes choices based on the utility function, trying to maximize the utility. If there are *I* individuals and *J* products in the market, the utility function of individual *i* associated with the product *j* can be written as:

$$U_{ij} = V_{ij} + \varepsilon_{ij} \qquad (C.1)$$

V is the systematic part of the utility, which is observed and can be modeled. ε is the error term which is unobserved and calculated randomly.

Assuming that the individual *i* is deciding between two products: *j* and *k*. The probability of *i* choosing product *j* is equal to the probability of *j*'s utility being larger than *k*'s utility:

$$P_{ij} = P\left\lfloor V_{ij} + \varepsilon_{ij} \succ V_{ik} + \varepsilon_{ik} \right\rfloor = P\left\lfloor \varepsilon_{ik} \prec V_{ij} + \varepsilon_{ij} - V_{ik} \right\rfloor \qquad (C.2)$$

DCA approach can be followed by various probabilistic choice models such as the logit model, the probit model or the mixed logit model. Michalek et al. used logit models which assumes that the unobserved error component of utility ε is independently and identicaly distributed (iid) for each alternative (i.e ε follows the extreme value distribution). Even though it is a widely used model in marketing literature, it introduces several sources of error. The logit models allow representing systematic part of taste variation (i.e. the part which can be related to the observed factors) but do not allow handling the random taste variation (i.e. related to factors that are not observed). Secondly, logit models cannot handle situations in which unobserved factors are correlated over time. Lastly, in some situations substitution patterns are extremely unrealistic because of a property called IIA – independence from irrelevant alternatives. In the logit model, the ratio of two probabilities depends only on the observed factors that are relating the alternatives and does not depend on anything else. For example, the logit model will assume the probability of choosing between an iPod and a Creative Zen is independent of the price of a Sony mp3 player which is not true. The logit model implies proportional substitution across alternatives, i.e. if 10% is drawn from one alternative, 10% is drawn from all the alternatives.

Mixed Logit solves all three problems mentioned with the logit model.

$$U_{ni} = \beta_n Z_{ni} + \varepsilon_{ni} \qquad \varepsilon_{ni} \sim \text{iid extreme value} \qquad \beta_n \sim f(\beta_n \mid \theta) \qquad (C.3)$$

Unlike the logit model, in mixed logit; the coefficients vary randomly over people in the population. In the logit model, these coefficients are fixed. Varying coefficients allow representing different people with different tastes. The term $f(\beta_n \mid \theta)$ describes the density of the coefficients in the population and θ's are the parameters of the population distributions. Each individual person has their own taste, i.e. knows his own β_n and ε_{nj} for all j and chooses the alternative if $U_{ni} > U_{nj}$. In this case the researcher is aware of Z_{ni} values but does not know the β_n values. If s/he did know the β_n; this will be a standard logit model in which the V is known and the error terms are iid extreme value. That would be the probability conditional on β_n:

$$L_{ni}(\beta) = \frac{e^{\beta_n^` z_{ni}}}{\sum_j e^{\beta_n^` z_{nj}}} \qquad (C.4)$$

Since the researcher does not know the β_n values, he cannot condition on β_n. What is done instead is to recognize that there is a distribution of these β_ns in the population and integrate that formula over all possible formulas of β_n. So the choice probability will be:

$$P_{ni} = \int L_{ni}(\beta) f(\beta \mid \theta) d\beta \qquad (C.5)$$

The result is the desired choice probability. The simulated probability is:

$$P_{ni} = \sum_r L_{ni}(\beta^T)/R \qquad (C.6)$$

You take a draw of β from its density; calculate the logit probability; repeat this many times. Sum up the results and then divide it by that number of repetitions. The result is the mixed logit probability. The distributions of the β s are determined by the researcher.

Results and the code from the ME589 project that was mentioned were integrated to the existing code by Michalek. Each consumer has a different value for β since they have different tastes, the model assumes that they have some distribution over the population. Sampling from f will give us values of β such that large sets of such samples will match the desired probability distribution. So we have two random variables in our model, which is the case for mixed logit models in general: $\beta's$ and $\varepsilon's$. The random utility for a given product is:

$$U(x) = u(B,z) + \varepsilon \qquad \text{random vector } B \in \Re^K \qquad (C.7)$$

Under a real market situation, certain portions of the population will be interested in similar products. Certain demographic factors will have an affect of the consumer's decision to purchase a certain product. The next step is incorporating demographics into the formulation. In order to do that; $L \in \mathrm{N}$ consumers are considered and the corresponding values for these consumers are stored in the vector $l \in \Re^L$. Additionally; K unobserved demographic variables are considered which are written in the vector $d \in \Re^K$. So the utility function becomes:

$$u(l, d, \xi, z) = (a + B^T l + Cd)^T z + \xi \qquad (C.8)$$

The coefficients that are crucial in determining the utility of a consumer for a given product are calculated using the demographic variables. If we were to expand this utility function, we would see that every demographic variable is related with every product characteristic.

In order to model these different values associated with various customer groups, data from the existing market is required. This group of data should then be evaluated to come up with reasonable values in the matrices L (demographics), Z (product characteristics), B (observed demographic interaction coefficients), a (constants) and C (unobserved demographic interaction coefficients). These matrices and values are used in calculating the utility of each product for each individual as stated in previous sections.

APPENDIX D: BM and BLP Models

Steven Berry, James Levinsohn and Ariel Pakes have been working on techniques for analyzing demand and supply in the U.S. automobile industry. Basically two papers [7, 8] were used as the source of the date used in our testing and integrated in our algorithm in Matlab. They consider 23 vehicle characteristics (Z) including price and 9 demographic variables (L). We were only changing the price, horsepower and fuel economy and keeping the others constant since they were the only ones that we can get results for from our engineering model. Berry, Levinshon and Pakes include their resulting values for their interaction coefficients: B matrix and C matrix in their papers and we used those in our matlab code. If there is no interaction found between a certain demographic characteristic and an attribute, the corresponding interaction value is 0 in matrix B. We used the vehicle characteristics and corresponding coefficients that are included in our study and left the other ones out.

The quantity is equal to the size of the car-buying population s times the probability of purchasing that vehicle type j:

$$q_j = sP_j = s \times \int L_{ji}(\beta) f(\beta \mid \theta) d\beta \qquad (3.12)$$

where P_j is calculated through simulation in MATLAB.

The initial aim was to increase the number of varying characteristics by adding the *power requirement for accessories* as a design variable. The plan was to estimate a good characteristic value for the *Number of Power Accessories* characteristic in the BLP model by dividing the power requirement with a reasonable number. However, when we tried using the BLP model, we were not able to get reasonable results –we were getting negative profits. So instead, we decided to use the BM model and introduce a variation on the coefficients to be able to model some variation of taste over the population.

Boyd and Mellman determined coefficients of vehicle attributes including price, fuel economy, and acceleration time (as well as other attributes) such that the predicted purchasing behavior would most closely match observed data from 1977-1978. They used two methods to determine the logit model: one with separate coefficients for price and fuel economy, and one collapsing these attributes into a value they call "price50", the purchasing price plus the price of fuel for the first 50,000 miles. In the first one, the utility equation developed was:

$$u_j = \beta_1 p_j + \beta_2 \left(\frac{100}{z_{1j}} \right) + \beta_3 \left(\frac{60}{z_{2j}} \right)$$

where $\beta_1 = -2.86 \times 10^{-4}$; $\beta_2 = -0.339$; $\beta_3 = 0.375$